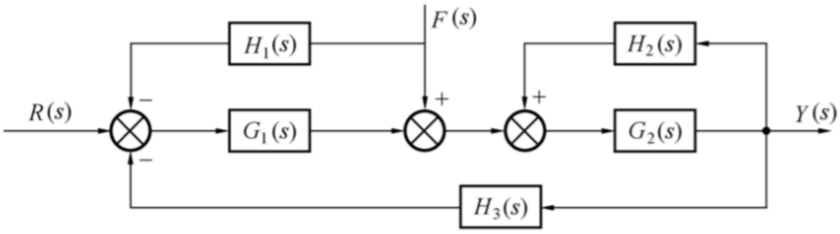
自动控制理论 A 作业 3

2019 年 9 月 19 日

2.11　画出题 2.8 图所示系统的信号流图，用梅森公式求 $\frac{Y(s)}{R(s)}$ 及 $\frac{Y(s)}{F(s)}$。



(a)

2.11 (a) 求 ~~R(s)~~ $\frac{Y(s)}{R(s)}$，令 $F(s)=0$，画出信号流图如

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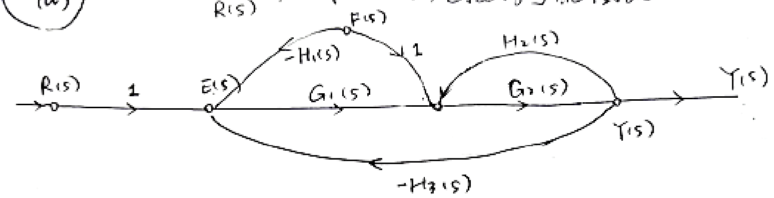


上方小回路 $L_1 = G_2(s)H_2(s)$

下方大回路 $L_2 = -G_1(s)G_2(s)H_3(s)$

两回路有接触

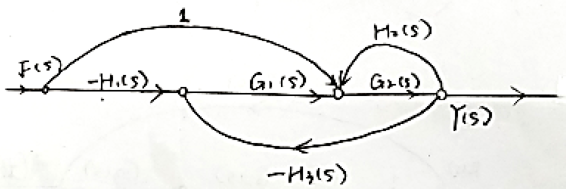
前向通路 $P_1 = G_1(s)G_2(s)$

$\Delta = 1 - G_2H_2 + G_1G_2H_3$

因 P_1 与 L_1、L_2 均接触，故 $\Delta_1 = 1$

故 $\frac{Y(s)}{R(s)} = \frac{G_1G_2}{1-G_2H_2+G_1G_2H_3}$

求 $\frac{Y(s)}{F(s)}$，令 $R(s)=0$



$L_1 = G_2H_2$

$L_2 = -G_1G_2H_3$　两回路有接触

$P_1 = -H_1G_1G_2$

$P_2 = G_2$

$\Delta = 1 - L_1 - L_2 = 1 - G_2H_2 + G_1G_2H_3$

$\Delta_1 = 1$，$\Delta_2 = 1$.

故 $\frac{Y(s)}{F(s)} = \frac{-H_1G_1G_2+G_2}{1-G_2H_2+G_1G_2H_3}$

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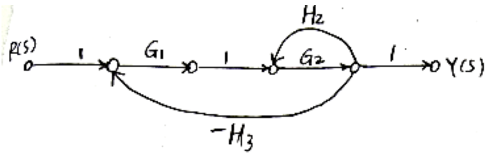


信号流图共有一条前向通路，通路增益 $P_1=G_1G_2$

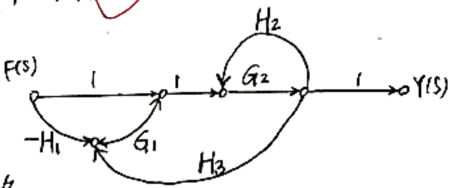
共有两个回路，回路增益为 $L_1=G_2H_2$，$L_2=-G_1G_2H_3$

