8.13　设描述线性定常离散系统的差分方程为

$$y(k+2)+3y(k+1)+2y(k)=u(k)$$

试选取

$$x_1(k)=y(k)$$
$$x_2(k)=y(k+1)$$

为一组状态变量，写出该系统的状态方程，并求其解。已知 $u(t)=1(t)$。

作业7.

$\boxed{8\text{-}13}$

解：状态方程 $\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$

设初始值为 $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix}$

$$X(z)=(zI-G)^{-1}zX(0)+(zI-G)^{-1}HU(z)$$

$$zI-G=\begin{bmatrix} z & -1 \\ 2 & z+3 \end{bmatrix} \qquad (zI-G)^{-1}=\begin{bmatrix} z+3 & 1 \\ -2 & z \end{bmatrix}\cdot\frac{1}{(z+1)(z+2)}$$

$$X(z)=(zI-G)^{-1}[zX(0)+HU(z)]=\frac{1}{(z+1)(z+2)}\cdot\begin{bmatrix} z+3 & 1 \\ -2 & z \end{bmatrix}\cdot\left\{z\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix}+\begin{bmatrix} 0 \\ 1 \end{bmatrix}\cdot\frac{z}{z-1}\right\}$$

$$=\frac{1}{(z+1)(z+2)}\begin{bmatrix} z(z+3)x_1(0)+zx_2(0)+\frac{z}{z-1} \\ -2zx_1(0)+z^2x_2(0)+\frac{z^2}{z-1} \end{bmatrix}$$

$$X(k)=Z^{-1}[X(z)]=\begin{bmatrix} \left[2x_1(0)+x_2(0)-\frac{1}{2}\right](-1)^k+\left[-x_1(0)-x_2(0)+\frac{1}{3}\right](-2)^k+\frac{1}{6} \\ \left[-2x_1(0)-x_2(0)+\frac{1}{2}\right](-1)^k+\left[2x_1(0)-x_2(0)-\frac{2}{3}\right](-2)^k+\frac{1}{6} \end{bmatrix}$$

8.16　试求取下列状态方程的离散化方程。

(1) $\dot{\boldsymbol{x}}=\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\boldsymbol{x}+\begin{bmatrix} 0 \\ 1 \end{bmatrix}u$　　　　(2) $\dot{\boldsymbol{x}}=\begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}\boldsymbol{x}+\begin{bmatrix} 0 \\ 1 \end{bmatrix}u$

$\boxed{8-16}$ 解: $G = e^{AT}$ $H = \int_0^T e^{At}\,dt\ B$

(1) $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ $e^{At} = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\int_0^T e^{At}\,dt = \begin{bmatrix} T & \frac{1}{2}T^2 \\ 0 & T \end{bmatrix}$ $G = e^{AT} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$

$H = (\int_0^T e^{AT}\,dt)B = \begin{bmatrix} \frac{1}{2}T^2 \\ T \end{bmatrix}$

$X(k+1) = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} X(k) + \begin{bmatrix} \frac{1}{2}T^2 \\ T \end{bmatrix} u$

(2) $(sI - A)^{-1} = \begin{bmatrix} s & -1 \\ 0 & s+2 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{s} & \frac{1}{s(s+2)} \\ 0 & \frac{1}{s+2} \end{bmatrix}$

$e^{At} = \mathcal{L}^{-1}[(sI - A)^{-1}] = \begin{bmatrix} 1 & \frac{1}{2} - \frac{1}{2}e^{-2t} \\ 0 & e^{-2t} \end{bmatrix}$

$G = e^{AT} = \begin{bmatrix} 1 & \frac{1}{2} - \frac{1}{2}e^{-2T} \\ 0 & e^{-2T} \end{bmatrix}$

$H = (\int_0^T e^{At}\,dt)B = \int_0^T (e^{At}B)\,dt = \begin{bmatrix} \frac{1}{2}T + \frac{1}{4}e^{-2T} - \frac{1}{4} \\ -\frac{1}{2}e^{-2T} + \frac{1}{2} \end{bmatrix}$

$\therefore X(k+1) = \begin{bmatrix} 1 & \frac{1}{2} - \frac{1}{2}e^{-2T} \\ 0 & e^{-2T} \end{bmatrix} X(k) + \begin{bmatrix} \frac{1}{2}T + \frac{1}{4}e^{-2T} - \frac{1}{4} \\ -\frac{1}{2}e^{-2T} + \frac{1}{2} \end{bmatrix} u$