信号分析与处理－作业4

1. 已知有限长序列 $x(n)$，$x(n)=\begin{cases}1 & n=0\\ 2 & n=1\\ -1 & n=2\\ 3 & n=3\end{cases}$ 又知 $DFT[x(n)]=X(k)=\sum_{n=0}^{3}x(n)W_4^{nk}$，$k=0,1,2,3$

令 $W_4=e^{-j\frac{2\pi}{4}}=e^{-j\frac{\pi}{2}}$，$W_4^2=e^{-j\pi}=-1$，$W_4^4=1$，$W_4^1=-j$；$W_4^{r+4l}=W_4^r$

$$\begin{pmatrix}X(0)\\X(1)\\X(2)\\X(3)\end{pmatrix}=\begin{pmatrix}W_4^0 & W_4^0 & W_4^0 & W_4^0\\ W_4^0 & W_4^1 & W_4^2 & W_4^3\\ W_4^0 & W_4^2 & W_4^4 & W_4^6\\ W_4^0 & W_4^3 & W_4^6 & W_4^9\end{pmatrix}\begin{pmatrix}x(1)\\x(2)\\x(3)\\x(4)\end{pmatrix}=\begin{pmatrix}1 & 1 & 1 & 1\\ 1 & -j & -1 & j\\ 1 & -1 & 1 & -1\\ 1 & j & -1 & -j\end{pmatrix}\begin{pmatrix}1\\2\\-1\\3\end{pmatrix}=\begin{pmatrix}5\\2+j\\-5\\2-j\end{pmatrix}$$

再求 $IDFT[X(k)]=x(n)$，$x(n)=\frac{1}{N}\sum_{k=0}^{N-1}X(k)e^{jn\frac{2\pi}{N}k}$，$N=4$，$n=0,1,2,3$

$$\begin{pmatrix}x(0)\\x(1)\\x(2)\\x(3)\end{pmatrix}=\frac{1}{4}\begin{pmatrix}W_4^0 & W_4^0 & W_4^0 & W_4^0\\ W_4^0 & W_4^{-1} & W_4^{-2} & W_4^{-3}\\ W_4^0 & W_4^{-2} & W_4^{-4} & W_4^{-6}\\ W_4^0 & W_4^{3} & W_4^{6} & W_4^{9}\end{pmatrix}\begin{pmatrix}X(0)\\X(1)\\X(2)\\X(3)\end{pmatrix}=\frac{1}{4}\begin{pmatrix}1 & 1 & 1 & 1\\ 1 & j & -1 & -j\\ 1 & -1 & 1 & -1\\ 1 & -j & -1 & j\end{pmatrix}\begin{pmatrix}5\\2+j\\-5\\2-j\end{pmatrix}=\begin{pmatrix}1\\2\\-1\\3\end{pmatrix}$$

经验证，结果正确。

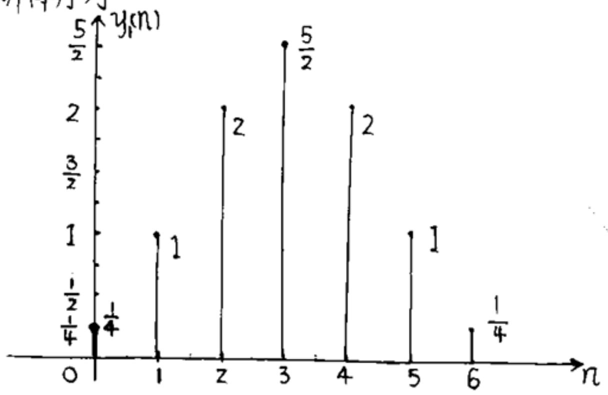
2. 由题可知 $x(n)=\begin{cases}\frac{1}{2} & n=0\\ 1 & n=1\\ 1 & n=2\\ \frac{1}{2} & n=3\end{cases}$，$N=4$