特征式 $\Delta=1-(L_1+L_2)=1-G_2H_2+G_1G_2H_3$

$\Delta_1=1$，$\Delta_2=1$．

故 $\dfrac{Y(S)}{R(S)}=\dfrac{1}{\Delta}\sum\limits_{K=1}^{2}P_K\Delta_K=\dfrac{G_1G_2}{1-G_2H_2+G_1G_2H_3}$

求 $\dfrac{Y(S)}{F(S)}$．信号流图如下：

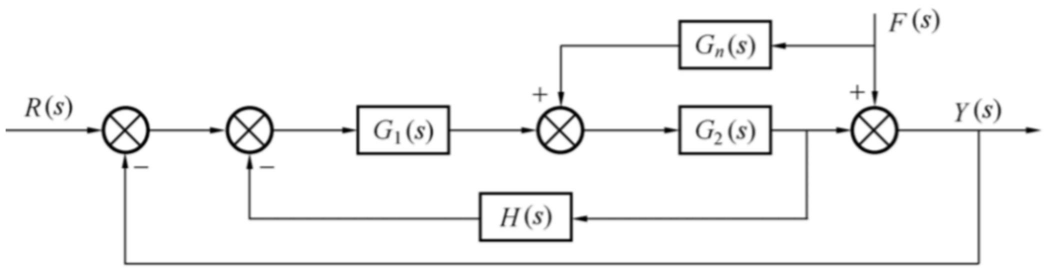


信号流图中共有两条前向通路．

增益 $P_1=G_2$，$P_2=-G_1G_2H_1$；共有两个回路，回路增益为 $L_1=G_2H_2$，$L_2=-G_1G_2H_3$

特征式为 $\Delta=1-G_2H_2+G_1G_2H_3$，$\Delta_1=1$，$\Delta_2=1$

$\therefore \dfrac{Y(S)}{F(S)}=\dfrac{1}{\Delta}\sum\limits_{K=1}^{2}P_K\Delta_K=\dfrac{G_2-G_1G_2H_1}{1-G_2H_2+G_1G_2H_3}$

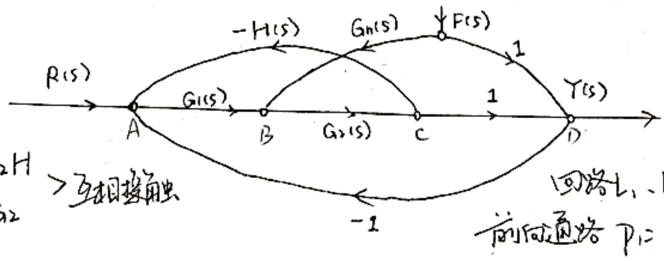


(b)

题 2.8 图

2.11(b)

求 $\frac{Y(s)}{R(s)}$



回路 $L_1 = -G_1G_2H$
$L_2 = -G_1G_2$ > 互相接触

$\Delta = 1 - L_1 - L_2$

前向通路 $P_1 = G_1G_2$ 与 L_1、L_2 均接触

$\Delta_1 = 1.$

$$\frac{Y(s)}{R(s)} = \frac{G_1G_2}{1+G_1G_2H+G_1G_2} \quad ①$$

回路 L_1、L_2 不变

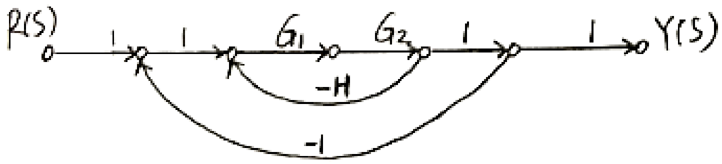
前向通路 $P_1 = 1$; $P_2 = G_nG_2$.

