1. 周期矩形信号 $f(t)$，有如下参数 τ, T, E

将其在 $t\in[-\frac{T}{2},\frac{T}{2}]$ 上进行傅立叶级数展开，$\omega_0=\frac{2\pi}{T}$

$$a_0=\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\,dt=\frac{1}{T}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} E\,dt=\frac{1}{T}E\tau$$

$$a_n=\frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\cos n\omega_0 t\,dt=\frac{2}{T}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} E\cos n\omega_0 t\,dt=\frac{4}{n\omega_0 T}E\sin n\omega_0\frac{\tau}{2}=\frac{2E\tau}{T}Sa\left(\frac{n\omega_0}{2}\tau\right)$$

而由于其为偶函数，$b_n=0$，由此 $C_n=\sqrt{a_n^2+b_n^2}=|a_n|$，谱线间隔由于 $\tau=\frac{T}{2}$，原有 $\omega_0=\frac{2\pi}{T}=\frac{\pi}{\tau}$

$Sa(\frac{n\omega_0}{2}\tau)$ 的分子为0，有 $\frac{n\omega_0}{2}\tau=k\pi$ 即 $n\omega_0=\frac{2k\pi}{\tau}$，零点间隔为 $\frac{2\pi}{\tau}$，$k=1,2,\cdots$，考虑直流分量

则谱线间隔应为 $\frac{\omega_0}{2\pi}$，而带宽为 $\frac{1}{\tau}$

(1) $f_1(t)$ 的谱线间隔为 $\cancel{2f_0=2000}$ $f_0=1000$ kHz，带宽为 $\frac{1}{\tau}=2000$ kHz

(2) $f_2(t)$ 的谱线间隔为 $f_0=\frac{1}{3}\times10^3$ kHz，~~带宽为 $\frac{2\pi}{\tau}=\frac{4}{3}\pi\times10^3$ kHz~~ 带宽为 $\frac{1}{\tau}=\frac{2}{3}\times10^3$ kHz

(3) $f_1(t)$ 的基波幅度 $A_{11}=\frac{2E_1\tau_1}{T_1}Sa\left(\frac{1\omega_0}{2}\tau_1\right)=\frac{E_1}{\pi}=\frac{2}{\pi}$

$f_2(t)$ 的基波幅度 $A_{21}=\frac{2E_2\tau_2}{T_2}Sa\left(\frac{1\omega_0}{2}\tau_2\right)=\frac{6}{\pi}$

则二者基波幅度之比 $\frac{A_{11}}{A_{21}}=\frac{1}{3}$

(4) $f_2(t)$ 三次谐波幅度 $A_{23}=\left|\frac{\sin\frac{3\pi}{2}}{\pi}\right|=\frac{2}{\pi}$，故 $f_1(t)$ 基波与 $f_2(t)$ 三次谐波之比为 $\frac{A_{11}}{A_{23}}=1$

2. 已知 $E=10V$，$f=10kHz$，对 $t\in[-\frac{T}{2},\frac{T}{2}]$ 内求其傅里叶级数

由于其为偶函数，$b_n=0$，令 $\omega_0=\frac{2\pi}{T}$

$$a_0=\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\,dt=\frac{1}{T}\int_{-\frac{T}{4}}^{\frac{T}{4}} E\cos\omega_0 t\,dt=\frac{E}{T}\cdot\frac{T}{2\pi}\sin\frac{2\pi}{T}t\Big|_{-\frac{T}{4}}^{\frac{T}{4}}=\frac{E}{\pi}$$

$$a_1=\frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\cos\omega_0 t\,dt=\frac{2}{T}\int_{-\frac{T}{4}}^{\frac{T}{4}} E\cos^2\omega_0 t\,dt=\frac{E}{2}$$

$$a_n=\frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\cdot\cos n\omega_0 t\,dt=\frac{2}{T}\int_{-\frac{T}{4}}^{\frac{T}{4}} E\cos\omega_0 t\cdot\cos n\omega_0 t\,dt=\frac{E}{T}\int_{-\frac{T}{4}}^{\frac{T}{4}}\cos(n+1)\omega_0 t+\cos(n-1)\omega_0 t\,dt$$

$$=\frac{E}{T}\left(\frac{1}{(n+1)\omega_0}\sin(n+1)\omega_0 t+\frac{1}{(n-1)\omega_0}\sin(n-1)\omega_0 t\right)\Big|_{-\frac{T}{4}}^{\frac{T}{4}}=\frac{E}{\pi}\left(\frac{1}{n+1}\sin(n+1)\frac{\pi}{2}+\frac{1}{n-1}\sin(n-1)\frac{\pi}{2}\right)$$

