Homework 3

(Due time: 10:00, Apr. 19, 2024)

1. Figure 1 shows a two degree of freedom manipulator. Let l_0, l_1, l_2 be the link length parameters and θ_1, θ_2 the joint angle variables of link 1 and link 2, respectively.

   (a) Express the position and orientation of frame C_3 relative to frame C_0 in terms of the joint angle variables and the link parameters.

   (b) Compute the spatial velocity of C_3 relative to C_0 as functions of the joint angles and the joint rates.

   (c) Compute the body velocity of C_3 relative to C_0 as functions of the joint angles and the joint rates.

   (d) *Optional:* Find the spatial velocity of the origin of C_3 and use this to check your answer for parts (b) and (c).

   You may want to use a symbolic math package, such as the one described in Appendix B of our textbook, to carry out the computations in this exercise.

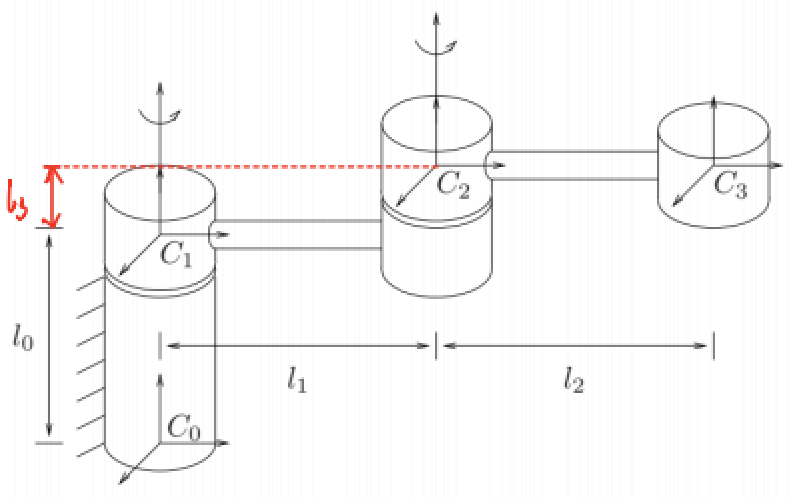


Fig.1. A two degree of freedom manipulator

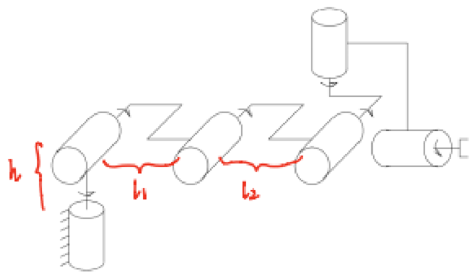
2. A coordinate frame, {B}, is located at the base of a robot manipulator. Frame {C} describes the position and orientation of a depth camera that was originally coincident with {B} then translated 7 units in x_B, translated -2 units in y_B, translated 5 units in z_B, rotated about z_c by -20 degrees, and rotated about y_c by -110 degrees. The camera detects an object having coordinates $q_c = [0.5\ 0.2\ 3.2]^T$. Determine the object coordinates in frame {B}, that is, q_b .

3. The position and velocity of an object are known to be $q_b(0) = [0 \quad 0.5 \quad 0]^T$ and $v_{q_b}(0) = [1.9 \quad 0.1 \quad -0.3]^T$ at time t_0. If the velocity is constant and

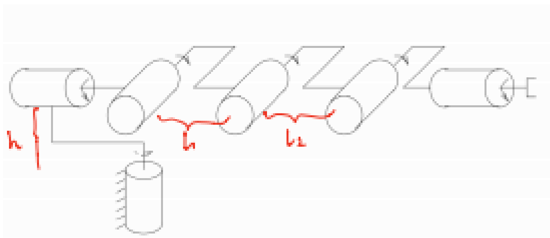
$$g_{ab} = \begin{bmatrix} 0.0722 & -0.963 & -0.259 & -5.00 \\ 0.954 & -0.00868 & 0.298 & -6.50 \\ -0.290 & -0.269 & 0.919 & 8.00 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then what is q_a after 5 units of time?

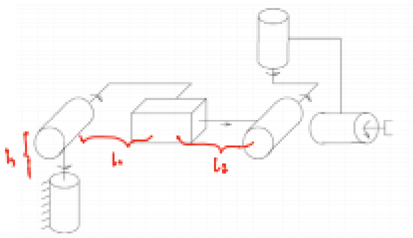
4. For each of the manipulators shown schematically in Figure 2, find the forward kinematics map and write the corresponding Matlab program function.



(i) Elbow manipulator



(ii) Inverse elbow manipulator



(iii) Stanford manipulator

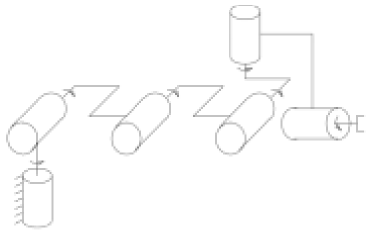
Fig 2. Sample manipulators. Revolute joints are represented by cylinders; prismatic joints are represented by rectangular boxes.

5. For each of the manipulators shown schematically in Figure 3,

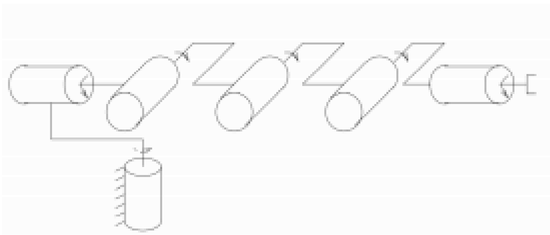
a) Solve the inverse kinematics problem using the Paden-Kahan subproblems.

b) Write Matlab functions for Paden-Kahan subproblem 1 to 3 in the text and use the functions to write Matlab programs solving the inverse kinematics.

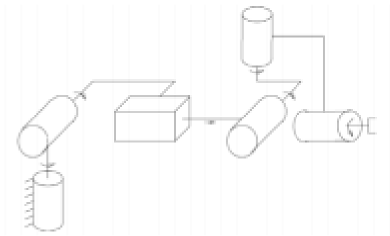
c) Verify your codes of the forward kinematics and the inverse kinematics by using two or more specific configuration examples.



(i) Elbow manipulator



(ii) Inverse elbow manipulator



(iii) Stanford manipulator

Fig 3. Sample manipulators. Revolute joints are represented by cylinders; prismatic joints are represented by rectangular boxes.