$\Delta = 1 - L_1 - L_2$

$\Delta_1 = 1 - L_1$, $\Delta_2 = 1$

$$\therefore \frac{Y(s)}{F(s)} = \frac{1+G_1G_2H+G_nG_2}{1+G_1G_2H+G_1G_2}$$

2.11(b). 求 $\frac{Y(S)}{R(S)}$. 信号流图如下:



信号流图中共有一条前向通路，增益 $P_1 = G_1G_2$.

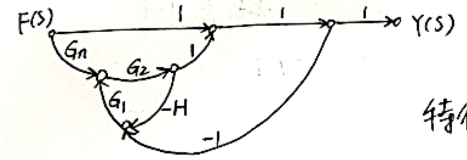
共有两条回路. 回路增益为 $L_1 = -G_1G_2H$, $L_2 = -G_1G_2$

特征式 $\Delta = 1 + G_1G_2 + G_1G_2H$

$\Delta_1 = 1$, $\Delta_2 = 1$

$$故 \quad \frac{Y(S)}{R(S)} = \frac{1}{\Delta}\sum_{k=1}^{2} P_k\Delta_k = \frac{G_1G_2}{1+G_1G_2+G_1G_2H}$$

2.11(b) 求 $\frac{Y(S)}{F(S)}$. 信号流图如下:



信号流图中共有两条前向通路 $P_1=1$, $P_2=-G_nG_2$

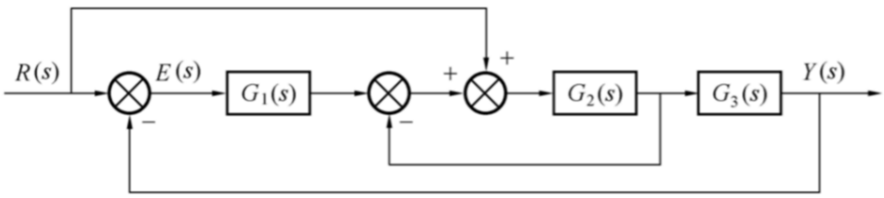
共有2条回路. 回路增益 $L_1=-G_2G_1$, $L_2=-G_1G_2H$

特征式为 $\Delta=1+G_1G_2+G_1G_2H$

$\Delta_1=1-L_2=1+G_1G_2H$, $\Delta_2=1$

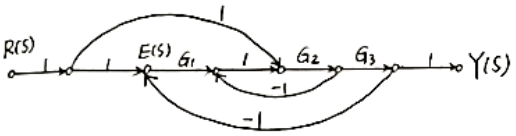
$$\therefore \frac{Y(S)}{F(S)}=\frac{1+G_nG_2+G_1G_2H}{1+G_1G_2+G_1G_2H}$$

2.12　画出题 2.9 图所示系统的信号流图，用梅森公式求 $\frac{Y(s)}{R(s)}$ 及 $\frac{E(s)}{R(s)}$。



(a)

2.12(a). 求$\frac{Y(S)}{R(S)}$. 信号流图如下:



信号流图中共有两条前向通路 $P_1=G_1G_2G_3$

$P_2=G_2G_3$

共有2条回路 $L_1=-G_2$

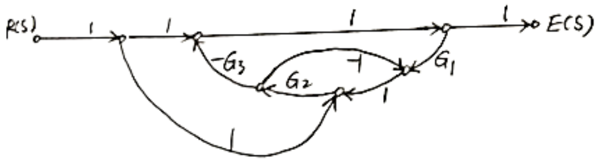
$L_2=-G_1G_2G_3$

特征式为 $\Delta=1+G_2+G_1G_2G_3$

$\Delta_1=1$, $\Delta_2=1$

$$\therefore \frac{Y(S)}{R(S)}=\frac{1}{\Delta}\sum_{k=1}^{2}P_k\Delta_k=\frac{G_2G_3+G_1G_2G_3}{1+G_2+G_1G_2G_3}$$

求 $\frac{E(S)}{R(S)}$. 信号流图如下:



信号流图中共有两条前向通路: $P_1=1$, $P_2=-G_2G_3$.

