1. 对于单个 via point 时使用 LFPB 进行轨迹生成, 计算中使用到的物理量定义如图??。

设最大加速度为 $a > 0$, 则 $(t_1, \theta_2), (t_3, \theta_2), (t_3, \theta_3)$ 附近加速度分别为 $sgn(\theta_2 - \theta_1)a, sgn(\frac{\theta_3-\theta_2}{t_3-t_2} - \frac{\theta_2-\theta_1}{t_2-t_1})a, sgn(\theta_3 - \theta_2)a$。

考虑 $t_1 \sim t_2 - \frac{t_{b2}}{2}$ 段。匀速阶段, 此部分延长线过 (t_2, θ_2), 交 t 轴于 $t_1 + \frac{t_{b1}}{2}$, 且速度为 $a_1 t_{b1}$, 因此 $a_1 t_{b1}(t_2 - t_1 - \frac{t_{b1}}{2}) = \theta_2 - \theta_{b1}$, 其中 θ_{b1} 为 $t_1 + t_{b1}$ 时刻的角度。解得 $t_{b1} = (t_2 - t_1) - \sqrt{(t_2 - t_1)^2 - \frac{2(t_2-t_1)}{a_1}}$, 则此时 $v_1 = \frac{\theta_2-\theta_1}{t_2-t_1-\frac{t_{b1}}{2}}$。

类似地，可以得到 $t_{b3} = (t_3 - t_2) - \sqrt{(t_3 - t_2)^2 + \frac{2(t_3-t_2)}{a_3}}$, 则此时 $v_2 = \frac{\theta_3-\theta_2}{t_3-t_2-\frac{t_{b3}}{2}}$。

另一方面, $t_{b2} = \frac{v_2-v_1}{2}$。由此, 轨迹可由以下表达式生成:

$$\theta(t) = \begin{cases} \theta_1 + \frac{a_1(t-t_1)^2}{2}, t \le t_1 + t_{b1} \\ \theta_1 + v_1(t - t_1 - \frac{t_{b1}}{2}), t_1 + t_{b1} \le t \le t_2 - \frac{t_{b2}}{2} \\ \theta_1 + v_1 t_a + v_1(t - t_2 + \frac{t_{b2}}{2}) + a_2\frac{(t-t_2+\frac{t_{b2}}{2})^2}{2}, t_2 - \frac{t_{b2}}{2} \le t \le t_2 + \frac{t_{b2}}{2} \\ \theta_3 - v_2(t - t_3 + \frac{t_{b3}}{2}), t_2 + \frac{t_{b2}}{2} \le t \le t_3 - t_{b3} \\ \theta_3 - \frac{a_3(t-t_3)^2}{2}, t \ge t_3 - t_{b3} \end{cases}$$

其中当 $t_2 - \frac{t_{b2}}{2} \le t \le t_2 + \frac{t_{b2}}{2}$ 时,$t_a = t_2 - t_1 - \frac{t_{b1}+t_{b2}}{2}$。$\theta(t)$ 表达式中, 前 2 项为 $t = t_2 - \frac{t_{b2}}{2}$ 即开始此段匀变速运动时的角度, 后 2 项由匀变速运动表达式 $x = v_0 t + \frac{at^2}{2}$ 得到。

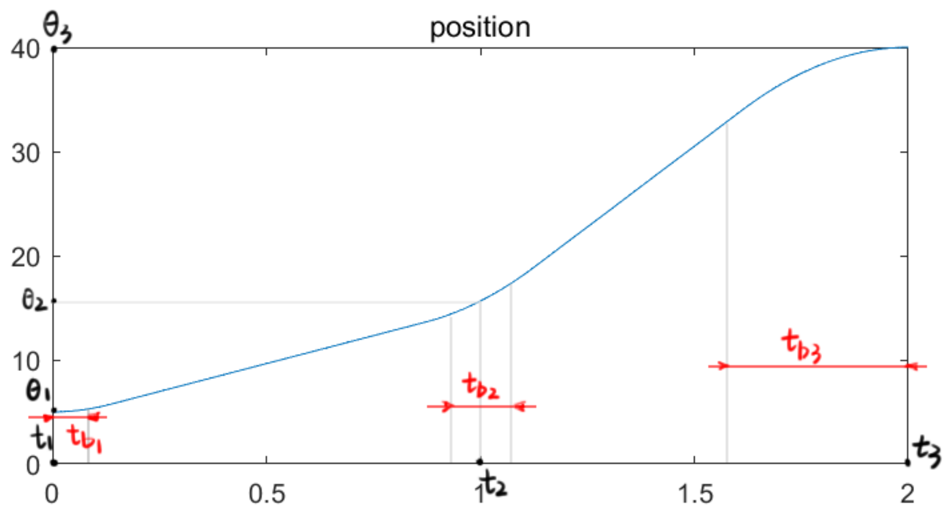

图 1: 物理量定义

速度与加速度如下:

$$\dot{\theta}(t)=\begin{cases}a_1(t-t_1), t\le t_1+t_{b1}\\ v_1, t_1+t_{b1}\le t\le t_2-\frac{t_{b2}}{2}\\ \frac{v_1+v_2}{2}+a_2(t-t_2), t_2-\frac{t_{b2}}{2}\le t\le t_2+\frac{t_{b2}}{2}\\ v_2, t_2+\frac{t_{b2}}{2}\le t\le t_3-t_{b3}\\ a_3(t-t_3), t\ge t_3-t_{b3}\end{cases}, \ddot{\theta}(t)=\begin{cases}a_1, t\le t_1+t_{b1}\\ 0, t_1+t_{b1}\le t\le t_2-\frac{t_{b2}}{2}\\ a_2, t_2-\frac{t_{b2}}{2}\le t\le t_2+\frac{t_{b2}}{2}\\ 0, t_2+\frac{t_{b2}}{2}\le t\le t_3-t_{b3}\\ a_3, t\ge t_3-t_{b3}\end{cases}$$

使用 Matlab 进行轨迹生成, 代码如下:

```matlab
clear;
clc;
% LFPB with a via point
% 最大加速度
max_acc = 80;
% 路径点时间与位置
t = [0, 1, 2];
pos = [5, 15, 40];

tb = zeros(1, 3);
vel = zeros(1, 2);
acc = zeros(1, 3);
acc(1) = max_acc * sign(pos(2)-pos(1));
acc(2) = max_acc * sign( (pos(3)-pos(2))/(t(3)-t(2)) - (pos(2)-pos(1))/(t(2)-t(1)) );
acc(3) = max_acc * -sign(pos(3)-pos(2));

% t1~t2
tb(1) = t(2)-t(1) - sqrt((t(2)-t(1))^2 - 2*(pos(2)-pos(1))/acc(1));
vel(1) = (pos(2)-pos(1)) / (t(2)-t(1) - tb(1)/2);
% t2~t3
tb(3) = (t(3)-t(2)) - sqrt((t(3)-t(2))^2 + 2*(pos(3)-pos(2))/acc(3));
vel(2) = (pos(3)-pos(2)) / (t(3)-t(2) - tb(3)/2);
% via point处变速用时
tb(2) = (vel(2) - vel(1)) / acc(2);

t_traj = t(1):0.01:t(3);
pos_traj = zeros(1, length(t_traj));
vel_traj = zeros(1, length(t_traj));
acc_traj = zeros(1, length(t_traj));
for i=1:length(t_traj)
    if t_traj(i) <= tb(1)
        pos_traj(i) = pos(1) + acc(1) * (t_traj(i)-t(1))^2 / 2;
        vel_traj(i) = acc(1) * (t_traj(i)-t(1));
        acc_traj(i) = acc(1);
    elseif t_traj(i) <= t(2)-tb(2)/2
        pos_traj(i) = pos(1) + vel(1) * (t_traj(i)-t(1)-tb(1)/2);
        vel_traj(i) = vel(1);
        acc_traj(i) = 0;
    elseif t_traj(i) <= t(2)+tb(2)/2
        % 前两项为pos2附近开始变速时的位置
        t_acc = (t(2)-t(1)-tb(1)/2-tb(2)/2);
        pos_traj(i) = pos(1) + vel(1)*t_acc + vel(1)*(t_traj(i)-(t(2)-tb(2)/2)) +
            acc(2)*(t_traj(i)-(t(2)-tb(2)/2))^2/2;
        vel_traj(i) = (vel(1)+vel(2))/2 + (t_traj(i)-t(2))*acc(2);
        acc_traj(i) = acc(2);
    elseif t_traj(i) <= t(3)-tb(3)
```

```
        % 前两项为pos2附近开始变速时的位置
        pos_traj(i) = pos(3) + vel(2) * (t_traj(i)-(t(3)-tb(3)/2));
        vel_traj(i) = vel(2);
        acc_traj(i) = 0;
    else
        pos_traj(i) = pos(3) + acc(3) * (t_traj(i) - t(3))^2 / 2;
        vel_traj(i) = acc(3) * (t_traj(i) - t(3));
        acc_traj(i) = acc(3);
    end
end

subplot(2,2,1);
plot(t_traj, pos_traj);
title("position");
subplot(2,2,2);
plot(t_traj, vel_traj);
title("velocity");
subplot(2,2,3);
plot(t_traj, acc_traj);
title("acceleration");
```

