

4.(i) 世界坐标系与末端坐标系定义如图 1。机器人 (ii)(iii) 中世界坐标系与末端坐标系的方向定义与 (i) 相同。正运动学代码如下:

```

1 syms t1 t2 t3 t4 t5 t6 % 关节角度
2 th = [t1, t2, t3, t4, t5, t6];
3
4 % 定义各关节参数
5 syms h l1 l2
6 q = [0 0 0 0 0 0; ...
7       0 0 l1 l1+l2 l1+l2 l1+l2; ...
8       h h h h h h];
9 w = [0 -1 -1 -1 0 0; ...
10      0 0 0 0 0 1; ...
11      1 0 0 0 1 0];
12
13 for i=1:6
14     v(:,i) = cross(q(1:3,i), w(1:3,i));
15 end
16
17 for i=1:6
18     w_hat = [ 0 -w(3,i) w(2,i); ...
19              w(3,i) 0 -w(1,i); ...
20             -w(2,i) w(1,i) 0];
21     ew = simplify(eye(3) + w_hat*sin(th(i)) + w_hat^2*(1-cos(th(i))));
22     e(:, :, i) = [ew simplify((eye(3)-ew)*w_hat*v(:,i)); 0 0 0 1];
23 end
24
25 gst0 = [ 0 0 1 0; ...
26          0 1 0 l1+l2; ...
27          -1 0 0 h; ...
28          0 0 0 1]; % 初始位姿,
29
30 gst = gst0;
31 for i=6:-1:1
32     gst = e(:, :, i)*gst;
33 end
34 simplify(gst);

```

得  $g_{st}(\theta) =$

$$\begin{bmatrix}
 s_1(s_5s_6c_{234} + s_{234}c_6) - c_1c_5s_6 & -s_1c_5c_{234} - c_1s_5 & -s_1(-s_5c_6c_{234} + s_{234}s_6) + c_1c_5c_6 & -s_1(l_1c_2 + l_2c_{23}) \\
 -c_1(s_5s_6c_{234} + s_{234}c_6) - s_1c_5s_6 & c_1c_5c_{234} - s_1s_5 & c_1(s_5c_6c_{234} - s_{234}s_6) + s_1c_5c_6 & c_1(l_1c_2 - l_2c_{23}) \\
 s_5s_6s_{234} - c_{234}c_6 & -s_{234}c_5 & -s_5c_6s_{234} - c_{234}s_6 & h - l_1s_2 - l_2s_{23} \\
 0 & 0 & 0 & 1
 \end{bmatrix}$$

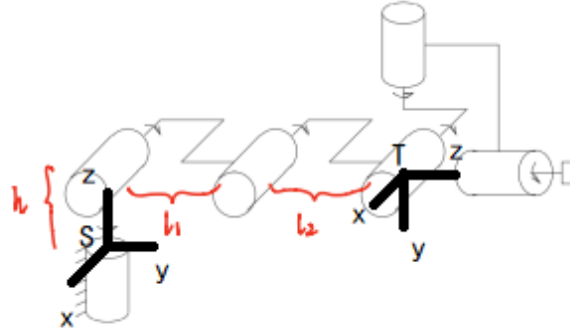


图 1: Elbow 坐标系定义

(ii) 修改机器人参数如下:

```

1  q = [0 0 0 0 0 0; ...
2      0 0 0 l1 l1+l2 l1+l2; ...
3      h h h h h h];
4  w = [0 0 -1 -1 -1 0; ...
5      0 1 0 0 0 1; ...
6      1 0 0 0 0 0];