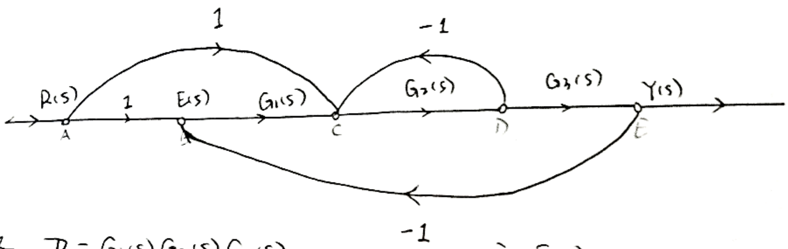
共有两个回路: $L_1=-G_2$, $L_2=-G_1G_2G_3$

特征式 $\Delta=1+G_2+G_1G_2G_3$

$\Delta_1=1+G_2$, $\Delta_2=1$

$$\therefore \frac{E(S)}{R(S)}=\frac{1+G_2-G_2G_3}{1+G_2+G_1G_2G_3}$$

2.12 (a)



求 $\frac{Y(s)}{R(s)}$

前向通路 $P_1=G_1(s)G_2(s)G_3(s)$

$P_2=G_2(s)G_3(s)$

回路 $L_1=-G_2(s)$

$L_2=-G_1(s)G_2(s)G_3(s)$ 有接触

故 $\Delta=1-L_1-L_2$

$\Delta_1=1$, $\Delta_2=1$

故 $$\frac{Y(s)}{R(s)}=\frac{G_1G_2G_3+G_2G_3}{1+G_2+G_1G_2G_3}$$

求 $\frac{E(s)}{R(s)}$

前向通路 $P_1=1$

$P_2=-G_2G_3$

回路 $L_1=-G_2$

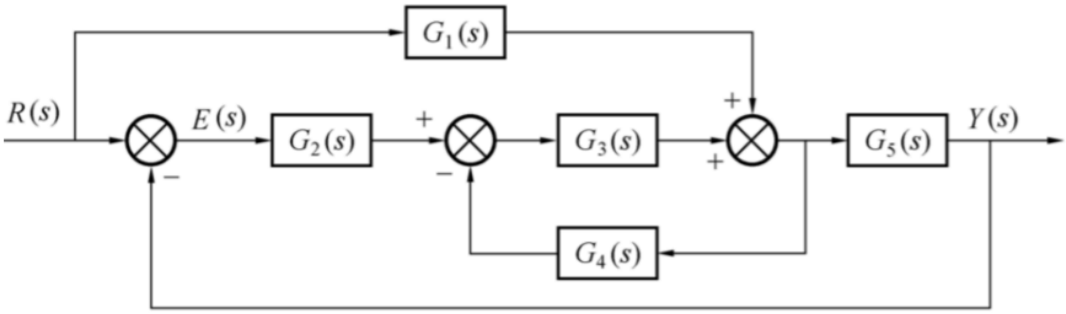
$L_2=-G_1G_2G_3$

$\Delta=1-L_1-L_2$

$\Delta_1=1-L_1$, $\Delta_2=1$

$$\therefore \frac{E(s)}{R(s)}=\frac{1+G_2-G_2G_3}{1+G_2+G_1G_2G_3}$$

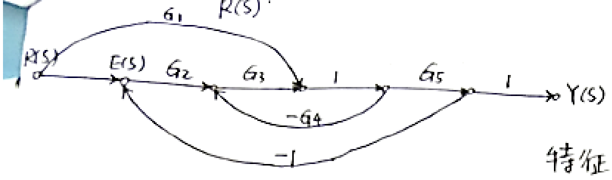
G1(s)



(b)

题 2.9 图

12(b) 求 $\frac{Y(S)}{R(S)}$:



共有两个前向通路: $P_1=G_2G_3G_5$, $P_2=G_1G_5$

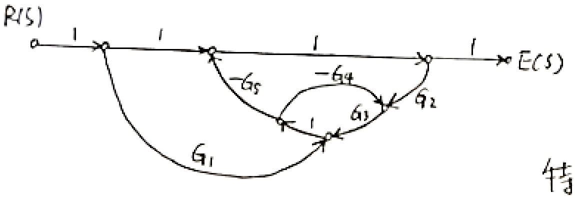
共有两个回路: $L_1=-G_3G_4$
$L_2=-G_2G_3G_5$

特征式 $\Delta=1+G_3G_4+G_2G_3G_5$

$\Delta_1=1$, $\Delta_2=1$

$\therefore \frac{Y(S)}{R(S)}=\frac{G_2G_3G_5+G_1G_5}{1+G_3G_4+G_2G_3G_5}$

求 $\frac{E(S)}{R(S)}$:



共有2条前向通路 $P_1=1$, $P_2=-G_1G_5$

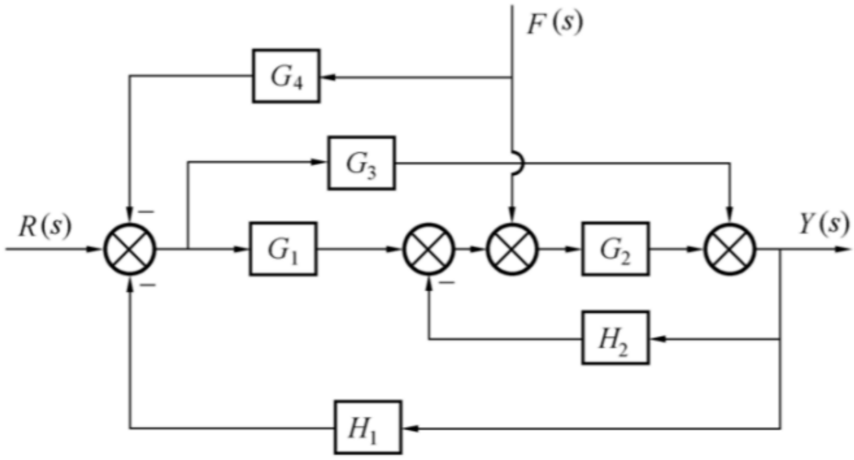
共有2个回路 $L_1=-G_2G_3G_5$, $L_2=-G_3G_4$

特征式 $\Delta=1+G_3G_4+G_2G_3G_5$

$\Delta_1=1+G_3G_4$, $\Delta_2=1$

$\therefore \frac{E(S)}{R(S)}=\frac{1+G_3G_4-G_1G_5}{1+G_3G_4+G_2G_3G_5}$

2.13　化简方框图，求$\dfrac{Y(s)}{R(s)}$及$\dfrac{Y(s)}{F(s)}$。

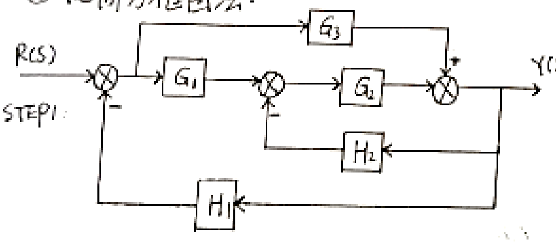


题 2.13 图

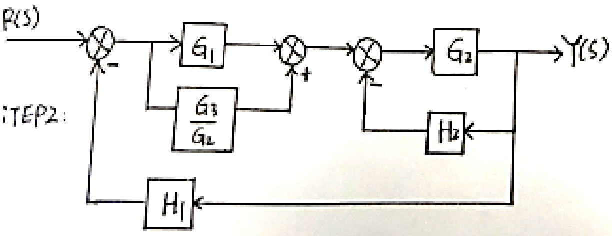
2.13 求 $\dfrac{Y(s)}{R(s)}$，令$F(s)=0$

①化简方框图法：

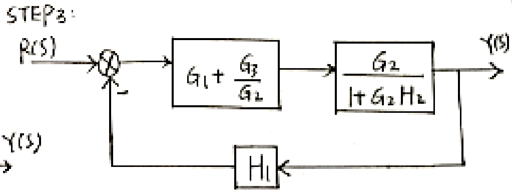
STEP1:



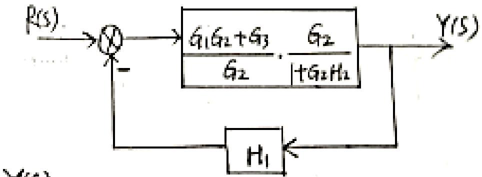
STEP2:



STEP3:



STEP4:

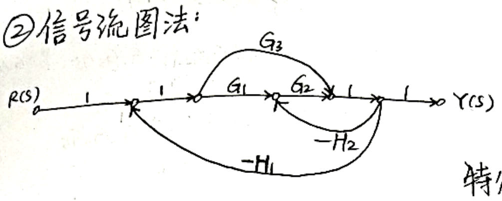


$$\therefore \frac{Y(s)}{R(s)} = \frac{\dfrac{G_1G_2+G_3}{1+G_2H_2}}{1+\dfrac{G_1G_2H_1+G_3H_1}{1+G_2H_2}}$$

$$= \frac{G_1G_2+G_3}{1+G_2H_2+G_3H_1+G_1G_2}$$

page3

②信号流图法：



共有两条前向通路：$P_1=G_1G_2$，$P_2=G_3$

共有三个回路：$L_1=-G_2H_2$，

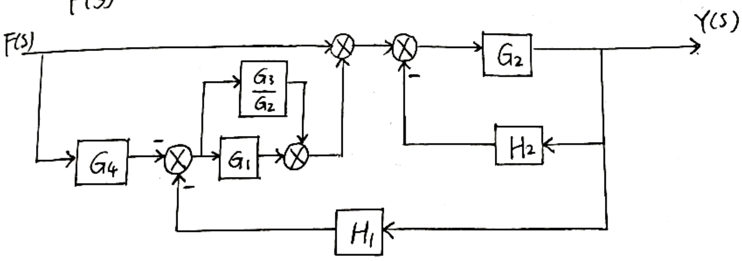
$L_2=-G_1G_2H_1$，$L_3=-G_3H_1$

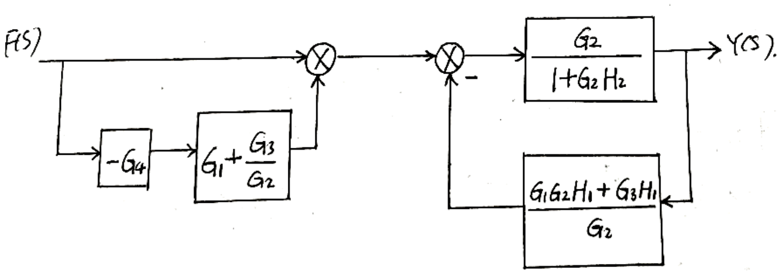
特征式 $\Delta=1+H_2G_2+G_1G_2H_1+G_3H_1$

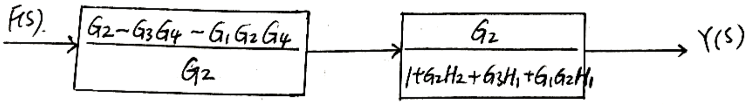
$\Delta_1=1$，$\Delta_2=1$

$$\therefore \frac{Y(s)}{R(s)}=\frac{1}{\Delta}\sum_{k=1}^{3}P_k\Delta_k=\frac{G_1G_2+G_3}{1+G_2H_2+G_1G_2H_1+G_3H_1}$$

