4.(i) 世界坐标系与末端坐标系定义如图 1。机器人 (ii)(iii) 中世界坐标系与末端坐标系的方向定义与 (i) 相同。正运动学代码如下:

```
syms t1 t2 t3 t4 t5 t6  % 关节角度
th = [t1, t2, t3, t4, t5, t6];

% 定义各关节参数
syms h l1 l2
q = [0    0    0        0        0        0; ...
     0    0    l1   l1+l2   l1+l2   l1+l2; ...
     h    h    h        h        h        h];
w = [0 -1 -1 -1 0   0; ...
     0  0  0  0 0   1; ...
     1  0  0  0 1   0];

for i=1:6
    v(:,i) = cross(q(1:3,i), w(1:3,i));
end

for i=1:6
    w_hat = [        0  -w(3,i)    w(2,i); ...
              w(3,i)         0   -w(1,i); ...
             -w(2,i)    w(1,i)         0];
    ew = simplify(eye(3) + w_hat*sin(th(i)) + w_hat^2*(1-cos(th(i))));
    e(:,:,i) = [ew simplify((eye(3)-ew)*w_hat*v(:,i)); 0 0 0 1];
end

gst0 = [ 0 0 1       0; ...
         0 1 0 l1+l2; ...
        -1 0 0       h; ...
         0 0 0       1]; % 初始位姿,

gst = gst0;
for i=6:-1:1
    gst = e(:,:,i)*gst;
end
simplify(gst);
```

得 $g_{st}(\theta) =$

$$\begin{bmatrix} s_1(s_5 s_6 c_{234} + s_{234} c_6) - c_1 c_5 s_6 & -s_1 c_5 c_{234} - c_1 s_5 & -s_1(-s_5 c_6 c_{234} + s_{234} s_6) + c_1 c_5 c_6 & -s_1(l_1 c_2 + l_2 c_{23}) \\ -c_1(s_5 s_6 c_{234} + s_{234} c_6) - s_1 c_5 s_6 & c_1 c_5 c_{234} - s_1 s_5 & c_1(s_5 c_6 c_{234} - s_{234} s_6) + s_1 c_5 c_6 & c_1(l_1 c_2 - l_2 c_{23}) \\ s_5 s_6 s_{234} - c_{234} c_6 & -s_{234} c_5 & -s_5 c_6 s_{234} - c_{234} s_6 & h - l_1 s_2 - l_2 s_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

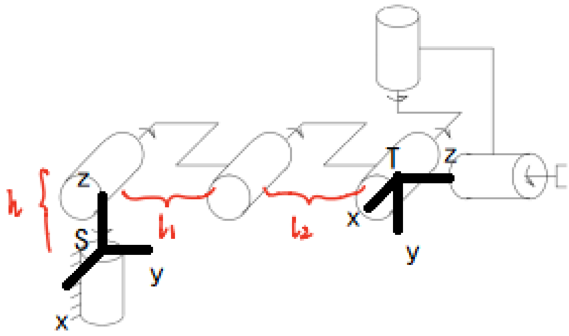

图 1: Elbow 坐标系定义

(ii) 修改机器人参数如下:

```
q = [0   0   0        0        0        0;  ...
     0   0   0       l1    l1+l2    l1+l2;  ...
     h   h   h        h        h        h];
w = [0   0 -1 -1 -1   0;  ...
     0   1  0  0  0   1;  ...
     1   0  0  0  0   0];
```