```

得  $g_{st}(\theta) =$

$$\begin{bmatrix} -(s_2c_6c_{345} + s_2c_6)c_1 + s_1c_6s_{345} & -s_1c_{345} - c_1s_2s_{345} & (-s_2s_6c_{345} + c_2c_6)c_1 + s_1s_6s_{345} & -l_1(c_1s_2s_3 + s_1c_3) - l_2(s_1c_3 + c_1s_2s_{34}) \\ -(s_2c_6c_{345} + c_2s_6)s_1 - c_1c_6s_{345} & c_1c_{345} - s_1s_2s_{345} & (-s_2s_6c_{345} + c_2c_6)s_1 - c_1s_6s_{345} & l_1(c_1c_3 - s_1s_2s_3) + l_2(c_1c_3 - s_1s_2s_{34}) \\ s_2s_6 - c_2c_6c_{345} & -c_2s_{345} & -s_2c_6 - s_6c_2c_{345} & h - c_2(l_1s_3 + l_2s_{34}) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(iii) 对代码进行修改以实现平移关节:

```

1  syms t1 t2 t3 t4 t5 t6 % 关节角度
2  th = [t1, t2, t3, t4, t5, t6];
3
4  % 关节类型, 1为旋转
5  rot = [1, 1, 0, 1, 1, 1];
6
7  % 定义各关节参数
8  % 对平移关节, 将v写在q中, w取[0;0;0]
9  % 对旋转关节, q与w正常赋值
10 syms h l1 l2
11 q = [0 0 0 0 0 0; ...
12      0 0 0 l1+l2 l1+l2 l1+l2; ...
13      h h 1 h h h];
14 w = [0 -1 0 -1 0 0; ...
15      0 0 0 0 0 1; ...
16      1 0 0 0 1 0];
17
18 for i=1:6
19     if rot(i) == 1
20         v(:,i) = cross(q(1:3,i), w(1:3,i));
21     else
22         v(:,i) = q(:,1);
23     end

```

```

24     end
25
26     for i=1:6
27         if rot(i) == 1
28             w_hat = [ 0 -w(3,i) w(2,i); ...
29                     w(3,i) 0 -w(1,i); ...
30                     -w(2,i) w(1,i) 0];
31             ew = simplify(eye(3) + w_hat*sin(th(i)) + w_hat^2*(1-cos(th(i))));
32             e(:, :, i) = [ew simplify((eye(3)-ew)*w_hat*v(:, i)); 0 0 0 1];
33         else
34             e(:, :, i) = [eye(3) q(1:3,i)*th(i); 0 0 0 1];
35         end
36     end
37
38     gst0 = [ 0 0 1 0; ...
39             0 1 0 l1+l2; ...
40             -1 0 0 h; ...
41             0 0 0 1]; % 初始位姿
42
43     gst = gst0;
44     for i=6:-1:1
45         gst = e(:, :, i)*gst;
46     end
47     simplify(gst);

```

得  $g_{st}(\theta) =$

$$\begin{bmatrix}
 (s_5 s_6 c_{24} + c_6 s_{24}) s_1 - c_1 c_5 s_6 & -c_1 s_5 - s_1 c_5 c_{24} & (-s_5 c_6 c_{24} + s_6 s_{24}) s_1 + c_1 c_5 c_6 & -(l_1 c_2 + l_2 c_{24} + \theta_3 c_2) s_1 \\
 -(s_5 s_6 c_{24} + c_6 s_{24}) c_1 - s_1 c_5 s_6 & -s_1 s_5 + c_1 c_5 c_{24} & (s_5 c_6 c_{24} - s_6 s_{24}) c_1 + s_1 c_5 c_6 & (l_1 c_2 + l_2 c_{24} + \theta_3 c_2) c_1 \\
 s_5 s_6 s_{24} - c_6 c_{24} & -s_{24} c_5 & -s_5 c_6 s_{24} - s_6 c_{24} & h - l_1 s_2 - l_2 s_{24} - \theta_3 s_2 \\
 0 & 0 & 0 & 1
 \end{bmatrix}$$

5. 定义子问题求解函数:

```

1  function Subs = add % add为文件名
2      Subs.Sub1 = @Sub1;
3      Subs.Sub2 = @Sub2;
4      Subs.Sub3 = @Sub3;
5  end
6
7  % e^{\hat{xi}\theta} p = q, r \ xi
8  % p, q, r = [x;y;z;1]
9  % xi = [v w]^T R6
10 function theta = Sub1(p, q, r, xi)
11     u = p - r; v = q - r; % [R3; 0]
12     w = xi(4:6); % R3
13     u = u(1:3); v = v(1:3); % conv [R3;0] into R3
14     u1 = u - w * w' * u; v1 = v - w * w' * v;
15     theta = atan2(w'*cross(u1, v1), u1'*v1);
16 end
17
18 % r xi1, xi2
19 % solve=0/1
20 function [theta1, theta2] = Sub2(p, q, r, xi1, xi2, solve)

