机器人学导论 作业 3（1-3 题）

210320621 吴俊达 更新于 2024.5.21

第 1 题

(a) 由指数积公式，$g_{c3}=e^{\xi_1\theta_1}e^{\xi_2\theta_2}g_{c3}(0)$

其中 $g_{c3}(0)=\begin{bmatrix}1&0&0&0\\0&1&0&l_1+l_2\\0&0&1&l_0+l_3\\0&0&0&1\end{bmatrix}$，$\xi_1=\begin{bmatrix}-\omega\times q\\\omega\end{bmatrix}=\begin{bmatrix}0&0&0&0&0&1\end{bmatrix}^T$，

$$\xi_2=\begin{bmatrix}-\omega\times q\\\omega\end{bmatrix}=\begin{bmatrix}l_1&0&0&0&0&1\end{bmatrix}^T$$

则 $e^{\xi_1\theta_1}=\begin{bmatrix}\cos\theta_1&-\sin\theta_1&0&0\\\sin\theta_1&\cos\theta_1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$，$e^{\xi_2\theta_2}=\begin{bmatrix}\cos\theta_2&-\sin\theta_2&0&l_1\sin\theta_2\\\sin\theta_2&\cos\theta_2&0&l_1(1-\cos\theta_2)\\0&0&1&0\\0&0&0&1\end{bmatrix}$

将上述 $e^{\xi_1\theta_1},e^{\xi_2\theta_2},g_{c3}(0)$ 代入即可求解。利用 MATLAB 计算得到（结果更新如下）

$$g_{c3}=\begin{bmatrix}\cos(\theta_1+\theta_2)&-\sin(\theta_1+\theta_2)&0&-l_2\sin(\theta_1+\theta_2)-l_1\sin\theta_1\\\sin(\theta_1+\theta_2)&\cos(\theta_1+\theta_2)&0&l_2\cos(\theta_1+\theta_2)+l_1\cos\theta_1\\0&0&1&l_0+l_3\\0&0&0&1\end{bmatrix}$$

(b) $g_{c0c1}(\theta_1)=\begin{bmatrix}\cos\theta_1&-\sin\theta_1&0&0\\\sin\theta_1&\cos\theta_1&0&0\\0&0&1&l_0\\0&0&0&1\end{bmatrix}$，$V_{c0c1}^s=\dot{\theta}_1\begin{bmatrix}0&0&0&0&0&1\end{bmatrix}^T$

【 $v=-\dot{R}R^T p+R^T\dot{p},\omega=(\dot{R}R^T)^\vee$ 】

$g_{c1c2}(\theta_2)=\begin{bmatrix}\cos\theta_2&-\sin\theta_2&0&0\\\sin\theta_2&\cos\theta_2&0&l_1\\0&0&1&l_3\\0&0&0&1\end{bmatrix}$，$V_{c1c2}^s=\dot{\theta}_2\begin{bmatrix}l_1&0&0&0&0&1\end{bmatrix}^T$

$$Ad_{g_{c0c1}}=\begin{bmatrix}R_{c0c1}&\begin{pmatrix}0\\0\\l_0\end{pmatrix}^\wedge R_{c0c1}\\0&R_{c0c1}\end{bmatrix}=\begin{bmatrix}R_{c0c1}&\begin{matrix}-l_0\sin\theta_1&-l_0\cos\theta_1&0\\l_0\cos\theta_1&-l_0\sin\theta_1&0\\0&0&0\end{matrix}\\0&R_{c0c1}\end{bmatrix}$$

则$V^s_{c0c2} = V^s_{c0c1} + Ad_{g_{c0c1}} V^s_{c1c2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \dot{\theta}_1 + \begin{bmatrix} l_1 \cos\theta_1 \\ l_1 \sin\theta_1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \dot{\theta}_2$。由$V^s_{c2c3} = 0$，因此

$$V^s_{c0c3} = V^s_{c0c2} + Ad_{g_{c0c2}} V^s_{c2c3} = V^s_{c0c2} = \begin{bmatrix} l_1 \cos\theta_1 \dot{\theta}_2 \\ l_1 \sin\theta_1 \dot{\theta}_2 \\ 0 \\ 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{bmatrix}。$$

(c) 由$V^b_{c2c3} = 0$，所以$V^b_{c0c3} = Ad_{g^{-1}_{c2c3}} V^b_{c0c2} + V^b_{c2c3} = Ad_{g^{-1}_{c2c3}} V^b_{c0c2}$