得 $g_{st}(\theta) =$

$$\begin{bmatrix} -(s_2c_6c_{345}+s_2c_6)c_1+s_1c_6s_{345} & -s_1c_{345}-c_1s_2s_{345} & (-s_2s_6c_{345}+c_2c_6)c_1+s_1s_6s_{345} & -l_1(c_1s_2s_3+s_1c_3)-l_2(s_1c_{34}+c_1s_2s_{34}) \\ -(s_2c_6c_{345}+c_2s_6)s_1-c_1c_6s_{345} & c_1c_{345}-s_1s_2s_{345} & (-s_2s_6c_{345}+c_2c_6)s_1-c_1s_6s_{345} & l_1(c_1c_3-s_1s_2s_3)+l_2(c_1c_{34}-s_1s_2s_{34}) \\ s_2s_6-c_2c_6c_{345} & -c_2s_{345} & -s_2c_6-s_6c_2c_{345} & h-c_2(l_1s_3+l_2s_{34}) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(iii) 对代码进行修改以实现平移关节:

```
syms t1 t2 t3 t4 t5 t6  % 关节角度
th = [t1, t2, t3, t4, t5, t6];

% 关节类型, 1为旋转
rot = [1, 1, 0, 1, 1, 1];

% 定义各关节参数
% 对平移关节, 将v写在q中, w取[0;0;0]
% 对旋转关节, q与w正常赋值
syms h l1 l2
q = [0    0    0       0       0       0;  ...
     0    0    0   l1+l2   l1+l2   l1+l2;  ...
     h    h    1       h       h       h];
w = [0 -1   0 -1 0   0;  ...
     0  0   0  0 0   1;  ...
     1  0   0  0 1   0];

for i=1:6
    if rot(i) == 1
        v(:,i) = cross(q(1:3,i), w(1:3,i));
    else
        v(:,i) = q(:,1);
    end
```

```matlab
    end

    for i=1:6
        if rot(i) == 1
            w_hat = [      0   -w(3,i)   w(2,i); ...
                      w(3,i)       0    -w(1,i); ...
                     -w(2,i)   w(1,i)       0];
            ew = simplify(eye(3) + w_hat*sin(th(i)) + w_hat^2*(1-cos(th(i))));
            e(:,:,i) = [ew simplify((eye(3)-ew)*w_hat*v(:,i)); 0 0 0 1];
        else
            e(:,:,i) = [eye(3) q(1:3,i)*th(i); 0 0 0 1];
        end
    end

    gst0 = [ 0 0 1      0; ...
             0 1 0  l1+l2; ...
            -1 0 0      h; ...
             0 0 0      1]; % 初始位姿

    gst = gst0;
    for i=6:-1:1
        gst = e(:,:,i)*gst;
    end
    simplify(gst);
```

得 $g_{st}(\theta) =$

$$\begin{bmatrix} (s_5s_6c_{24}+c_6s_{24})s_1-c_1c_5s_6 & -c_1s_5-s_1c_5c_{24} & (-s_5c_6c_{24}+s_6s_{24})s_1+c_1c_5c_6 & -(l_1c_2+l_2c_{24}+\theta_3c_2)s_1 \\ -(s_5s_6c_{24}+c_6s_{24})c_1-s_1c_5s_6 & -s_1s_5+c_1c_5c_{24} & (s_5c_6c_{24}-s_6s_{24})c_1+s_1c_5c_6 & (l_1c_2+l_2c_{24}+\theta_3c_2)c_1 \\ s_5s_6s_{24}-c_6c_{24} & -s_{24}c_5 & -s_5c_6s_{24}-s_6c_{24} & h-l_1s_2-l_2s_{24}-\theta_3s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5. 定义子问题求解函数:

```matlab
function Subs = add  % add为文件名
    Subs.Sub1 = @Sub1;
    Subs.Sub2 = @Sub2;
    Subs.Sub3 = @Sub3;
end

% e^{\hat{\xi}\theta}p = q, r \xi
% p, q, r = [x;y;z;1]
% xi = [v w]^{T} R6
function theta = Sub1(p, q, r, xi)
    u = p - r; v = q - r; % [R3; 0]
    w = xi(4:6); % R3
    u = u(1:3); v = v(1:3); % conv [R3;0] into R3
    u1 = u - w * w' * u; v1 = v - w * w' * v;
    theta = atan2(w'*cross(u1, v1), u1'*v1);
end

% r   xi1, xi2
% solve=0/1
function [theta1, theta2] = Sub2(p, q, r, xi1, xi2, solve)
```

```
    u = p - r; v = q - r; % [R3; 0]
    u = u(1:3); v = v(1:3);
    w1 = xi1(4:6); w2 = xi2(4:6); % R3
    a = ((w1'*w2)*w2'*u - w1'*v) / ((w1'*w2)^2 - 1);
    b = ((w1'*w2)*w1'*v - w2'*u) / ((w1'*w2)^2 - 1);
    y = sqrt((norm(u)^2 - a^2 - b^2 - 2*a*b*w1'*w2) / (norm(cross(w1, w2))^2));
    if solve == 0
        y = -y;  % change the solve
    end
    z = a*w1 + b*w2 + y*cross(w1,w2);
    c = [z; 0] + r; % [R3; 0]
    subs = add;
    theta2 = subs.Sub1(p, c, r, xi2);
    theta1 = subs.Sub1(c, q, r, xi1);
end

function theta = Sub3(p, q, r, xi, dis, solve)
    u = p - r; v = q - r; % [R3; 0]
    w = xi(4:6); % R3
    u = u(1:3); v = v(1:3); % conv [R3;0] into R3
    u1 = u - w * w' * u; v1 = v - w * w' * v;
    dis1_2 = dis^2 - norm(w'*(p(1:3) - q(1:3)))^2;
    theta0 = atan2( dot(w, cross(u1, v1)), dot(u1, v1) );

    if solve ~= 0
        theta = theta0 + acos((norm(u1)^2 + norm(v1)^2 - dis1_2) / (2 * norm(u1) *
            norm(v1)));
    else
        theta = theta0 - acos((norm(u1)^2 + norm(v1)^2 - dis1_2) / (2 * norm(u1) *
            norm(v1)));
    end
end
```