```

```

21 u = p - r; v = q - r; % [R3; 0]
22 u = u(1:3); v = v(1:3);
23 w1 = xi1(4:6); w2 = xi2(4:6); % R3
24 a = ((w1'*w2)*w2'*u - w1'*v) / ((w1'*w2)^2 - 1);
25 b = ((w1'*w2)*w1'*v - w2'*u) / ((w1'*w2)^2 - 1);
26 y = sqrt((norm(u)^2 - a^2 - b^2 - 2*a*b*w1'*w2) / (norm(cross(w1, w2))^2));
27 if solve == 0
28     y = -y; % change the solve
29 end
30 z = a*w1 + b*w2 + y*cross(w1,w2);
31 c = [z; 0] + r; % [R3; 0]
32 subs = add;
33 theta2 = subs.Sub1(p, c, r, xi2);
34 theta1 = subs.Sub1(c, q, r, xi1);
35 end
36
37 function theta = Sub3(p, q, r, xi, dis, solve)
38     u = p - r; v = q - r; % [R3; 0]
39     w = xi(4:6); % R3
40     u = u(1:3); v = v(1:3); % conv [R3;0] into R3
41     u1 = u - w * w' * u; v1 = v - w * w' * v;
42     dis1_2 = dis^2 - norm(w*(p(1:3) - q(1:3)))^2;
43     theta0 = atan2( dot(w, cross(u1, v1)), dot(u1, v1) );
44
45     if solve ~= 0
46         theta = theta0 + acos((norm(u1)^2 + norm(v1)^2 - dis1_2) / (2 * norm(u1) *
47             norm(v1)));
48     else
49         theta = theta0 - acos((norm(u1)^2 + norm(v1)^2 - dis1_2) / (2 * norm(u1) *
50             norm(v1)));
51     end
52 end

```

(i) 对 Elbow 机器人:

令  $g_d = g_{st}(\theta) = e^{\hat{\xi}_1\theta_1} \dots e^{\hat{\xi}_6\theta_6} g_{st}(0), g_d g_{st}(0)^{-1} = g_1$ , 则取  $q_1 = \xi_1 \cap \xi_2$ ,  $p_3 = \xi_4 \cap \xi_5 \cap \xi_6$ , 此时  $g_1 p_3 = e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3} p_3$ ,

$\|g_1 p_3 - q_1\| = \|e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} (e^{\hat{\xi}_3\theta_3} p_3 - q_1)\| = \|e^{\hat{\xi}_3\theta_3} p_3 - q_1\|$ ,

利用 Paden-Kahan 子问题中 Subproblem 3(下文简称为 Sub 3) 可解出  $\theta_3$ 。

由  $e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} (e^{\hat{\xi}_3\theta_3} p_3) = g_1 p_3$ , 由 Sub 2 解出  $\theta_1, \theta_2$ 。

令  $g_2 = e^{\hat{\xi}_4\theta_4} e^{\hat{\xi}_5\theta_5} e^{\hat{\xi}_6\theta_6} = (e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3})^{-1} g_1$ , 取  $p_5 \in \xi_6, p_5 \notin \xi_4, \xi_5$ , 则  $e^{\hat{\xi}_4\theta_4} e^{\hat{\xi}_5\theta_5} p_5 = g_2 p_5$ , 由 Sub 2 解出  $\theta_4, \theta_5$ 。

令  $g_3 = (e^{\hat{\xi}_1\theta_1} \dots e^{\hat{\xi}_5\theta_5})^{-1} g_1$ , 取  $p_6 \notin \xi_6$ , 则  $g_3 p_6 = e^{\hat{\xi}_6\theta_6} p_6$ , 由 Sub 1 解出  $\theta_6$ 。

逆运动学代码:

```

1 subs = add;
2 % 定义机器人参数
3 h = 0.3;
4 l1 = 0.5; l2 = 0.5;
5 % 旋转轴与变换矩阵
6 rot = [1, 1, 1, 1, 1, 1];
7 q = [0 0 0 0 0 0; ...
8     0 0 l1 l1+l2 l1+l2 l1+l2; ...
9     h h h h h h];

