积分得到的是值 100

1. 应用冲激信号的抽样特性（筛选特性），求下列表示式的函数值。

(1) $\int_{-\infty}^{\infty} f(t-t_0)\delta(t)dt = f(t-t_0)\Big|_{t=0} = f(-t_0)$

(2) $\int_{-\infty}^{\infty} f(t_0-t)\delta(t)dt = f(t_0-t)\Big|_{t=0} = f(t_0)$

(3) $\int_{-\infty}^{\infty} \delta(t-t_0)u(t-2t_0)dt = u(t-2t_0)\Big|_{t=t_0} = u(-t_0) = \begin{cases} 0 & t_0>0 \\ 1 & t_0<0 \end{cases}$

(4) $\int_{-\infty}^{\infty} (t+\sin t)\delta(t-\frac{\pi}{6})dt = (t+\sin t)\Big|_{t=\frac{\pi}{6}} = \frac{1}{2}+\frac{\pi}{6}$

(5) $\int_{-\infty}^{\infty} e^{-j\omega t}[\delta(t)-\delta(t-t_0)]dt$

$= \int_{-\infty}^{\infty} e^{-j\omega t}\delta(t)dt - \int_{-\infty}^{\infty} e^{-j\omega t}\delta(t-t_0)dt = e^0 - e^{-j\omega t_0} = 1-e^{-j\omega t_0}$

2. 判断信号 $f(t) = 2\cos(10t+5) - \sin(6t-3)$ 是否为周期信号（要求写出步骤）？如是周期信号，计算$f(t)$的基波周期。

解 $2\cos(10t+5)$ $T_1 = \frac{2\pi}{10} = \frac{\pi}{5}$, $\frac{T_1}{T_2} = \frac{3}{5}$ 为有理数，

$-\sin(6t-3)$ $T_2 = \frac{2\pi}{6} = \frac{\pi}{3}$ 故 $f(t)$ 是周期信号

$f(t)$的基波周期为 $T_1$、$T_2$ 的最小公倍数 $T=\pi$

3. 已知信号$f_1(t) = u(t+1) - u(t-1)$，$f_2(t) = \delta(t+5) + \delta(t-5)$，画出下列各卷积波形。

计算卷积积分

(1) $s_1(t) = f_1(t) * f_2(t)$

(2) $s_2(t) = \{[f_1(t) * f_2(t)][u(t+5) - u(t-5)]\} * f_2(t)$

相乘

公式. $f(t)*\delta(t)=f(t)$

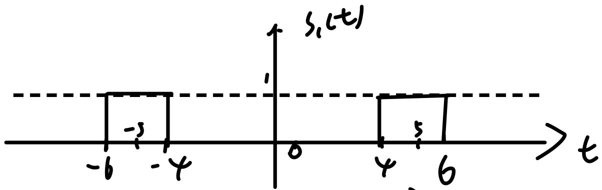
$f(t-t_1)*\delta(t-t_2) = f(t-t_1-t_2)$

解.

(1) $s_1(t) = f_1(t)*f_2(t) = u(t+1)\delta(t+5)+u(t+1)\delta(t-5)-u(t-1)\delta(t+5)-u(t-1)\delta(t-5)$

$= u(t+6)+u(t-4)-u(t+4)-u(t-6)$

$\quad -6 \qquad 4 \qquad -4 \qquad 6$



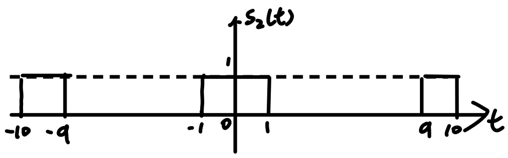

(2) $s_2(t) = \{s_1(t)[u(t+5)-u(t-5)]\}*f_2(t) = \{[s_1(t)u(t+5)]-[s_1(t)u(t-5)]\}*f_2(t)$

$= \{[u(t+5)+u(t-4)-u(t+4)-u(t-6)]-[u(t-5)-u(t-6)]\}*f_2(t)$

$= [u(t+5)+u(t-4)-u(t+4)-u(t-5)]*[\delta(t+5)+\delta(t-5)]$

$= u(t+10)+u(t+1)-u(t+9)-u(t)+u(t)+u(t-9)-u(t-1)-u(t-10)$

$= u(t+10)-u(t+9)+u(t+1)-u(t-1)+u(t-9)-u(t-10)$




★单位阶跃信号相乘

$u(t-t_1)u(t-t_2) = u(t-t_0)$, $t_0=\max\{t_1,t_2\}$

4. 证明：$\sin(t),\sin(2t),\ldots,\sin(nt)$（$n$为正整数）是在区间$(0,2\pi)$的正交函数集。

然后回答：(1) 该函数集在区间$(0,2\pi)$是否为完备的正交函数集，为什么？

(2) 该函数集在区间$\left(0,\frac{\pi}{2}\right)$是否为正交函数集，为什么？

（所有证明和计算都要求写出具体步骤）

证明. 只需证 $\int_0^{2\pi}\sin(\alpha t)\sin(\beta t)dt=0\ \forall\alpha,\beta\in Z_+,\alpha\neq\beta$ $\int_0^{2\pi}\sin(kt)\sin(kt)dt\neq0$

利用 $\sin A\sin B=\frac{1}{2}[\cos(A-B)-\cos(A+B)]$ 原式 $=\frac{1}{2}\int_0^{2\pi}\cos|\alpha-\beta|tdt-\frac{1}{2}\int_0^{2\pi}\cos(\alpha+\beta)tdt$