(i) 对 Elbow 机器人:

令 $g_d = g_{st}(\theta) = e^{\hat{\xi_1}\theta_1} \dots e^{\hat{\xi_6}\theta_6} g_{st}(0), g_d g_{st}(0)^{-1} = g_1$, 则取 $q_1 = \xi_1 \bigcap \xi_2$, $p_3 = \xi_4 \bigcap \xi_5 \bigcap \xi_6$, 此时 $g_1 p_3 = e^{\hat{\xi_1}\theta_1} e^{\hat{\xi_2}\theta_2} e^{\hat{\xi_3}\theta_3} p_3$,

$||g_1 p_3 - q_1|| = ||e^{\hat{\xi_1}\theta_1} e^{\hat{\xi_2}\theta_2} (e^{\hat{\xi_3}\theta_3} p_3 - q_1)|| = ||e^{\hat{\xi_3}\theta_3} p_3 - q_1||$,

利用 Paden-Kahan 子问题中 Subproblem 3(下文简写为 Sub 3) 可解出 θ_3。

由 $e^{\hat{\xi_1}\theta_1} e^{\hat{\xi_2}\theta_2} (e^{\hat{\xi_3}\theta_3} p_3) = g_1 p_3$, 由 Sub 2 解出 θ_1, θ_2。

令 $g_2 = e^{\hat{\xi_4}\theta_4} e^{\hat{\xi_5}\theta_5} e^{\hat{\xi_6}\theta_6} = (e^{\hat{\xi_1}\theta_1} e^{\hat{\xi_2}\theta_2} e^{\hat{\xi_3}\theta_3})^{-1} g_1$, 取 $p_5 \in \xi_6, p_5 \notin \xi_4, \xi_5$, 则 $e^{\hat{\xi_4}\theta_4} e^{\hat{\xi_5}\theta_5} p_5 = g_2 p_5$, 由 Sub 2 解出 θ_4, θ_5。

令 $g_3 = (e^{\hat{\xi_1}\theta_1} \dots e^{\hat{\xi_5}\theta_5})^{-1} g_1$, 取 $p_6 \notin \xi_6$, 则 $g_3 p_6 = e^{\hat{\xi_6}\theta_6} p_6$, 由 Sub 1 解出 θ_6。