```

```

10 w = [0 -1 -1 -1 0 0; ...
11       0 0 0 0 0 1; ...
12       1 0 0 0 1 0];
13 for i=1:6
14     if rot(i) == 1
15         v(:,i) = cross(q(1:3,i), w(1:3,i));
16     else
17         v(:,i) = q(:,1);
18     end
19 end
20 % 初始位姿
21 gst0 = [ 0 0 1 0; ...
22          0 1 0 l1+l2; ...
23          -1 0 0 h; ...
24          0 0 0 1];
25
26 gst = [ 0.3484 -0.9264 0.1425 -0.3867; ...
27         -0.9164 -0.3048 0.2594 0.8686; ...
28         -0.1969 -0.2210 -0.9552 0.2252; ...
29         0 0 0 1.0000];
30 % 选解(0-7)
31 for sol = 0:7
32
33     % 求解
34     p3 = [0; l1+l2; h; 1];
35     q1 = [0; 0; h; 1];
36     g1 = gst*gst0^(-1);
37     theta3 = subs.Sub3(p3, q1, [q(1:3,3); 1], [v(1:3,3);w(1:3,3)], norm(g1*p3-
38         q1), bitand(sol,4));
39
40     w_hat = [ 0 -w(3,3) w(2,3); ...
41              w(3,3) 0 -w(1,3); ...
42              -w(2,3) w(1,3) 0];
43     ew3_ = (eye(3) + w_hat*sin(theta3) + w_hat^2*(1-cos(theta3)));
44     ew3 = [ew3_ ((eye(3)-ew3_)*w_hat*v(:,3)); 0 0 0 1];
45
46     [theta1, theta2] = subs.Sub2(ew3*p3, g1*p3, q1, [v(1:3,1);w(1:3,1)] ,[v
47         (1:3,2);w(1:3,2)], bitand(sol,2));
48
49     w_hat = [ 0 -w(3,2) w(2,2); ...
50              w(3,2) 0 -w(1,2); ...
51              -w(2,2) w(1,2) 0];
52     ew2_ = (eye(3) + w_hat*sin(theta2) + w_hat^2*(1-cos(theta2)));
53     ew2 = [ew2_ ((eye(3)-ew2_)*w_hat*v(:,2)); 0 0 0 1];
54     w_hat = [ 0 -w(3,1) w(2,1); ...
55              w(3,1) 0 -w(1,1); ...
56              -w(2,1) w(1,1) 0];
57     ew1_ = (eye(3) + w_hat*sin(theta1) + w_hat^2*(1-cos(theta1)));
58     ew1 = [ew1_ ((eye(3)-ew1_)*w_hat*v(:,1)); 0 0 0 1];
59     g2 = (ew1*ew2*ew3)^(-1)*g1;
60     p5 = [0; l1+l2+0.2; h; 1];
61     [theta4, theta5] = subs.Sub2(p5, g2*p5, p3, [v(1:3,4);w(1:3,4)] ,[v(1:3,5)
62         ;w(1:3,5)], bitand(sol,1));
63
64     w_hat = [ 0 -w(3,4) w(2,4); ...
65              w(3,4) 0 -w(1,4); ...
66              -w(2,4) w(1,4) 0];

```

```

64     ew4_ = (eye(3) + w_hat*sin(theta4) + w_hat^2*(1-cos(theta4)));
65     ew4 = [ew4_ ((eye(3)-ew4_)*w_hat*v(:,4)); 0 0 0 1];
66     w_hat = [      0 -w(3,5)  w(2,5); ...
67             w(3,5)      0 -w(1,5); ...
68             -w(2,5)  w(1,5)      0];
69     ew5_ = (eye(3) + w_hat*sin(theta5) + w_hat^2*(1-cos(theta5)));
70     ew5 = [ew5_ ((eye(3)-ew5_)*w_hat*v(:,5)); 0 0 0 1];
71     g3 = (ew1*ew2*ew3*ew4*ew5)^(-1)*g1;
72     p6 = [0.1; l1+l2+0.2; h; 1];
73     theta6 = subs.Sub1(p6, g3*p6, p3, [v(1:3,6);w(1:3,6)]);
74
75     [theta1, theta2, theta3, theta4, theta5, theta6] * 180 / pi
76     end

```

取  $h = 0.3$ ,  $l1 = l2 = 0.5$ , 输入关节角度经正运动学计算得到齐次矩阵, 代入上述代码 `gst` 中, 逆运动学得到 8 组解 (均化为角度制):

