Homework

1. Show that the function that takes $((x_1, x_2),(y_1, y_2)) \in \mathrm{R}^2 \times R^2$ to $|x_1 y_1| + |x_2 y_2|$ is not an inner product on R^2.

2. Suppose V is a real inner product space, show that

   (a) Show that the inner product $\langle u+v, u-v \rangle = \|u\|^2 - \|v\|^2$ for every $u, v \in V$.

   (b) Show that if $u, v \in V$ have the same norm, then $u+v$ is orthogonal to $u-v$.

   (c) Use part(b) to show that the diagonals of a rhombus are perpendicular to each other.

3. Suppose $u, v \in V$. Prove that the inner product $\langle u, v \rangle = 0$ if and only if $\|u\| \le \|u + av\|$ for all $a \in R$.

4. Suppose n is a positive integer. Prove that $\frac{1}{\sqrt{2\pi}}, \frac{\cos x}{\sqrt{\pi}}, \frac{\cos 2x}{\sqrt{\pi}}, \cdots, \frac{\cos nx}{\sqrt{\pi}}, \frac{\sin x}{\sqrt{\pi}}, \frac{\sin 2x}{\sqrt{\pi}}, \cdots, \frac{\sin nx}{\sqrt{\pi}}$ is an orthonormal list of vectors in $C[-\pi, \pi]$, the linear space of continuous real-valued functions on $[-\pi, \pi]$ with inner product $\langle f, g \rangle = \int_{-\pi}^{\pi} f(x) g(x) dx$.

5. On $P^2[x]$, the linear space of polynomial functions of degree ≤ 2, consider the inner product given by $\langle p, q \rangle = \int_0^1 p(x) q(x) dx$. Apply the Gram–Schmidt Procedure to the basis $1, x, x^2$ to produce an orthonormal basis of $P^2[x]$

6. Find a polynomial $q \in P^2[x]$ such that $p\left(\frac{1}{2}\right) = \int_0^1 p(x) q(x) dx$ for every $p(x) \in P^2[x]$.

7. Suppose U is the subspace of R^4 defined by $U = span\big((1,2,3,-4),(-5,4,3,2)\big)$ Find an orthonormal basis of U and an orthonormal basis of its orthogonal complement $U^\perp$

8. In R^4, let $U = span\big((1,1,0,0),(1,1,1,2)\big)$. Find $u \in U$ such that $\|u - (1,2,3,4)\|_2$ is as small as possible.