逆运动学代码:

```
    subs = add;
    % 定义机器人参数
    h = 0.3;
    l1 = 0.5; l2 = 0.5;
    % 旋转轴与变换矩阵
    rot = [1, 1, 1, 1, 1, 1];
    q = [0    0    0     0       0       0; ...
         0    0    l1    l1+l2   l1+l2   l1+l2; ...
         h    h    h     h       h       h];
```

```
w = [0 -1 -1 -1 0   0; ...
     0  0  0  0 0   1; ...
     1  0  0  0 1   0];
for i=1:6
    if rot(i) == 1
        v(:,i) = cross(q(1:3,i), w(1:3,i));
    else
        v(:,i) = q(:,1);
    end
end
% 初始位姿
gst0 = [ 0 0 1      0; ...
         0 1 0  l1+l2; ...
        -1 0 0      h; ...
         0 0 0      1];

gst = [ 0.3484   -0.9264    0.1425   -0.3867; ...
       -0.9164   -0.3048    0.2594    0.8686; ...
       -0.1969   -0.2210   -0.9552    0.2252; ...
             0         0         0    1.0000];
% 选解(0~7)
for sol = 0:7

    % 求解
    p3 = [0; l1+l2; h; 1];
    q1 = [0; 0; h; 1];
    g1 = gst*gst0^(-1);
    theta3 = subs.Sub3(p3, q1, [q(1:3,3); 1], [v(1:3,3);w(1:3,3)], norm(g1*p3-q1), bitand(sol,4));

    w_hat = [        0  -w(3,3)   w(2,3); ...
               w(3,3)         0  -w(1,3); ...
              -w(2,3)    w(1,3)        0];
    ew3_ = (eye(3) + w_hat*sin(theta3) + w_hat^2*(1-cos(theta3)));
    ew3 = [ew3_ ((eye(3)-ew3_)*w_hat*v(:,3)); 0 0 0 1];

    [theta1, theta2] = subs.Sub2(ew3*p3, g1*p3, q1, [v(1:3,1);w(1:3,1)] ,[v(1:3,2);w(1:3,2)], bitand(sol,2));

    w_hat = [        0  -w(3,2)   w(2,2); ...
               w(3,2)         0  -w(1,2); ...
              -w(2,2)    w(1,2)        0];
    ew2_ = (eye(3) + w_hat*sin(theta2) + w_hat^2*(1-cos(theta2)));
    ew2 = [ew2_ ((eye(3)-ew2_)*w_hat*v(:,2)); 0 0 0 1];
    w_hat = [        0  -w(3,1)   w(2,1); ...
               w(3,1)         0  -w(1,1); ...
              -w(2,1)    w(1,1)        0];
    ew1_ = (eye(3) + w_hat*sin(theta1) + w_hat^2*(1-cos(theta1)));
    ew1 = [ew1_ ((eye(3)-ew1_)*w_hat*v(:,1)); 0 0 0 1];
    g2 = (ew1*ew2*ew3)^(-1)*g1;
    p5 = [0; l1+l2+0.2; h; 1];
    [theta4, theta5] = subs.Sub2(p5, g2*p5, p3, [v(1:3,4);w(1:3,4)] ,[v(1:3,5);w(1:3,5)], bitand(sol,1));

    w_hat = [        0  -w(3,4)   w(2,4); ...
               w(3,4)         0  -w(1,4); ...
              -w(2,4)    w(1,4)        0];
```

```
        ew4_ = (eye(3) + w_hat*sin(theta4) + w_hat^2*(1-cos(theta4)));
        ew4 = [ew4_ ((eye(3)-ew4_)*w_hat*v(:,4)); 0 0 0 1];
        w_hat = [      0   -w(3,5)    w(2,5); ...
                     w(3,5)       0   -w(1,5); ...
                    -w(2,5)    w(1,5)       0];
        ew5_ = (eye(3) + w_hat*sin(theta5) + w_hat^2*(1-cos(theta5)));
        ew5 = [ew5_ ((eye(3)-ew5_)*w_hat*v(:,5)); 0 0 0 1];
        g3 = (ew1*ew2*ew3*ew4*ew5)^(-1)*g1;
        p6 = [0.1; l1+l2+0.2; h; 1];
        theta6 = subs.Sub1(p6, g3*p6, p3, [v(1:3,6);w(1:3,6)]);

        [theta1, theta2, theta3, theta4, theta5, theta6] * 180 / pi
    end
```

取 h = 0.3, l1 = l2 = 0.5, 输入关节角度经正运动学计算得到齐次矩阵, 代入上述代码 gst 中, 逆运动学得到 8 组解 (均化为角度制):

输入 [24,-13,35,44,76,13], 输出如下:

23.9986 -12.9995 34.9956 -135.9815 104.0027 -167.0109

23.9986 -12.9995 34.9956 44.0185 75.9973 12.9891

-156.0014 158.0039 34.9956 100.9859 -104.0027 12.9891

-156.0014 158.0039 34.9956 -79.0141 -75.9973 -167.0109

23.9986 21.9961 325.0044 -100.9859 104.0027 -167.0109

23.9986 21.9961 325.0044 79.0141 75.9973 12.9891

-156.0014 -167.0005 325.0044 135.9815 -104.0027 12.9891

-156.0014 -167.0005 325.0044 -44.0185 -75.9973 -167.0109

(ii)（存疑）对 Inverse Elbow 机器人:

令 $g_d = g_{st}(\theta) = e^{\hat{\xi_1}\theta_1}\dots e^{\hat{\xi_6}\theta_6}g_{st}(0), g_d g_{st}(0)^{-1} = g_1$, 则取 $q_1 = \xi_1 \bigcap \xi_2$, $p_4 = \xi_5 \bigcap \xi_6$, 此时 $g_1 p_4 = e^{\hat{\xi_1}\theta_1}e^{\hat{\xi_2}\theta_2}e^{\hat{\xi_3}\theta_3}e^{\hat{\xi_4}\theta_4}p_4$,

$||g_1 p_4 - q_1|| = ||e^{\hat{\xi_1}\theta_1}e^{\hat{\xi_2}\theta_2}e^{\hat{\xi_3}\theta_3}(e^{\hat{\xi_4}\theta_4}p_4 - q_1)|| = ||e^{\hat{\xi_4}\theta_4}p_4 - q_1||$,

由 Sub 3 可解出 θ_4。

取 $p_3 = \xi_4 \bigcap \xi_6$, 类似地有 $||e^{\hat{\xi_3}\theta_3}p_3 - q_1|| = ||g_1 p_3 - q_1||$,

由 Sub 3 解出 θ_3。

此时 $e^{\hat{\xi_1}\theta_1}e^{\hat{\xi_2}\theta_2}(e^{\hat{\xi_3}\theta_3}e^{\hat{\xi_4}\theta_4})p_4 = g_1 p_4$, 由 Sub 2 解出 θ_1, θ_2。

取 $p_6 \notin \xi_5, \xi_6$, 令 $(e^{\hat{\xi_1}\theta_1}\dots e^{\hat{\xi_4}\theta_4})^{-1}g_1 = g_2$, 则 $g_2 p_6 = e^{\hat{\xi_5}\theta_5}e^{\hat{\xi_6}\theta_6}p_6$, 由 Sub 2 解出 θ_5, θ_6。