输入 `[24,-13,35,44,76,13]`, 输出如下:

```

23.9986 -12.9995 34.9956 -135.9815 104.0027 -167.0109
23.9986 -12.9995 34.9956 44.0185 75.9973 12.9891
-156.0014 158.0039 34.9956 100.9859 -104.0027 12.9891
-156.0014 158.0039 34.9956 -79.0141 -75.9973 -167.0109
23.9986 21.9961 325.0044 -100.9859 104.0027 -167.0109
23.9986 21.9961 325.0044 79.0141 75.9973 12.9891
-156.0014 -167.0005 325.0044 135.9815 -104.0027 12.9891
-156.0014 -167.0005 325.0044 -44.0185 -75.9973 -167.0109

```

(ii) (存疑) 对 Inverse Elbow 机器人:

令  $g_d = g_{st}(\theta) = e^{\hat{\xi}_1\theta_1} \dots e^{\hat{\xi}_6\theta_6} g_{st}(0)$ ,  $g_d g_{st}(0)^{-1} = g_1$ , 则取  $q_1 = \xi_1 \cap \xi_2$ ,

$p_4 = \xi_5 \cap \xi_6$ , 此时  $g_1 p_4 = e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3} e^{\hat{\xi}_4\theta_4} p_4$ ,

$\|g_1 p_4 - q_1\| = \|e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3} (e^{\hat{\xi}_4\theta_4} p_4 - q_1)\| = \|e^{\hat{\xi}_4\theta_4} p_4 - q_1\|$ ,

由 Sub 3 可解出  $\theta_4$ 。

取  $p_3 = \xi_4 \cap \xi_6$ , 类似地有  $\|e^{\hat{\xi}_3\theta_3} p_3 - q_1\| = \|g_1 p_3 - q_1\|$ ,

由 Sub 3 解出  $\theta_3$ 。

此时  $e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} (e^{\hat{\xi}_3\theta_3} e^{\hat{\xi}_4\theta_4}) p_4 = g_1 p_4$ , 由 Sub 2 解出  $\theta_1, \theta_2$ 。

取  $p_6 \notin \xi_5, \xi_6$ , 令  $(e^{\hat{\xi}_1\theta_1} \dots e^{\hat{\xi}_4\theta_4})^{-1} g_1 = g_2$ , 则  $g_2 p_6 = e^{\hat{\xi}_5\theta_5} e^{\hat{\xi}_6\theta_6} p_6$ , 由 Sub 2 解出  $\theta_5, \theta_6$ 。

逆运动学代码:

```

1     subs = add;
2     % 定义机器人参数
3     h = 0.3;

```

```

4      l1 = 0.5; l2 = 0.5;
5      % 旋转轴与变换矩阵
6      rot = [1, 1, 1, 1, 1, 1];
7      q = [0 0 0 0 0 0; ...
8           0 0 0 l1 l1+l2 l1+l2; ...
9           h h h h h h];
10     w = [0 0 -1 -1 -1 0; ...
11          0 1 0 0 0 1; ...
12          1 0 0 0 0 0];
13     for i=1:6
14         if rot(i) == 1
15             v(:,i) = cross(q(1:3,i), w(1:3,i));
16         else
17             v(:,i) = q(:,1);
18         end
19     end
20     % 初始位姿
21     gst0 = [ 0 0 1 0; ...
22            0 1 0 l1+l2; ...
23            -1 0 0 h; ...
24            0 0 0 1];
25
26     gst = [ 0.5216 -0.1854 0.8328 -0.6581
27            -0.8527 -0.0820 0.5159 0.6437
28            -0.0273 -0.9792 -0.2009 0.1650
29            0 0 0 1.0000];
30     % 选解 (0~7)
31     for sol = 0:7
32         % 求解
33         p4 = [0; l1+l2; h; 1];
34         q1 = [0; 0; h; 1];
35         g1 = gst*gst0^(-1);
36         theta4 = subs.Sub3(p4, q1, [q(1:3,4); 1], [v(1:3,4);w(1:3,4)], norm(g1*p4-
37             q1), bitand(sol,3));
38
39         w_hat = [ 0 -w(3,4) w(2,4); ...
40                 w(3,4) 0 -w(1,4); ...
41                 -w(2,4) w(1,4) 0];
42         ew4_ = (eye(3) + w_hat*sin(theta4) + w_hat^2*(1-cos(theta4)));
43         ew4 = [ew4_ ((eye(3)-ew4_)*w_hat*v(:,4)); 0 0 0 1];
44
45         theta3 = subs.Sub1(ew4*p4, g1*p4, [q(1:3,3); 1], [v(1:3,3);w(1:3,3)]);
46         p3 = [0; l1; h; 1];
47
48         w_hat = [ 0 -w(3,3) w(2,3); ...
49                 w(3,3) 0 -w(1,3); ...
50                 -w(2,3) w(1,3) 0];
51         ew3_ = (eye(3) + w_hat*sin(theta3) + w_hat^2*(1-cos(theta3)));
52         ew3 = [ew3_ ((eye(3)-ew3_)*w_hat*v(:,3)); 0 0 0 1];
53         [theta1, theta2] = subs.Sub2(ew3*ew4*p4, g1*p4, q1, [v(1:3,1);w(1:3,1)], [
54             v(1:3,2);w(1:3,2)], bitand(sol,2));
55
56         w_hat = [ 0 -w(3,2) w(2,2); ...
57                 w(3,2) 0 -w(1,2); ...
58                 -w(2,2) w(1,2) 0];
59         ew2_ = (eye(3) + w_hat*sin(theta2) + w_hat^2*(1-cos(theta2)));
60         ew2 = [ew2_ ((eye(3)-ew2_)*w_hat*v(:,2)); 0 0 0 1];