而$V^b_{c0c2} = V^b_{c1c2} + Ad_{g^{-1}_{c1c2}} V^b_{c0c1}$，其中$V^b_{c1c2} = Ad_{g^{-1}_{c1c2}} V^s_{c1c2}$

$$Ad_{g^{-1}_{c1c2}} = \begin{bmatrix} R^T_{c1c2} & -R^T_{c1c2} \begin{pmatrix} 0 \\ l_1 \\ l_3 \end{pmatrix}^{\wedge} \\ 0 & R^T_{c1c2} \end{bmatrix} = \begin{bmatrix} \cos\theta_2 & \sin\theta_2 & 0 & -l_3 \sin\theta_2 & l_3 \cos\theta_2 & -l_1 \\ -\sin\theta_2 & \cos\theta_2 & 0 & -l_3 \cos\theta_2 & -l_3 \sin\theta_2 & 0 \\ 0 & 0 & 1 & l_1 \cos\theta_2 & l_1 \sin\theta_2 & 0 \\ & & & \cos\theta_2 & \sin\theta_2 & 0 \\ & 0 & & -\sin\theta_2 & \cos\theta_2 & 0 \\ & & & 0 & 0 & 1 \end{bmatrix}$$

$$g_{c2c3} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}，则 Ad_{g^{-1}_{c2c3}} = \begin{bmatrix} R^T_{c2c3} & -R^T_{c2c3} \begin{pmatrix} 0 \\ l_2 \\ 0 \end{pmatrix}^{\wedge} \\ 0 & R^T_{c2c3} \end{bmatrix} = \begin{bmatrix} & 0 & 0 & -l_2 \\ I & 0 & 0 & 0 \\ & l_2 & 0 & 0 \\ 0 & & I & \end{bmatrix},$$

$R^T \dot{p} = 0$，

$$R^T \dot{R} = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 & 0 \\ -\sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\dot{\theta}_1 \sin\theta_1 & -\dot{\theta}_1 \cos\theta_1 & 0 \\ \dot{\theta}_1 \cos\theta_1 & -\dot{\theta}_1 \sin\theta_1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\dot{\theta}_1 & 0 \\ \dot{\theta}_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow V^b_{c0c1} = \dot{\theta}_1 \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T$$

则 $V_{c0c2}^{b}=Ad_{g_{c1c2}^{-1}}\left(\begin{bmatrix}0\\0\\0\\0\\0\\\dot{\theta}_1\end{bmatrix}+\begin{bmatrix}l_1\dot{\theta}_2\\0\\0\\0\\0\\\dot{\theta}_2\end{bmatrix}\right)=\begin{bmatrix}\cos\theta_2 l_1\dot{\theta}_2-l_1(\dot{\theta}_1+\dot{\theta}_2)\\-\sin\theta_2 l_1\dot{\theta}_2\\0\\0\\0\\\dot{\theta}_1+\dot{\theta}_2\end{bmatrix}$，进而

$$V_{c0c3}^{b}=\begin{bmatrix}\cos\theta_2 l_1\dot{\theta}_2-(l_2+l_1)(\dot{\theta}_1+\dot{\theta}_2)\\-\sin\theta_2 l_1\dot{\theta}_2\\0\\0\\0\\\dot{\theta}_1+\dot{\theta}_2\end{bmatrix}。$$

【此处和提供的标准答案不同，似乎有算错之处，但我没检查出来】

第 2 题

齐次变换矩阵为 $g_{bc}=\begin{bmatrix}R & p\\0 & 1\end{bmatrix}$，其中 $p=\begin{bmatrix}7\\-2\\5\end{bmatrix}$

$$R=R_z(-20°)R_y(-110°)=\begin{bmatrix}\cos(-20°) & -\sin(-20°) & 0\\\sin(-20°) & \cos(-20°) & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos(-110°) & 0 & \sin(-110°)\\0 & 1 & 0\\-\sin(-110°) & 0 & \cos(-110°)\end{bmatrix}$$

利用 MATLAB 计算得：

```
R =


   -0.3214    0.3420   -0.8830
    0.1170    0.9397    0.3214
    0.9397         0   -0.3420
```

又知点坐标为 $q_c=\begin{bmatrix}0.5\\0.2\\3.2\end{bmatrix}$，则由 $\begin{bmatrix}q_b\\1\end{bmatrix}=g_{bc}\begin{bmatrix}q_c\\1\end{bmatrix}$，解得 $q_b=\begin{bmatrix}4.0820\\-0.7251\\4.3754\end{bmatrix}$。

第 3 题

在 5 个单位时间后，$q_b = 5\begin{bmatrix} 1.9 \\ 0.1 \\ -0.3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.5 \\ 0 \end{bmatrix} = \begin{bmatrix} 9.5 \\ 1 \\ -1.5 \end{bmatrix}$

则 $\begin{bmatrix} q_a \\ 1 \end{bmatrix} = g_{ab} \begin{bmatrix} q_b \\ 1 \end{bmatrix} = \begin{bmatrix} 0.0722 & -0.963 & -0.259 & -5.00 \\ 0.954 & -0.00868 & 0.298 & -6.50 \\ -0.290 & -0.269 & 0.919 & 8.00 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 9.5 \\ 1 \\ -1.5 \\ 1 \end{bmatrix} = \begin{bmatrix} -4.9247 \\ 1.6303 \\ 3.4725 \\ 1 \end{bmatrix}$,

因此 $q_a = \begin{bmatrix} -4.9247 \\ 1.6303 \\ 3.4725 \end{bmatrix}$。