求 $\frac{Y(s)}{F(s)}$ ①化简方框图法：

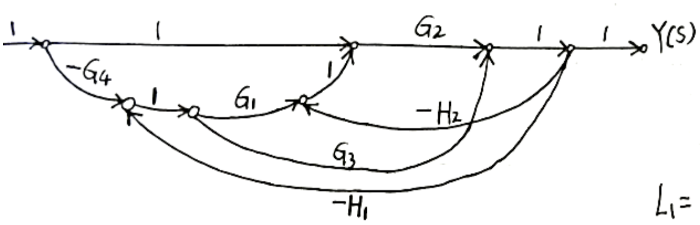






$$\therefore \frac{Y(s)}{R(s)}=\frac{G_2-G_3G_4-G_1G_2G_4}{1+G_2H_2+G_3H_1+G_1G_2H_1}$$

信号流图法：



信号流图中共有三条前向通路

共有三个回路

$P_1=G_2$，$P_2=-G_4G_1G_2$，

$P_3=-G_4G_3$

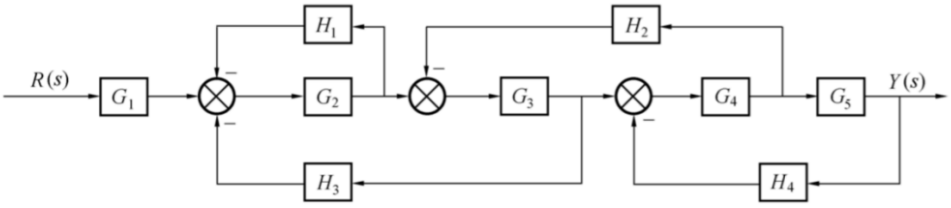
$L_1=-G_2H_2$，$L_2=-G_2H_1G_1$，$L_3=-H_1G_3$

特征式 $\Delta=1+G_2H_2+G_1G_2H_1+G_3H_1$

$\Delta_1=1$，$\Delta_2=1$，$\Delta_3=1$

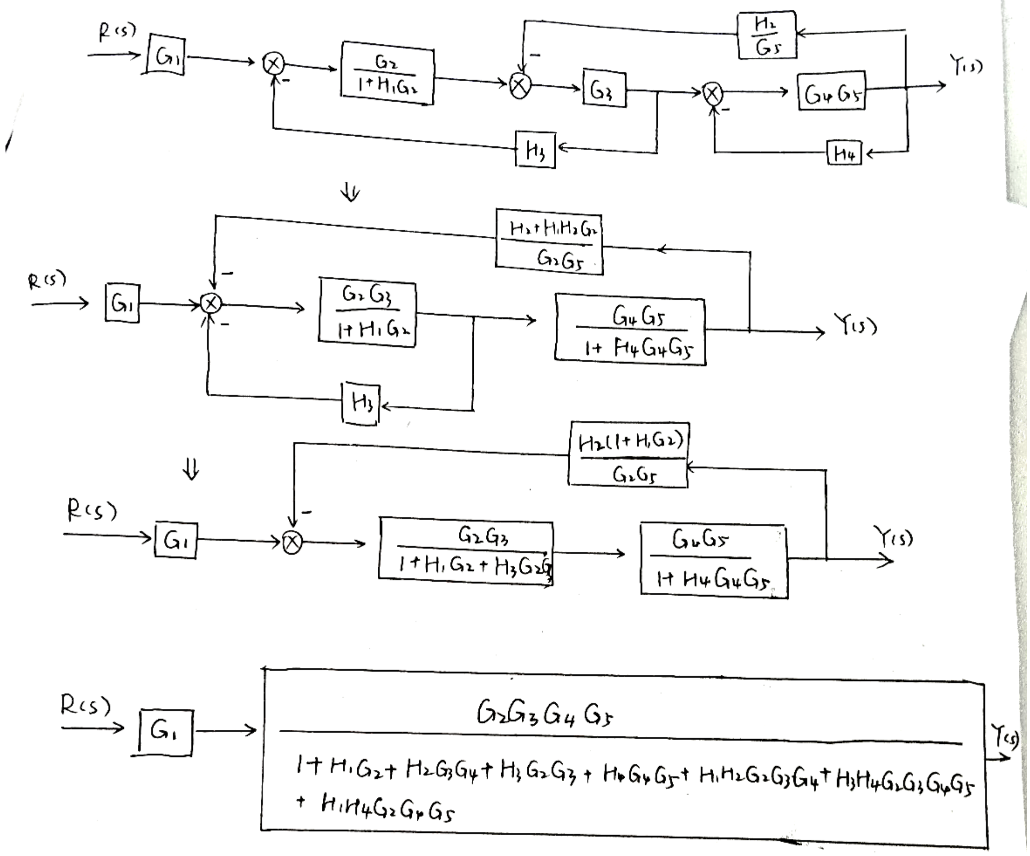
$$\therefore \frac{Y(s)}{F(s)}=\frac{1}{\Delta}\sum_{k=1}^{3}P_k\Delta_k=\frac{G_2-G_1G_2G_4-G_3G_4}{1+G_2H_2+G_1G_2H_1+G_3H_1}$$