```

```

59     w_hat = [      0  -w(3,1)  w(2,1); ...
60             w(3,1)      0  -w(1,1); ...
61             -w(2,1)  w(1,1)      0];
62     ew1_ = (eye(3) + w_hat*sin(theta1) + w_hat^2*(1-cos(theta1)));
63     ew1 = [ew1_ ((eye(3)-ew1_)*w_hat*v(:,1)); 0 0 0 1];
64     g2 = (ew1*ew2*ew3*ew4)^(-1)*g1;
65
66     p6 = [0.1; l1+l2+0.2; h; 1];
67     [theta5, theta6] = subs.Sub2(p6, g2*p6, p4, [v(1:3,5);w(1:3,5)] ,[v(1:3,6)
68         ;w(1:3,6)], bitand(sol,4));
end

```

(iii) 对 Stanford 机器人:

令  $g_d = g_{st}(\theta) = e^{\hat{\xi}_1\theta_1} \dots e^{\hat{\xi}_6\theta_6} g_{st}(0)$ ,  $g_d g_{st}(0)^{-1} = g_1$ , 则取  $q_1 = \xi_1 \cap \xi_2$ ,  $p_3 = \xi_4 \cap \xi_5 \cap \xi_6$ , 此时  $g_1 p_3 = e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3} p_3$ ,  $\theta_3 = \|g_1 p_3 - q_1\| - l_1 - l_2$ 。由  $e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} (e^{\hat{\xi}_3\theta_3} p_3) = g_1 p_3$ , 由 Sub 2 解出  $\theta_1, \theta_2$ 。  
 令  $g_2 = e^{\hat{\xi}_4\theta_4} e^{\hat{\xi}_5\theta_5} e^{\hat{\xi}_6\theta_6} = (e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3})^{-1} g_1$ , 取  $p_5 \in \xi_6, p_5 \notin \xi_4, \xi_5$ , 则  $e^{\hat{\xi}_4\theta_4} e^{\hat{\xi}_5\theta_5} p_5 = g_2 p_5$ , 由 Sub 2 解出  $\theta_4, \theta_5$ 。  
 令  $g_3 = (e^{\hat{\xi}_1\theta_1} \dots e^{\hat{\xi}_5\theta_5})^{-1} g_1$ , 取  $p_6 \notin \xi_6$ , 则  $g_3 p_6 = e^{\hat{\xi}_6\theta_6} p_6$ , 由 Sub 1 解出  $\theta_6$ 。

