自动控制理论 A 作业 5

2019 年 10 月 10 日

6.4 信号拉氏变换式如下，求其对应的 Z 变换。

(1) $E(s) = \dfrac{1}{(s+a)(s+b)}$

(2) $E(s) = \dfrac{K}{s(s+a)}$

(3) $E(s) = \dfrac{s+1}{s^2}$

解

6.4 解：(1) $E(s) = \dfrac{1}{(s+a)(s+b)}$

$= \dfrac{1}{b-a}\left(\dfrac{1}{s+a} - \dfrac{1}{s+b}\right)$

作拉氏反变换得 $e(t) = \dfrac{1}{b-a}(e^{-at} - e^{-bt})$

$Z[e^{-at}] = \dfrac{z}{z-e^{-aT}}$

$Z[e^{-bt}] = \dfrac{z}{z-e^{-bT}}$

故 $E(z) = \dfrac{1}{b-a}\left(\dfrac{z}{z-e^{-aT}} - \dfrac{z}{z-e^{-bT}}\right) = \dfrac{(e^{-aT}-e^{-bT})z}{(b-a)[z^2-(e^{-aT}+b^{-bT})z+e^{-(a+b)T}]}$

T

(2) $E(s) = \dfrac{K}{s(s+a)} = \dfrac{K}{a}\cdot\dfrac{a}{s(s+a)} = \dfrac{K}{a}\left(\dfrac{1}{s} - \dfrac{1}{s+a}\right)$

作拉氏反变换得 $e(t) = \dfrac{K}{a}[1(t) - e^{-at}]$

$Z[1(t)] = \dfrac{z}{z-1}$

$Z[e^{-at}] = \dfrac{z}{z-e^{-aT}}$

故 $E(z) = \dfrac{K}{a}\left(\dfrac{z}{z-1} - \dfrac{z}{z-e^{-aT}}\right)$

$= \dfrac{Kz(1-e^{-aT})}{az^2-(1+e^{-aT})az+ae^{-aT}}$

(3) $E(s) = \dfrac{s+1}{s^2} = \dfrac{1}{s} + \dfrac{1}{s^2}$

作拉氏反变换得 $e(t) = 1(t) + t$

$Z[1(t)] = \dfrac{z}{z-1}$

$Z[t] = \dfrac{Tz}{(z-1)^2}$

故 $E(z) = \dfrac{z}{z-1} + \dfrac{Tz}{(z-1)^2} = \dfrac{z^2+(T-1)z}{(z-1)^2}$

6.6　求下列各式的 Z 反变换。

(2) $X(z)=\dfrac{z}{(z-1)(z-2)}$

(4) $X(z)=\dfrac{z}{(z-1)^2(z-2)^2}$

(6) $X(z)=\dfrac{z^3+2z^2+1}{z^3-1.5z^2+0.5z}$

(2) 解:

$$X(z)=\frac{z}{z-2}-\frac{z}{z-1}$$

查表得:

$$x(k)=2^k-1$$

$$x^*(t)=\sum_{k=0}^{\infty}(2^k-1)\delta(t-kT)$$

(4) 解:

- 极点$z_1=1$, $z_2=2$, 均为二阶极点，则

$$x(kT)=\frac{1}{(2-1)!}\frac{\mathrm{d}}{\mathrm{d}z}\left[\frac{z}{(z-2)^2}z^{k-1}\right]_{z=1}+\frac{1}{(2-1)!}\frac{\mathrm{d}}{\mathrm{d}z}\left[\frac{z}{(z-1)^2}z^{k-1}\right]_{z=2}$$
$$=-2^{k+1}+k\cdot 2^{k-1}+k+2$$

$$k=0,1,2,\cdots$$

- 故

$$x^*(t)=\sum_{k=0}^{\infty}\left(-2^{k+1}+k\cdot 2^{k-1}+k+2\right)\delta(t-kT)$$

(6) 解:

$$X(z)=\frac{z^3+2z^2+1}{z(z-0.5)(z-1)}$$

- 极点$z_1=1$, $z_2=0.5, z_3=0$, 均为一阶极点

$$x(kT)=\frac{z^3+2z^2+1}{(z-1)(z-0.5)}z^{k-1}\Big|_{z=0}+\frac{z^3+2z^2+1}{z(z-0.5)}z^{k-1}\Big|_{z=1}+\frac{z^3+2z^2+1}{z(z-1)}z^{k-1}\Big|_{z=0.5}$$
$$=8-13\cdot 2^{-k}$$

$$k=2,3,4,\cdots$$

当k=0,1时，由长除法可得系数.