其中, $|\alpha-\beta|$, $\alpha+\beta$ 均为正整数 故 $\cos|\alpha-\beta|t$ $T_1=\frac{2\pi}{|\alpha-\beta|}$, $\cos(\alpha+\beta)t\cdot T_2=\frac{2\pi}{\alpha+\beta}$

积分区间 $(0,2\pi)$ 为 $T_1$、$T_2$的整数倍 而余弦函数在单个周期上的积分为零(对称性)

⇒原式=0

$\int_0^{2\pi}\sin(kt)\sin(kt)dt=\frac{1}{2}\int_0^{2\pi}(1-\cos2kt)dt=\pi-\frac{1}{2}\int_0^{2\pi}\cos2ktdt=\pi\neq0$

从而该函数集在$(0,2\pi)$为正交函数集

(1) 不是, $\int_0^{2\pi}\sin t\cos t\,dt=\frac{1}{2}\int_0^{2\pi}\sin2t\,dt=-\frac{1}{4}\cos2t\Big|_0^{2\pi}=-\frac{1}{4}(\cos4\pi-\cos0)=0$

说明在区间$(0,2\pi)$内, $\sin t$ 与 $\cos t$正交, 故$\{\sin t\ldots\sin nt\}$在$(0,2\pi)$内不是完备的正交函数集

(2) 不是, $\int_0^{\frac{\pi}{2}}\sin t\sin2t\,dt=\frac{1}{2}\int_0^{\frac{\pi}{2}}\cos(t)dt-\frac{1}{2}\int_0^{\frac{\pi}{2}}\cos(3t)dt=\frac{1}{2}\sin t\Big|_0^{\frac{\pi}{2}}-\frac{1}{6}\sin3t\Big|_0^{\frac{\pi}{2}}=\frac{1}{2}-\frac{1}{6}(-1)=\frac{2}{3}\neq0$

说明区间$(0,\frac{\pi}{2})$中, $\sin t$与$\sin2t$不正交

故该函数集在$(0,\frac{\pi}{2})$不是正交函数集

5. 已知连续信号$f_1(t)=\begin{cases}2, & 1<t<3\\ 0, & 其他\end{cases}$，$f_2(t)=\begin{cases}1, & 0<t<1\\ -1, & 1<t<2\\ 0, & 其他\end{cases}$

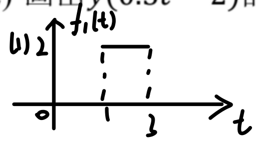
$x_1(t)*x_2(t)=\frac{d}{dt}x_1(t)*\int_{-\infty}^{t}x_2(\tau)d\tau$

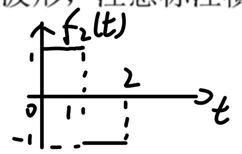
$r(t)\xrightarrow{\frac{d}{dt}}u(t)\xrightarrow{\frac{d}{dt}}\delta(t)$

(1) 求卷积函数$y(t)=f_1(t)*f_2(t)$，并画出其概略图。

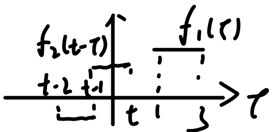
(2) 画出$y(0.5t-2)$的波形，注意标注横、纵坐标刻度，并附上简要的步骤说明。

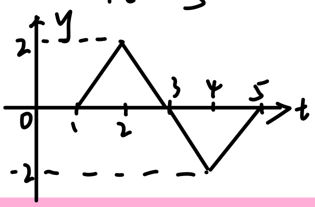
解. (1)





$f_1(t)*f_2(t)=\int_{-\infty}^{+\infty}f_1(\tau)f_2(t-\tau)d\tau$





① $t\le1$ 或 $t\ge5$时, $f_1(t)*f_2(t)=0$

② $t\in(1,2)$时 $f_1(t)*f_2(t)=1\times2\,(t-1)=2t-2$

③ $t\in(2,3)$时 $f_1(t)*f_2(t)=1\times2+(-2)\,(t-2)=-2t+6$

④ $t\in(3,4)$时 $f_1(t)*f_2(t)=-2+2[3-(t-1)]=-2+6-2t+2=6-2t$

⑤ $t\in(4,5)$时 $f_1(t)*f_2(t)=-2\,[3-(t-2)]=2t-10$

法二. $f_1(t)=2[u(t-1)-u(t-3)]$ $f_2(t)=u(t)-2u(t-1)+u(t-2)$

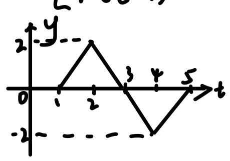
$f_1(t)*f_2(t)=\frac{d}{dt}f_1(t)*\int_{-\infty}^{t}f_2(\tau)d\tau=2[\delta(t-1)-\delta(t-3)]*[r(t)-2r(t-1)+r(t-2)]$

$=2[\delta(t-1)r(t)-2\delta(t-1)r(t-1)+\delta(t-1)r(t-2)-\delta(t-3)r(t)+2\delta(t-3)r(t-1)-\delta(t-3)r(t-2)]$

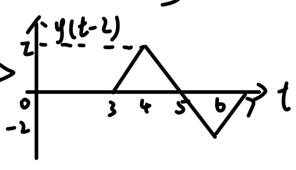
$=2[r(t-1)-2r(t-2)+\underline{r(t-3)-r(t-3)}+2r(t-4)-r(t-5)]$

$=2[r(t-1)-2r(t-2)+2r(t-4)-r(t-5)]$ 图同上.

(2)



向右平移两个单位



时间轴展缩