Homework 2

(Due time: 10:00, Apr. 5, 2024)

Note: If there is no extra statement, we assume that Frame {A}:$\{X_A, Y_A, Z_A\}$ is a spatial frame, and Frame {B}:$\{X_B, Y_B, Z_B\}$ is a body frame attached to a rigid body.

1. A vector v is rotated about Z_A by θ degrees and is subsequently rotated about X_A by φ degrees. Give the rotation matrix R_{ab} that accomplished these rotations in the given order. If $\theta = 30$ degrees and $\varphi = 45$ degrees, calculate the rotation matrix.

2. A frame {B} is located initially coincident with a frame {A}. We rotate {B} about Z_B by θ degrees, and then we rotate the resulting frame about X_B by φ degrees. Find the rotation matrix R_{ab} that will change the description of vectors from {B} to {A}. If $\theta = 30$ degrees and $\varphi = 45$ degrees, calculate the rotation matrix.

3. Find the homogeneous transformation matrix g_{ab} of the frame {B} with respect to the frame {A}, which is generated by the following sequence of rigid motion.

   (1) Rotation about Z_A by 90 degrees;

   (2) Rotation about X_A by -90 degrees;

   (3) Translation to the vector $(3,7,9)^T$.

4. The following matrix represents a homogeneous transformation matrix. Find the four unknown entries in the first column, a, b, c, d.

$$g = \begin{bmatrix} a & 0 & -1 & 0 \\ b & 0 & 0 & 1 \\ c & -1 & 0 & 2 \\ d & 0 & 0 & 1 \end{bmatrix}$$

5. Find the homogeneous transformation matrix g_{ab} of the frame {B} with respect to the frame {A}, which is generated by the following sequence of rigid motion.

   (1) Translation along the vector $(3,7,9)^T$;

   (2) Rotated about X_B by -90 degrees;

   (3) Rotated about Z_B by 90 degrees.

6. The following frame definitions are given as known

$$g_{ua} = \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 & 11 \\ 1/2 & \sqrt{3}/2 & 0 & -1 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$g_{ba} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 & 10 \\ 0 & 1/2 & \sqrt{3}/2 & -20 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$g_{cu} = \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 & -3 \\ \sqrt{3}/4 & 3/4 & -1/2 & -3 \\ 1/4 & \sqrt{3}/4 & \sqrt{3}/2 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Solve for g_{cb}.

7. (a) Referring to Fig. 2, give the value of g_{ab} ;

   (b) Referring to Fig. 2, give the value of g_{ac};

   (c) Referring to Fig. 2, give the value of g_{bc};

   (d) Referring to Fig. 2, give the value of g_{ca}.

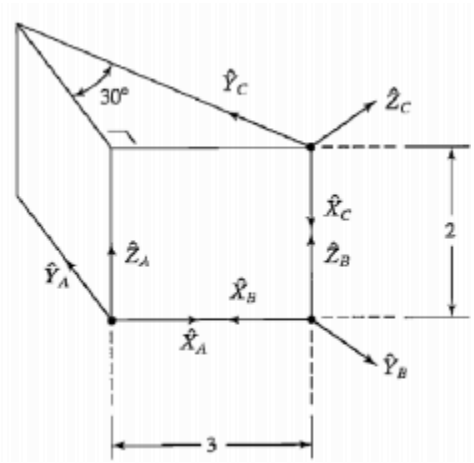

Fig.2. Frames at the corners of a wedge.

8. Given a homogeneous transformation matrix and we assume that it rotates around a certain axis.

$$g = \begin{bmatrix} 0 & 1 & 0 & 10 \\ 0 & 0 & -1 & 20 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

Find the equivalent rotational axis ω, rotation angle θ, and $\xi = [v^T, \omega^T]^T$.

9. Matlab programing

(a) Write a Matlab program function that transforms rigid transformation matrices from the **ZYZ convention** to the **exponential** representation, named **zyz2exp**; and write a Matlab program function that transforms rigid transformation matrices from the **exponential representation** to the **ZYZ** convention, named **exp2zyz**. Submit your Matlab code and give two examples to verify your program.

(b) Write a Matlab program function that transforms rigid transformation matrices from the **RPY convention** to the **exponential** representation, named **rpy2exp**; and write a program function that transforms rigid transformation matrices from the **exponential representation** to the **RPY** convention, named **exp2rpy**. Submit your Matlab code and give two examples to verify your program.