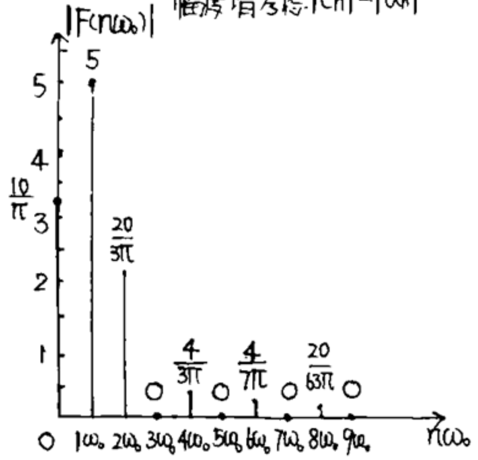
由于 $C_n=\sqrt{a_n^2+b_n^2}=|a_n|$，且
$$a_n=\begin{cases}\frac{E}{2} & n=1\\ 0 & n=3,5,7\cdots2k+1\cdots\quad k=1,2\cdots\\ \frac{E}{\pi}\frac{2}{n^2-1}\cos\left(\frac{n-2}{2}\pi\right) & n=2,4,6\cdots2k\cdots\quad k=1,2\cdots\end{cases}$$

一、第六题

$a_0=\frac{E}{\pi}=\frac{10}{\pi}$

$$a_n=\begin{cases}5 & n=1\\ 0 & n=3.5.7\cdots 2K+1\cdots \quad K=1.2\cdots\\ \frac{20}{\pi}\frac{1}{n^2-1}\cos(\frac{n-2}{2}\pi) & n=2.4.6\cdots 2K\cdots \quad K=1.2\cdots\end{cases}$$ 单位 V

幅度谱考虑 $|C_n|=|a_n|$



3.(a)

由题设 $g(t)=\frac{df(t)}{dt}=-E\cdot\delta(t+\frac{T}{2})-E\cdot\delta(t-\frac{T}{2})+\frac{2E}{T}(u(t+\frac{T}{2})-u(t-\frac{T}{2}))$

$$G(\omega)=\int_{-\infty}^{+\infty}g(t)e^{-j\omega t}dt=-E\int_{-\infty}^{+\infty}\delta(t+\tfrac{T}{2})e^{-j\omega t}dt-E\int_{-\infty}^{+\infty}\delta(t-\tfrac{T}{2})e^{-j\omega t}dt+\int_{-\frac{T}{2}}^{\frac{T}{2}}\frac{2E}{T}e^{-j\omega t}dt$$

$$=-E(e^{j\omega\frac{T}{2}}+e^{-j\omega\frac{T}{2}})+\frac{2E}{T}\cdot\frac{1}{-j\omega}e^{-j\omega t}\Big|_{-\frac{1}{2}T}^{\frac{1}{2}T}$$

$$=-2E\cos\frac{\omega T}{2}+\frac{2E}{T}\cdot\frac{1}{-j\omega}(e^{-j\omega\frac{1}{2}T}-e^{j\omega\frac{1}{2}T})=-2E\cos\frac{\omega T}{2}+\frac{4E}{\omega T}\sin\frac{\omega T}{2}$$

由于 $f(t)\xrightarrow{\mathcal{F}}F(\omega)$，$\frac{df(t)}{dt}\xrightarrow{\mathcal{F}}G(\omega)$. 故 $F(\omega)=\frac{1}{j\omega}G(\omega)+\pi G(0)\delta(\omega)$

而 $G(0)=(-2E\cos\frac{\omega T}{2}+\frac{4E}{\omega T}\sin\frac{\omega T}{2})\Big|_{\omega=0}=\lim_{\omega\to 0}\frac{4E}{\omega T}\sin\frac{\omega T}{2}-2E=0$

故 $F(\omega)=\frac{1}{j\omega}(-2E\cos\frac{\omega T}{2}+2ESa(\frac{\omega T}{2}))$

3(b) 由题，该函数绝对可积、满足狄里赫利条件．令 $\omega_0=\frac{2\pi}{T}$

$$F(j\omega)=\int_{-\infty}^{+\infty}f(t)e^{-j\omega t}dt=E\int_0^T\sin\frac{2\pi}{T}e^{-j\omega t}dt=E\int_0^T\frac{1}{2j}(e^{j\omega_0 t}-e^{-j\omega_0 t})e^{-j\omega t}dt$$

$$=\frac{E}{2j}\int_0^T e^{-j(\omega-\omega_0)t}-e^{-j(\omega+\omega_0)t}dt=\frac{E}{2j}(-\frac{1}{j(\omega-\omega_0)}e^{-j(\omega-\omega_0)t}+\frac{1}{j(\omega+\omega_0)}e^{-j(\omega+\omega_0)t})\Big|_0^T$$

$$=\frac{E}{2}(\frac{1}{\omega-\omega_0}(e^{-j(\omega-\omega_0)T}-1)-\frac{1}{\omega+\omega_0}(e^{-j(\omega+\omega_0)T}-1))$$

$$=\frac{E\omega_0}{\omega^2-\omega_0^2}(e^{-j\omega T}-1)$$

4. 已知 $\mathcal{F}(f_1(t))=F_1(\omega)$，由此 $f_2(t)=f_1(-(t-t_0))$

$\mathcal{F}(f_1(-t))=\frac{1}{|-1|}F(\frac{1}{-1}\omega)=F(-\omega)$

$\mathcal{F}(f_2(t))=\mathcal{F}(f_1(-(t-t_0)))=F(-\omega)\cdot e^{-j\omega t_0}$