2.16　已知系统方框图如题 2.16 图所示，试分别用方框图化简规则和信号流图的梅森公式求系统传递函数$\dfrac{Y(s)}{R(s)}$。



题 2.16 图

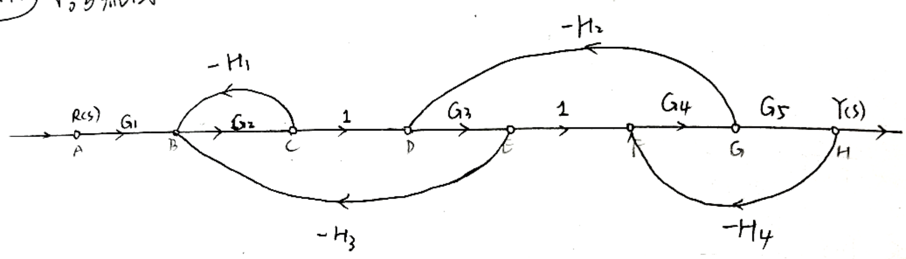
2.16 方框图化简



故 $\dfrac{Y(s)}{R(s)}$

$$= \frac{G_1G_2G_3G_4G_5}{1+H_1G_2+H_2G_3G_4+H_3G_2G_3+H_4G_4G_5+H_1H_2G_2G_3G_4+H_3H_4G_2G_3G_4G_5+H_1H_4G_2G_4G_5}$$

2.16 信号流图



前向通路 $P_1 = G_1 G_3 G_4 G_5 G_2$

回路 $L_1 = -H_1 G_2$

$L_2 = -G_3 G_4 H_2$

$L_3 = -H_3 G_3 G_2$

$L_4 = -H_4 G_4 G_5$

$\Delta = 1 - L_1 - L_2 - L_3 - L_4 + L_1 L_2 + L_3 L_4 + L_1 L_4$

$\Delta_1 = 1$

$$\therefore \frac{Y(s)}{R(s)} = \frac{G_1 G_3 G_4 G_5 G_2}{(1 + G_2 H_1 + G_3 G_4 H_2 + H_3 G_3 G_2 + H_4 G_4 G_5 + H_1 H_2 G_2 G_3 G_4 + H_3 H_4 G_3 G_4 G_5 G_2 + H_1 H_4 G_4 G_5 G_2)}$$

本次作业中存在的问题：

【1】化简。每道题求什么就应该得到一个相应的表达式，不要用文字来描述。要注意答题规范的问题。

【2】信号流图画法不熟。相邻的几个相加点、或是相邻的相加点和分支点，在判断回路时，都是不可以视作同一个节点的。有些同学对梅森公式的使用还不是很清楚。应用公式出错