(1) 求 $x(n)$ 与 $x(n)$ 的线性卷积，$y_1(n)=\sum_{m=-\infty}^{+\infty}x(m)\cdot x(n-m)$

得 $y_1(n)=x(0)\cdot x(n)+x(1)\cdot x(n-1)+x(2)\cdot x(n-2)+x(3)x(n-3)$

根据已给的 $x(n)$，可得 $y_1(n)=\frac{1}{4}\delta(n)+1\cdot\delta(n-1)+2\cdot\delta(n-2)+\frac{5}{2}\delta(n-3)+2\delta(n-4)+1\delta(n-5)+\frac{1}{4}\delta(n-6)$

可画出所得序列



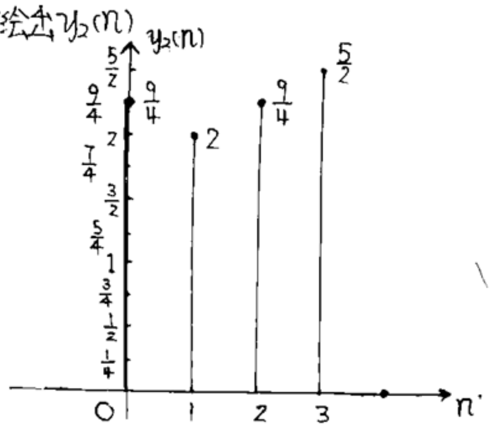
(2) 求 $x(n)$ 与 $x(n)$ 的4点圆卷积、圆周卷积、

$$y_2(n)=x(n)\circledast x(n)=\sum_{m=0}^{3}x(m)\cdot x((n-m))_4R_4(n)$$

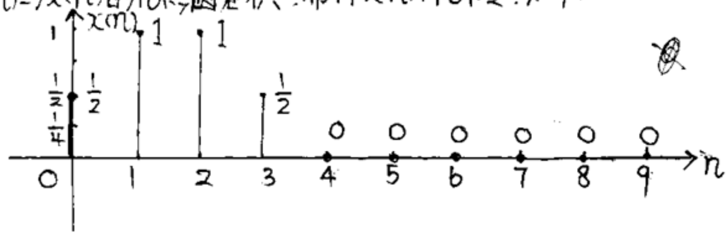
即 $y_2(n)=x(0)\cdot x((n))_4\cdot R_4(n)+x(1)\cdot x((n-1))_4\cdot R_4(n)+x(2)\cdot x((n-2))_4 R_4(n)+x(3)\cdot x((n-3))_4\cdot R_4(n)$

可得 $y_2(n)=\frac{9}{4}\delta(n)+2\delta(n-1)+\frac{9}{4}\delta(n-2)+\frac{5}{2}\delta(n-3)$

由此可绘出 $y_2(n)$



(3) 求 $x(n)$ 与 $x(n)$ 的10点圆卷积，需将 $x(n)$ 补6个零，如下



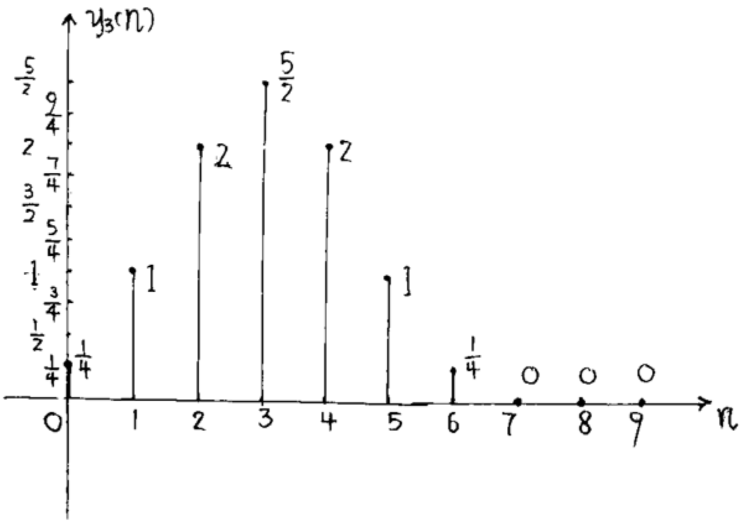
此时 $y_3(n)=x(n)\circledast x(n)=\sum_{m=0}^{9}x(m)\cdot x((n-m))_{10}\cdot R_{10}(n)$

由于 $x(4)=x(5)=x(6)=x(7)=x(8)=x(9)=0$.

