Homework

1. Let $P^{n\times n}$ be a set of all $n\times n$ matrices,

   (1) Please show that the set of all the matrices that are exchangeable with $A\in P^{n\times n}$ is a subspace of $P^{n\times n}$. If this subspace is denoted as $C(A)$. It means $C(A)=\{B\in P^{n\times n} \mid AB=BA, A\in P^{n\times n}\}$.

   (2) When $A=\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \cdots & 1 \end{bmatrix}$, please find $C(A)$.

   (3) If $A=\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 2 & \cdots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \cdots & n \end{bmatrix}$, please find the dimension and a basis of $C(A)$.

2. Please find a basis and the dimension of the solution space of the following system of linear equations

$$\begin{cases} 3x_1+2x_2-5x_3+4x_4=0 \\ 3x_1-x_2+3x_3-3x_4=0 \\ 3x_1+5x_2-13x_3+11x_4=0 \end{cases}$$