逆运动学代码:

```
    subs = add;
    % 定义机器人参数
    h = 0.3;
```

```
l1 = 0.5; l2 = 0.5;
% 旋转轴与变换矩阵
rot = [1, 1, 1, 1, 1, 1];
q = [0  0   0       0       0       0; ...
     0  0   0       l1   l1+l2  l1+l2; ...
     h  h   h       h       h       h];
w = [0  0 -1 -1 -1  0; ...
     0  1  0  0  0  1; ...
     1  0  0  0  0  0];
for i=1:6
    if rot(i) == 1
        v(:,i) = cross(q(1:3,i), w(1:3,i));
    else
        v(:,i) = q(:,1);
    end
end
% 初始位姿
gst0 = [ 0 0 1      0; ...
    0 1 0 l1+l2; ...
    -1 0 0      h; ...
    0 0 0       1];

gst = [ 0.5216    -0.1854     0.8328    -0.6581
    -0.8527    -0.0820     0.5159     0.6437
    -0.0273    -0.9792    -0.2009     0.1650
    0          0          0       1.0000];
% 选解(0~7)
for sol = 0:7
    % 求解
    p4 = [0; l1+l2; h; 1];
    q1 = [0; 0; h; 1];
    g1 = gst*gst0^(-1);
    theta4 = subs.Sub3(p4, q1, [q(1:3,4); 1], [v(1:3,4);w(1:3,4)], norm(g1*p4-
        q1), bitand(sol,3));

    w_hat = [        0  -w(3,4)    w(2,4); ...
        w(3,4)         0   -w(1,4); ...
        -w(2,4)    w(1,4)         0];
    ew4_ = (eye(3) + w_hat*sin(theta4) + w_hat^2*(1-cos(theta4)));
    ew4 = [ew4_ ((eye(3)-ew4_)*w_hat*v(:,4)); 0 0 0 1];

    theta3 = subs.Sub1(ew4*p4, g1*p4, [q(1:3,3); 1], [v(1:3,3);w(1:3,3)]);
    p3 = [0; l1; h; 1];

    w_hat = [        0  -w(3,3)    w(2,3); ...
        w(3,3)         0   -w(1,3); ...
        -w(2,3)    w(1,3)         0];
    ew3_ = (eye(3) + w_hat*sin(theta3) + w_hat^2*(1-cos(theta3)));
    ew3 = [ew3_ ((eye(3)-ew3_)*w_hat*v(:,3)); 0 0 0 1];
    [theta1, theta2] = subs.Sub2(ew3*ew4*p4, g1*p4, q1, [v(1:3,1);w(1:3,1)] ,[
        v(1:3,2);w(1:3,2)], bitand(sol,2));

    w_hat = [        0  -w(3,2)    w(2,2); ...
        w(3,2)         0   -w(1,2); ...
        -w(2,2)    w(1,2)         0];
    ew2_ = (eye(3) + w_hat*sin(theta2) + w_hat^2*(1-cos(theta2)));
    ew2 = [ew2_ ((eye(3)-ew2_)*w_hat*v(:,2)); 0 0 0 1];
```

```
        w_hat = [         0  -w(3,1)    w(2,1); ...
            w(3,1)         0    -w(1,1); ...
            -w(2,1)    w(1,1)          0];
        ew1_ = (eye(3) + w_hat*sin(theta1) + w_hat^2*(1-cos(theta1)));
        ew1 = [ew1_ ((eye(3)-ew1_)*w_hat*v(:,1)); 0 0 0 1];
        g2 = (ew1*ew2*ew3*ew4)^(-1)*g1;

        p6 = [0.1; l1+l2+0.2; h; 1];
        [theta5, theta6] = subs.Sub2(p6, g2*p6, p4, [v(1:3,5);w(1:3,5)] ,[v(1:3,6);w(1:3,6)], bitand(sol,4));
    end
```

(iii) 对 Stanford 机器人:

令 $g_d = g_{st}(\theta) = e^{\hat{\xi}_1\theta_1}\dots e^{\hat{\xi}_6\theta_6}g_{st}(0), g_d g_{st}(0)^{-1} = g_1$，则取 $q_1 = \xi_1 \bigcap \xi_2$, $p_3 = \xi_4 \bigcap \xi_5 \bigcap \xi_6$, 此时 $g_1 p_3 = e^{\hat{\xi}_1\theta_1}e^{\hat{\xi}_2\theta_2}e^{\hat{\xi}_3\theta_3}p_3$,
$\theta_3 = ||g_1 p_3 - q_1|| - l_1 - l_2$。由 $e^{\hat{\xi}_1\theta_1}e^{\hat{\xi}_2\theta_2}(e^{\hat{\xi}_3\theta_3}p_3) = g_1 p_3$, 由 Sub 2 解出 θ_1, θ_2。
令 $g_2 = e^{\hat{\xi}_4\theta_4}e^{\hat{\xi}_5\theta_5}e^{\hat{\xi}_6\theta_6} = (e^{\hat{\xi}_1\theta_1}e^{\hat{\xi}_2\theta_2}e^{\hat{\xi}_3\theta_3})^{-1}g_1$, 取 $p_5 \in \xi_6, p_5 \notin \xi_4, \xi_5$, 则 $e^{\hat{\xi}_4\theta_4}e^{\hat{\xi}_5\theta_5}p_5 = g_2 p_5$, 由 Sub 2 解出 θ_4, θ_5。
令 $g_3 = (e^{\hat{\xi}_1\theta_1}\dots e^{\hat{\xi}_5\theta_5})^{-1}g_1$, 取 $p_6 \notin \xi_6$, 则 $g_3 p_6 = e^{\hat{\xi}_6\theta_6}p_6$, 由 Sub 1 解出 θ_6。