即 $y_3(n)=x(0)\cdot x((n))_{10}R_{10}(n)+x(1)\cdot x((n-1))_{10}R_{10}(n)+x(2)\cdot x((n-2))_{10}R_{10}(n)+x(3)\cdot x((n-3))_{10}\cdot R_{10}(n)$

$=\frac{1}{4}\delta(n)+1\cdot\delta(n-1)+2\cdot\delta(n-2)+\frac{5}{2}\cdot\delta(n-3)+2\cdot\delta(n-4)+1\cdot\delta(n-5)+\frac{1}{4}\delta(n-6)+0\cdot\delta(n-7)+0\cdot\delta(n-8)+0\cdot\delta(n-9)$

由此可画出 $y_3(n)$



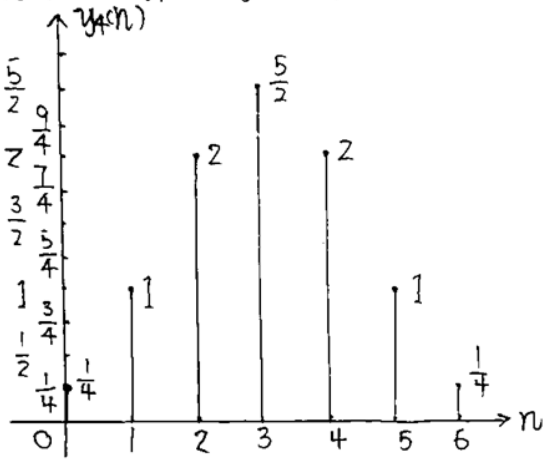
要求$x(n)$与$x(n)$的圆周卷积和线性卷积相同，由题$x(n)$圆卷积需补零
其补零个数应为有效移位数$4-1=3$，故$L_{min}=4+4-1=7$，其最小值为7.

$$x(n) ⊛ x(n) = y_4(n) = \sum_{m=0}^{6} x(m)\,x((n-m))_7 R_7(n)$$，且$x(4)=x(5)=x(6)=0$

即 $y_4(n) = x(0)\cdot x((n))_7 R_7(n) + x(1)x((n-1))_7 R_7(n) + x(2)\cdot x((n-2))_7 R_7(n) + x(3)\cdot x((n-3))_7 \cdot R_7(n)$

$$= \frac{1}{4}\delta(n) + 1\cdot\delta(n-1) + 2\cdot\delta(n-2) + \frac{5}{2}\delta(n-3) + 2\delta(n-4) + 1\cdot\delta(n-5) + \frac{1}{4}\delta(n-6)$$

由此可绘出$y_4(n)$，可知$y_4(n)$与$y_1(n)$一致，L最小值应为6.



3.

(1) 已知$y'(t)+3y(t)=2\cdot x'(t)$，可设$y(t) \xrightarrow{F} Y(\omega)$，$x(t) \xrightarrow{F} X(\omega)$

将上述方程化为$j\omega Y(\omega)+3Y(\omega)=2j\omega X(\omega)$，可得$H(\omega)=\frac{Y(\omega)}{X(\omega)}=\frac{2j\omega}{j\omega+3}=2-6\cdot\frac{1}{j\omega+3}$

则系统冲激响应的傅里叶变换为$H(\omega)$

$h(t)=F^{-1}(H(\omega))$，又知$F(\delta(t))=1$，$F(e^{-3t}\cdot u(t))=\frac{1}{j\omega+3}$

即$\underline{h(t)=2\delta(t)-6e^{-3t}\cdot u(t)}$

单位阶跃信号$u(t)$的傅里叶变换为$\pi\delta(\omega)+\frac{1}{j\omega}$

单位阶跃响应$c(t)$的傅里叶变换为$C(\omega)=H(\omega)\cdot(\pi\delta(\omega)+\frac{1}{j\omega})=\frac{2j\omega}{j\omega+3}(\pi\delta(\omega)+\frac{1}{j\omega})$