逆运动学代码:

```

1     subs = add;
2     % 定义机器人参数
3     h = 0.3;
4     l1 = 0.5; l2 = 0.5;
5     % 旋转轴与变换矩阵
6     rot = [1, 1, 0, 1, 1, 1];
7     q = [0  0  0  0  0  0; ...
8         0  0  1  l1+l2  l1+l2  l1+l2; ...
9         h  h  0  h  h  h];
10    w = [0 -1  0 -1  0  0; ...
11        0  0  0  0  0  1; ...
12        1  0  0  0  1  0];
13
14    for i=1:6
15        if rot(i) == 1
16            v(:,i) = cross(q(1:3,i), w(1:3,i));
17        else
18            v(:,i) = q(:,1);
19        end
20    end
21
22    % 初始位姿
23    gst0 = [ 0  0  1  0; ...
24            0  1  0  l1+l2; ...
25            -1  0  0  h; ...
26            0  0  0  1];
27
28    gst = [ 0.7027  0.6895  0.1754 -1.0633
29           -0.6322  0.4921  0.5984  0.7445
30           0.3263 -0.5314  0.7818 -0.2244
31           0  0  0  1.0000
32    ];

```



```

31 % 选解 (0~3)
32 for sol = 0:3
33 %sol = 0;
34 % 求解
35 p3 = [0; l1+l2; h; 1];
36 q1 = [0; 0; h; 1];
37 g1 = gst*gst0^(-1);
38 theta3 = norm(g1*p3 - q1) - l1 - l2;
39
40 ew3 = [eye(3) q(1:3,i)*theta3; 0 0 0 1];
41 [theta1, theta2] = subs.Sub2(ew3*p3, g1*p3, q1, [v(1:3,1);w(1:3,1)], [v
(1:3,2);w(1:3,2)], bitand(sol,1));
42
43 w_hat = [ 0 -w(3,2) w(2,2); ...
44 w(3,2) 0 -w(1,2); ...
45 -w(2,2) w(1,2) 0];
46 ew2_ = (eye(3) + w_hat*sin(theta2) + w_hat^2*(1-cos(theta2)));
47 ew2 = [ew2_ ((eye(3)-ew2_)*w_hat*v(:,2)); 0 0 0 1];
48 w_hat = [ 0 -w(3,1) w(2,1); ...
49 w(3,1) 0 -w(1,1); ...
50 -w(2,1) w(1,1) 0];
51 ew1_ = (eye(3) + w_hat*sin(theta1) + w_hat^2*(1-cos(theta1)));
52 ew1 = [ew1_ ((eye(3)-ew1_)*w_hat*v(:,1)); 0 0 0 1];
53 g2 = (ew1*ew2*ew3)^(-1)*g1;
54 p5 = [0; l1+l2+0.2; h; 1];
55 [theta4, theta5] = subs.Sub2(p5, g2*p5, p3, [v(1:3,4);w(1:3,4)], [v(1:3,5)
;w(1:3,5)], bitand(sol,2));
56
57 w_hat = [ 0 -w(3,4) w(2,4); ...
58 w(3,4) 0 -w(1,4); ...
59 -w(2,4) w(1,4) 0];
60 ew4_ = (eye(3) + w_hat*sin(theta4) + w_hat^2*(1-cos(theta4)));
61 ew4 = [ew4_ ((eye(3)-ew4_)*w_hat*v(:,4)); 0 0 0 1];
62 w_hat = [ 0 -w(3,5) w(2,5); ...
63 w(3,5) 0 -w(1,5); ...
64 -w(2,5) w(1,5) 0];
65 ew5_ = (eye(3) + w_hat*sin(theta5) + w_hat^2*(1-cos(theta5)));
66 ew5 = [ew5_ ((eye(3)-ew5_)*w_hat*v(:,5)); 0 0 0 1];
67 g3 = (ew1*ew2*ew3*ew4*ew5)^(-1)*g1;
68 p6 = [0.1; l1+l2+0.2; h; 1];
69 theta6 = subs.Sub1(p6, g3*p6, p3, [v(1:3,6);w(1:3,6)]);
70 end

```

取  $h = 0.3$ ,  $l1 = l2 = 0.5$ , 逆运动学得到 4 组解:

输入 [55, 22, 0.4, 96, -53, 11], 输出如下:

55.0011 26.8127 0.4000 -86.5362 -127.0053 -166.9043

-124.9989 162.9847 0.4000 76.7387 127.0053 13.0957

55.0011 26.8127 0.4000 93.4638 -52.9947 13.0957

-124.9989 162.9847 0.4000 -103.2613 52.9947 -166.9043