逆运动学代码:

```
subs = add;
% 定义机器人参数
h = 0.3;
l1 = 0.5; l2 = 0.5;
% 旋转轴与变换矩阵
rot = [1, 1, 0, 1, 1, 1];
    q = [0    0    0        0        0        0; ...
         0    0    1   l1+l2    l1+l2    l1+l2; ...
         h    h    0        h        h        h];
    w = [0 -1   0 -1 0   0; ...
         0  0   0  0 0   1; ...
         1  0   0  0 1   0];
for i=1:6
    if rot(i) == 1
        v(:,i) = cross(q(1:3,i), w(1:3,i));
    else
        v(:,i) = q(:,1);
    end
end
% 初始位姿
gst0 = [ 0 0 1       0; ...
         0 1 0 l1+l2; ...
        -1 0 0       h; ...
         0 0 0       1];

gst = [     0.7027      0.6895      0.1754     -1.0633
   -0.6322      0.4921      0.5984      0.7445
    0.3263     -0.5314      0.7818     -0.2244
         0           0           0      1.0000
];
```

```
    % 选解(0~3)
    for sol = 0:3
    %sol = 0;
        % 求解
        p3 = [0; l1+l2; h; 1];
        q1 = [0; 0; h; 1];
        g1 = gst*gst0^(-1);
        theta3 = norm(g1*p3 - q1) - l1 - l2;

        ew3 = [eye(3) q(1:3,i)*theta3; 0 0 0 1];
        [theta1, theta2] = subs.Sub2(ew3*p3, g1*p3, q1, [v(1:3,1);w(1:3,1)], [v(1:3,2);w(1:3,2)], bitand(sol,1));

        w_hat = [        0  -w(3,2)    w(2,2); ...
                      w(3,2)         0   -w(1,2); ...
                     -w(2,2)    w(1,2)          0];
        ew2_ = (eye(3) + w_hat*sin(theta2) + w_hat^2*(1-cos(theta2)));
        ew2 = [ew2_ ((eye(3)-ew2_)*w_hat*v(:,2)); 0 0 0 1];
        w_hat = [        0  -w(3,1)    w(2,1); ...
                      w(3,1)         0   -w(1,1); ...
                     -w(2,1)    w(1,1)          0];
        ew1_ = (eye(3) + w_hat*sin(theta1) + w_hat^2*(1-cos(theta1)));
        ew1 = [ew1_ ((eye(3)-ew1_)*w_hat*v(:,1)); 0 0 0 1];
        g2 = (ew1*ew2*ew3)^(-1)*g1;
        p5 = [0; l1+l2+0.2; h; 1];
        [theta4, theta5] = subs.Sub2(p5, g2*p5, p3, [v(1:3,4);w(1:3,4)] ,[v(1:3,5);w(1:3,5)], bitand(sol,2));

            w_hat = [        0  -w(3,4)    w(2,4); ...
                          w(3,4)         0   -w(1,4); ...
                         -w(2,4)    w(1,4)          0];
            ew4_ = (eye(3) + w_hat*sin(theta4) + w_hat^2*(1-cos(theta4)));
            ew4 = [ew4_ ((eye(3)-ew4_)*w_hat*v(:,4)); 0 0 0 1];
            w_hat = [        0  -w(3,5)    w(2,5); ...
                          w(3,5)         0   -w(1,5); ...
                         -w(2,5)    w(1,5)          0];
            ew5_ = (eye(3) + w_hat*sin(theta5) + w_hat^2*(1-cos(theta5)));
            ew5 = [ew5_ ((eye(3)-ew5_)*w_hat*v(:,5)); 0 0 0 1];
            g3 = (ew1*ew2*ew3*ew4*ew5)^(-1)*g1;
            p6 = [0.1; l1+l2+0.2; h; 1];
            theta6 = subs.Sub1(p6, g3*p6, p3, [v(1:3,6);w(1:3,6)]);
    end
```

取 $h = 0.3$, $l1 = l2 = 0.5$, 逆运动学得到 4 组解:

输入 [55, 22, 0.4, 96, -53, 11], 输出如下:

55.0011 26.8127 0.4000 -86.5362 -127.0053 -166.9043

-124.9989 162.9847 0.4000 76.7387 127.0053 13.0957

55.0011 26.8127 0.4000 93.4638 -52.9947 13.0957

-124.9989 162.9847 0.4000 -103.2613 52.9947 -166.9043