即$C(\omega)=\frac{2}{j\omega+3}$，由于$f(\omega)\cdot\delta(\omega)=f(0)\delta(\omega)$，可知$H(0)=0$，即$C(\omega)$前一项为0，只有后一项

$$\underline{\underline{c(t)=F^{-1}(C(\omega))=2e^{-3t}u(t)}}$$

(2) 已知$y''(t)+y'(t)+y(t)=x'(t)+x(t)$，同样有$F(y(t))=Y(\omega)$，$F(x(t))=X(\omega)$可假设

将上述方程化为$(j\omega)^2Y(\omega)+j\omega\cdot Y(\omega)+Y(\omega)=j\omega X(\omega)+X(\omega)$，得$H(\omega)=\frac{j\omega+1}{(j\omega)^2+j\omega+1}$

可化 $$H(\omega)=\frac{(j\omega+\frac{1}{2})+\frac{\sqrt{3}}{3}\cdot\frac{\sqrt{3}}{2}}{(j\omega+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}=\frac{j\omega+\frac{1}{2}}{(j\omega+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}+\frac{\sqrt{3}}{3}\cdot\frac{\frac{\sqrt{3}}{2}}{(j\omega+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}$$

由此 $h(t)=\mathcal{F}^{-1}(H(\omega))$，而 $\mathcal{F}^{-1}\left(\dfrac{j\omega+\frac{1}{2}}{(j\omega+\frac{1}{2})^2+\frac{3}{4}}\right)=e^{-\frac{1}{2}t}\cdot\cos\frac{\sqrt{3}}{2}t\cdot u(t)$

$$\mathcal{F}^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{(j\omega+\frac{1}{2})^2+\frac{3}{4}}\right)=e^{-\frac{1}{2}t}\cdot\sin\frac{\sqrt{3}}{2}t\cdot u(t)$$

综上 $h(t)=e^{-\frac{1}{2}t}\cdot\cos\frac{\sqrt{3}}{2}t\cdot u(t)+\frac{\sqrt{3}}{3}\cdot e^{-\frac{1}{2}t}\cdot\sin\frac{\sqrt{3}}{2}t\cdot u(t)$。

而单位冲激响应的傅里叶变换 $C(\omega)=H(\omega)\cdot\left(\pi\delta(\omega)+\frac{1}{j\omega}\right)=\frac{j\omega+1}{(j\omega)^2+j\omega+1}\left(\pi\delta(\omega)+\frac{1}{j\omega}\right)$

同理 $\frac{j\omega+1}{(j\omega)^2+j\omega+1}\cdot\pi\delta(\omega)=\pi\delta(\omega)$

即 $C(\omega)=\pi\delta(\omega)+\frac{j\omega+1}{(j\omega)((j\omega)^2+j\omega+1)}=\pi\delta(\omega)+\frac{1}{j\omega}+\frac{-(j\omega)-1}{(j\omega)^2+j\omega+1}+\frac{1}{(j\omega)^2+j\omega+1}$

前两项 $\pi\delta(\omega)+\frac{1}{j\omega}=\mathcal{F}(u(t))$，第三项 $=-\frac{j\omega+\frac{1}{2}}{(j\omega+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}-\frac{\sqrt{3}}{3}\cdot\frac{\frac{\sqrt{3}}{2}}{(j\omega+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}$

$$=\mathcal{F}\left(-e^{-\frac{1}{2}t}\cos\frac{\sqrt{3}}{2}t\cdot u(t)-\frac{\sqrt{3}}{3}\cdot e^{-\frac{1}{2}t}\sin\frac{\sqrt{3}}{2}t\cdot u(t)\right)$$

~~即 $c(t)=u(t)-e^{-\frac{1}{2}t}\cdot\cos(\frac{\sqrt{3}}{2}t)\cdot u(t)-\frac{\sqrt{3}}{3}e^{-\frac{1}{2}t}\cdot\sin(\frac{\sqrt{3}}{2}t)\cdot u(t)$~~

第四项 $=\frac{1}{(j\omega)^2+j\omega+1}=\frac{2\sqrt{3}}{3}\frac{\frac{\sqrt{3}}{2}}{(j\omega+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}=\mathcal{F}\left(\frac{2\sqrt{3}}{3}\cdot e^{-\frac{1}{2}t}\cdot\sin(\frac{\sqrt{3}}{2}t)\cdot u(t)\right)$