运用傅里叶变换性质有：$\mathcal{F}(f_2(t))=F(-\omega)e^{-j\omega t_0}$

5. 由题 $f_2(t)=f_1(t-\frac{\tau}{2})\cos\omega_0 t \xlongequal{令g(t)=f_1(t-\frac{\tau}{2})} g(t)\cos\omega_0 t$

已知 $f_1(t)\xrightarrow{\mathcal{F}}F_1(\omega)$，$g(t)=f_1(t-\frac{\tau}{2})\xrightarrow{\mathcal{F}}F_1(\omega)\cdot e^{-j\omega\frac{\tau}{2}}=G(\omega)$

由时域乘积~~是频域卷积~~傅氏变换后为频域卷积

故 $g(t)\cdot\frac{1}{2}(e^{j\omega_0 t}+e^{-j\omega_0 t})\xrightarrow{\mathcal{F}}G(\omega)*\pi(\delta(\omega-\omega_0)+\delta(\omega+\omega_0))\frac{1}{2\pi}$

即 $\frac{1}{2}G(\omega)*(\delta(\omega-\omega_0)+\delta(\omega+\omega_0))=\frac{1}{2}\int_{-\infty}^{+\infty}G(\lambda)\cdot(\delta(\omega-\omega_0-\lambda)+\delta(\omega+\omega_0-\lambda))d\lambda$

$=\frac{1}{2}(G(\omega-\omega_0)+G(\omega+\omega_0))=\frac{1}{2}(F_1(\omega-\omega_0)\cdot e^{-j(\omega-\omega_0)\frac{\tau}{2}}+F_1(\omega+\omega_0)e^{-j(\omega+\omega_0)\frac{\tau}{2}})$

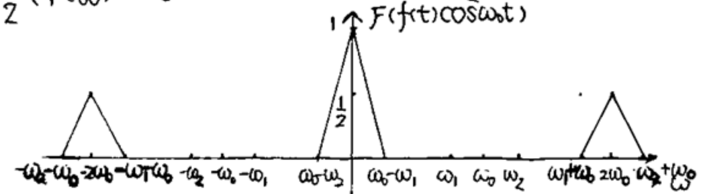
$=\frac{1}{2}\left(Sa^2(\frac{\omega-\omega_0}{4}\tau)e^{-j(\omega-\omega_0)\frac{\tau}{2}}+Sa^2(\frac{\omega+\omega_0}{4}\tau)e^{-j(\omega+\omega_0)\frac{\tau}{2}}\right)\frac{E\tau}{2}$ 15

$=\frac{E\tau}{4}e^{-j\omega\frac{\tau}{2}}\left(Sa^2(\frac{\omega-\omega_0}{4}\tau)e^{+j\omega_0\frac{\tau}{2}}+Sa^2(\frac{\omega+\omega_0}{4}\tau)e^{-j\omega_0\frac{\tau}{2}}\right)$

6. (1) $f(t)\cos\omega_0 t=f(t)\cdot\frac{1}{2}(e^{j\omega_0 t}+e^{-j\omega_0 t})$

$\mathcal{F}(f(t)\cos\omega_0 t)=\frac{1}{2\pi}(F(\omega)*\pi(\delta(\omega-\omega_0)+\delta(\omega+\omega_0)))$

$=\frac{1}{2}(F(\omega)*(\delta(\omega-\omega_0)+\delta(\omega+\omega_0)))=\frac{1}{2}(F(\omega-\omega_0)+F(\omega+\omega_0))$



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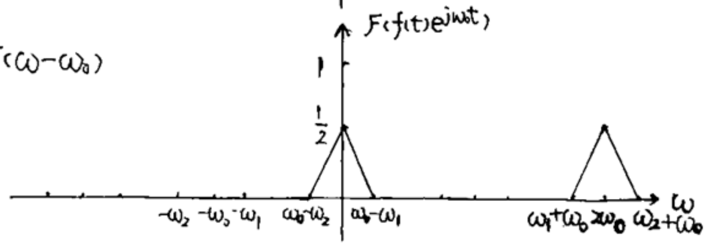
(2) $f(t)e^{j\omega_0 t}\xrightarrow{\mathcal{F}}F(\omega-\omega_0)$



(3) $f(t)\cos(\omega_1 t)\xrightarrow{\mathcal{F}}\frac{1}{2\pi}(F(\omega)*\pi(\delta(\omega-\omega_1)+\delta(\omega+\omega_1)))=\frac{1}{2}(F(\omega-\omega_1)+F(\omega+\omega_1))$