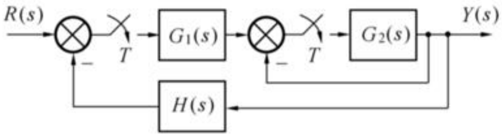
- 故

$$x^*(t)=\sum_{k=2}^{\infty}\left[8-13\cdot 2^{-k}\right]\delta(t-kT)+\delta(t)+3.5\delta(t-T)$$

注意此处的第一项,即 z=0 时式子里的z^{k-1}，由于 0 的非正数次方没有意义，所以 k=0 和 k=1 要单独拿出来算，而不能使用留数法。故此，用长除法算出第 1、2 项系数，k>2 时的项不受影响可用留数法表示。

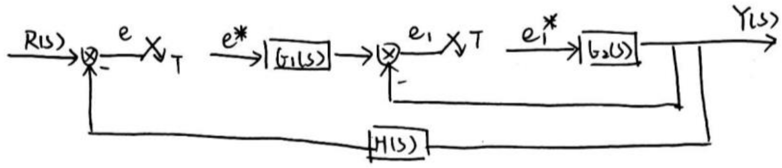
（这里我们认为 0 的 0 次方也没有意义。并且用长除法验算，k=1 时的系数与用留数法表示的不一样。）

6.10　求题 6.10 图所示系统的闭环脉冲传递函数。



题 6.10 图　闭环系统

6.10 解:



$Y(z) = E_1(z)G_2(z)$　　　$E_1(z) = \frac{Y(z)}{G_2(z)}$

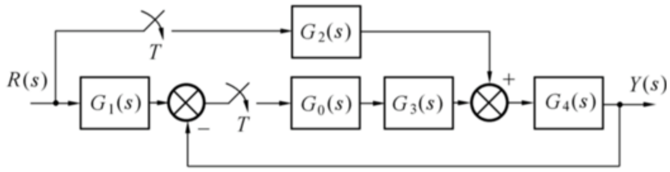
$E_1(z) = E(z)G_1(z) - E_1(z)G_2(z)$　　　$E_1(z) = \frac{G_1(z)}{1+G_2(z)}E(z)$

$E(z) = R(z) - HG_2(z)E_1(z)$　　　$\therefore Y(z) = \frac{G_1(z)G_2(z)}{1+G_2(z)}E(z)$

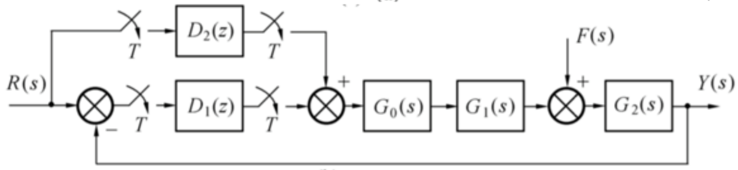
$E(z)\left[1 + HG_2(z)\frac{G_1(z)}{1+G_2(z)}\right] = R(z)$

$$\frac{Y(z)}{R(z)} = \frac{G_1(z)G_2(z)}{1+G_2(z)+HG_2(z)G_1(z)}$$

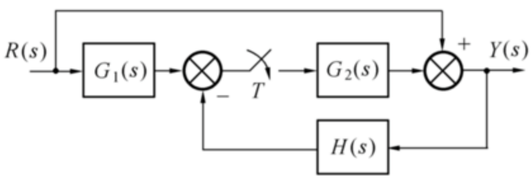
6.11　求题 6.11 图所示系统输出的 Z 变换 $Y(z)$。



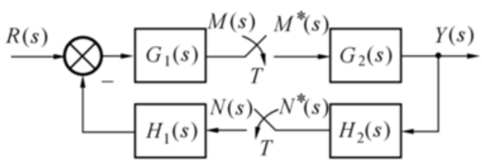
(a)