故 $c(t)=u(t)-e^{-\frac{1}{2}t}\cdot\cos(\frac{\sqrt{3}}{2}t)\cdot u(t)+\frac{\sqrt{3}}{3}\cdot e^{-\frac{1}{2}t}\sin(\frac{\sqrt{3}}{2}t)\cdot u(t)$ ✓

(3) 由题已知 $y'(t)+2y(t)=x''(t)+3x'(t)+3x(t)$，假设 $\mathcal{F}(y(t))=Y(\omega)$，$\mathcal{F}(x(t))=X(\omega)$

上述化为 $j\omega\cdot Y(\omega)+2Y(\omega)=(j\omega)^2X(\omega)+3j\omega X(\omega)$，得 $H(\omega)=\frac{Y(\omega)}{X(\omega)}=(j\omega+1)+\frac{1}{j\omega+2}$

$h(t)=\mathcal{F}^{-1}(H(\omega))=\delta'(t)+\delta(t)+e^{-2t}\cdot u(t)$ ✓

而 $C(\omega)=H(\omega)\cdot\left(\pi\delta(\omega)+\frac{1}{j\omega}\right)=\pi\cdot H(0)\cdot\delta(\omega)+\frac{1}{j\omega}H(\omega)=\frac{3}{2}\pi\delta(\omega)+1+\frac{1}{j\omega}\frac{3}{2}+\frac{1}{j\omega+2}\cdot(-\frac{1}{2})$

$c(t)=\mathcal{F}^{-1}(C(\omega))=\frac{3}{4}+\delta(t)+\frac{3}{4}\mathrm{sgn}(t)-\frac{1}{2}e^{-2t}u(t)$

$$=\frac{3}{2}u(t)+\delta(t)-\frac{1}{2}e^{-2t}u(t)$$ ✓

4. 已知 $H(\omega)=\frac{1}{j\omega+1}$，激励信号 $e(t)=\sin t+\sin 3t$

由此 $E(\omega)=\mathcal{F}(e(t))=j\pi\left(\delta(\omega+1)-\delta(\omega-1)+\delta(\omega+3)-\delta(\omega-3)\right)$

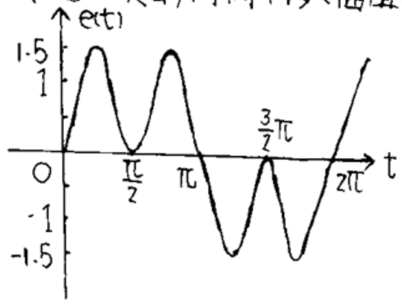
而 $R(\omega)=\mathcal{F}(r(t))=E(\omega)\cdot H(\omega)=\frac{j\pi}{j\omega+1}\left(\delta(\omega+1)-\delta(\omega-1)+\delta(\omega+3)-\delta(\omega-3)\right)$

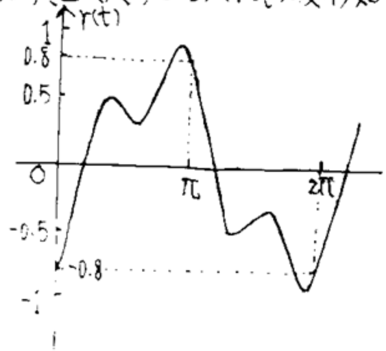
$$=\frac{j\pi}{1-j}\delta(\omega+1)-\frac{j\pi}{1+j}\delta(\omega-1)+\frac{j\pi}{1-3j}\delta(\omega+3)-\frac{j\pi}{1+3j}\delta(\omega-3)$$

接上题

$$r(t)=F^{-1}(R(\omega))=\frac{j}{2(1-j)}e^{-jt}-\frac{j}{2(1+j)}e^{jt}+\frac{j}{2(1-3j)}e^{-j3t}-\frac{j}{2(1+3j)}e^{j3t}$$
$$=\frac{j-1}{4}e^{-jt}+\frac{-j-1}{4}e^{jt}+\frac{j-3}{20}e^{-j3t}+\frac{-j-3}{20}e^{j3t}$$
$$=-\frac{1}{4}(e^{jt}+e^{-jt})+\frac{-j}{4}(e^{jt}-e^{-jt})-\frac{3}{20}(e^{j3t}+e^{-j3t})+\frac{-j}{20}(e^{j3t}-e^{-j3t})$$
$$=-\frac{1}{2}\cos t+\frac{1}{2}\sin t-\frac{3}{10}\cos 3t+\frac{1}{10}\sin 3t$$
$$=\frac{\sqrt{2}}{2}\sin(t-45^\circ)+\frac{\sqrt{10}}{10}\sin(3t-71.565^\circ)$$

由于前一项、后一项的时间平移、幅值变化不一,可知其已失真,$e(t)$、$r(t)$波形如下:





由此 传输发生失真,$r(t)$不仅幅值失真 还有相位失真

5. 已知 $H(\omega)=\begin{cases}1 & |\omega|<\frac{2\pi}{\tau}\\ 0 & |\omega|>\frac{2\pi}{\tau}\end{cases}$,且$X(\omega)=\tau Sa(\frac{\omega\tau}{2})$作为激励

由此 $h(t)=F^{-1}(H(\omega))=\frac{2}{\tau}Sa(\frac{2\pi}{\tau}t)$,而$x(t)=\begin{cases}1 & t<|\frac{\tau}{2}|\\ 0 & t>|\frac{\tau}{2}|\end{cases}=u(t+\frac{\tau}{2})-u(t-\frac{\tau}{2})$

故 $y(t)=h(t)*x(t)=\frac{2}{\tau}Sa(\frac{2\pi}{\tau}t)*\left(u(t+\frac{\tau}{2})-u(t-\frac{\tau}{2})\right)$
$$=\int_{-\infty}^{+\infty}\frac{2}{\tau}Sa(\frac{2\pi}{\tau}\lambda)\cdot u(t+\frac{\tau}{2}-\lambda)d\lambda-\int_{-\infty}^{+\infty}\frac{2}{\tau}Sa(\frac{2\pi}{\tau}\lambda)\cdot u(t-\frac{\tau}{2}-\lambda)d\lambda$$
$$=\int_{-\infty}^{t+\frac{\tau}{2}}\frac{2}{\tau}Sa(\frac{2\pi}{\tau}\lambda)d\lambda-\int_{-\infty}^{t-\frac{\tau}{2}}\frac{2}{\tau}Sa(\frac{2\pi}{\tau}\lambda)d\lambda$$
$$\xlongequal{令x=\frac{2\pi}{\tau}\lambda}\frac{1}{\pi}\int_{-\infty}^{\frac{2\pi}{\tau}(t+\frac{\tau}{2})}Sa(x)dx-\frac{1}{\pi}\int_{-\infty}^{\frac{2\pi}{\tau}(t-\frac{\tau}{2})}Sa(x)dx$$
$$=\frac{1}{\pi}\left(Si(\frac{2\pi}{\tau}(t+\frac{\tau}{2}))-Si(\frac{2\pi}{\tau}(t-\frac{\tau}{2}))\right)$$

6. 由频域指标可有:假设 $\omega_c=\omega_1=\omega_p=1000\,rad/s$,$\alpha_p=3dB$.

$\omega_s=\omega_2=5000\,rad/s$,$\alpha_s=20dB$

见背面

求巴特沃斯滤波器阶数

$$n=\frac{\lg\sqrt{10^{0.1\alpha_s}-1}}{\lg\frac{\omega_s}{\omega_c}}=1.428$$ ，故取 $n=2$，查巴特沃斯多项式表后

得 $$H(\bar{s})=\frac{1}{\bar{s}^2+\sqrt{2}\,\bar{s}+1}=\cancel{\frac{\omega_c^2}{s^2+\sqrt{2}\omega_c s+\omega_c^2}}=\cancel{\frac{10^6}{s^2+\sqrt{2}\times10^3 s+10^6}}$$

通过反归一化处理，令 $s=\bar{s}\omega_c$ 得

$$H(s)=\frac{\omega_c^2}{s^2+\sqrt{2}\omega_c\cdot s+\omega_c^2}=\frac{10^6}{s^2+\sqrt{2}\times10^3 s+10^6}$$