(b)



(e)



(f)

6.11 解：(a) $$Y(s)=\frac{R(s)G_1(s)G_0(s)G_3(s)G_4(s)+R(s)G_2(s)G_4(s)}{1+G_0(s)G_3(s)G_4(s)}$$

$$Y^*(s)=\frac{RG_1^*(s)G_0G_3G_4^*(s)+R^*(s)G_2G_4^*(s)}{1+G_0G_3G_4^*(s)}$$

$$Y(z)=\frac{RG_1(z)G_0G_3G_4(z)+R(z)G_2G_4(z)}{1+G_0G_3G_4(z)}$$

(b) 当 $F(s)=0$ 时 $$Y(s)=\frac{R(s)\left[D_1(s)+D_2(s)\right]G_0(s)G_1(s)G_2(s)}{1+D_1(s)G_0(s)G_1(s)G_2(s)}$$

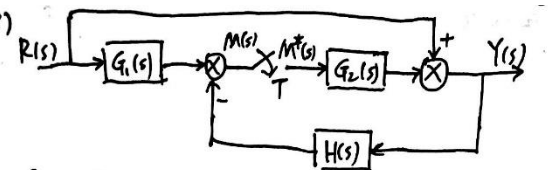
$R(s)=0$ 时 $$Y(s)=\frac{F(s)G_2(s)}{1+D_1(s)G_0(s)G_1(s)G_2(s)}$$

$$Y(s)=\frac{R(s)\left[D_1(s)+D_2(s)\right]G_0(s)G_1(s)G_2(s)+F(s)G_2(s)}{1+D_1(s)G_0(s)G_1(s)G_2(s)}$$

取样 $$Y^*(s)=\frac{R^*(s)\left[D_1^*(s)+D_2^*(s)\right]G_0G_1G_2^*(s)+FG_2^*(s)}{1+D_1^*(s)G_0G_1G_2^*(s)}$$

$$Y(z)=\frac{R(z)\left[D_1(z)+D_2(z)\right]G_0G_1G_2(z)+FG_2(z)}{1+D_1(z)G_0G_1G_2(z)}$$

(c)



$$\begin{cases} M^*(s)G_2(s)+R(s)=Y(s) \\ M(s)=G_1(s)R(s)-H(s)Y(s) \end{cases}$$

$\xRightarrow{\text{消去}Y(s)}$ $M(s)=G_1(s)R(s)-H(s)R(s)-H(s)M^*(s)G_2(s)$

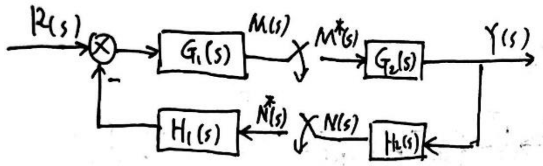
$$M(z)=G_1R(z)-HR(z)-G_2H(z)\cdot M(z)$$

$$M(z)=\frac{G_1R(z)-HR(z)}{1+G_2H(z)}$$

$$Y(z)=M(z)G_2(z)+R(z)$$

$$=\frac{\left[G_1R(z)-HR(z)\right]G_2(z)}{1+G_2H(z)}+R(z)$$

(f)



$$\begin{cases} M(s) = \left[R(s) - N^*(s) H_1(s)\right] G_1(s) \\ N(s) = Y(s) H_2(s) \\ Y(s) = G_2(s) M^*(s) \end{cases}$$

$$\Rightarrow \begin{cases} M(z) = G_1R(z) - H_1G_1(z) N(z) \\ N(z) = G_2H_2(z) M(z) \\ Y(z) = G_2(z) M(z) \end{cases}$$

$$M(z) = \frac{G_1R(z)}{1 + G_1H_1(z) \cdot G_2H_2(z)}$$

$$Y(z) = G_2(z) \cdot \frac{G_1R(z)}{1 + G_1H_1(z) \cdot G_2\cancel{H_2}(z)}$$