运行结果如图??。

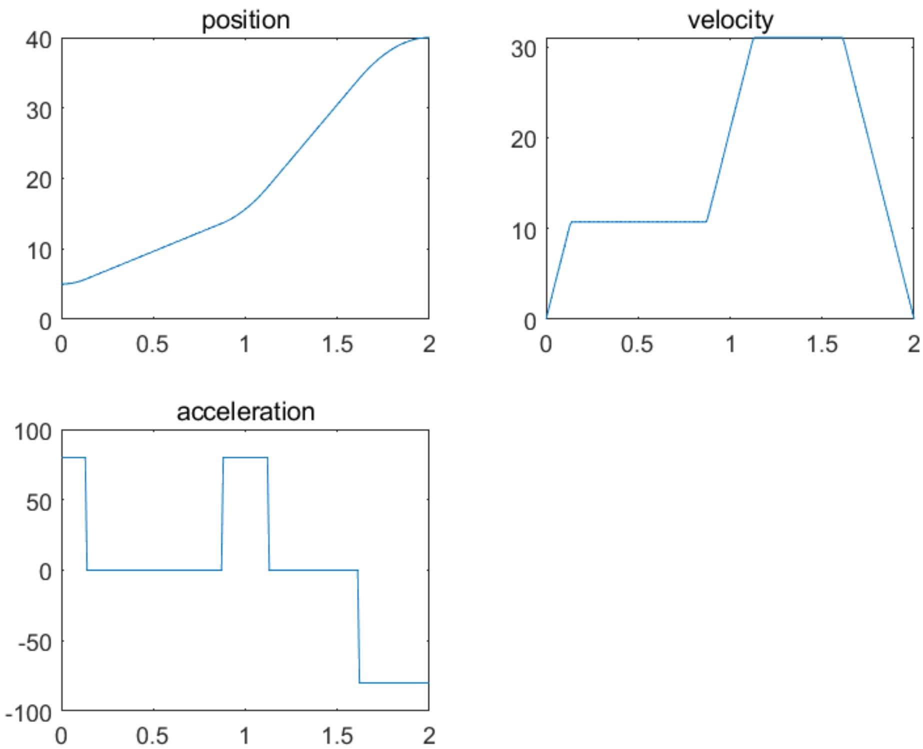

图 2: 轨迹生成结果

2. 记 $\dot\theta_{\theta=\theta_0}=\dot\theta_0,\dot\theta_{\theta=\theta_v}=\dot\theta_v,\dot\theta_{\theta=\theta_g}=\dot\theta_g$。

考虑两条三次轨迹满足如下表达式: $\dot\theta(t)=\begin{cases}\theta=a_{00}+a_{01}(t-t_0)+a_{02}(t-t_0)^2+a_{03}(t-t_0)^3, t_0\le t\le t_v\\ \theta=a_{10}+a_{11}(t-t_v)+a_{12}(t-t_v)^2+a_{13}(t-t_v)^3, t_v\le t\le t_g\end{cases}$

由于 $\theta_0,\dot\theta_0,\theta_v,\dot\theta_v,\theta_g,\dot\theta_g$ 已知, 则可得如下表达式:

$$\dot\theta(t)=\begin{cases}\theta_0=a_{00}\\ \dot\theta_0=a_{01}\\ \theta_v=a_{00}+a_{01}(t_v-t_0)+a_{02}(t_v-t_0)^2+a_{03}(t_v-t_0)^3\\ \dot\theta_v=a_{01}+2a_{02}(t_v-t_0)+3a_{03}(t_v-t_0)^2\\ \theta_v=a_{10}\\ \dot\theta_v=a_{11}\\ \theta_g=a_{10}+a_{11}(t_g-t_v)+a_{12}(t_g-t_v)^2+a_{13}(t_g-t_v)^3\\ \dot\theta_g=a_{11}+2a_{12}(t_g-t_v)+3a_{13}(t_g-t_v)^2\end{cases},$$

$$\begin{bmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\1&\Delta t_{v0}&\Delta t_{v0}^2&\Delta t_{v0}^3&0&0&0&0\\0&1&2\Delta t_{v0}&3\Delta t_{v0}^2&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&1&\Delta t_{gv}&\Delta t_{gv}^2&\Delta t_{gv}^3\\0&0&0&0&0&1&2\Delta t_{gv}&3\Delta t_{gv}^2\end{bmatrix}\begin{bmatrix}a_{00}\\a_{01}\\a_{02}\\a_{03}\\a_{10}\\a_{11}\\a_{12}\\a_{13}\end{bmatrix}=\begin{bmatrix}\theta_0\\\dot\theta_0\\\theta_v\\\dot\theta_v\\\theta_v\\\dot\theta_v\\\theta_g\\\dot\theta_g\end{bmatrix}$$

其中 $\Delta t_{v0}=t_v-t_0,\Delta t_{gv}=t_g-t_v$。记上式为 $Pa=b$, 可解出两条三次轨迹的各项系数。

Matlab 代码如下:

```
clear;
clc;

P = [
    1 0 0 0 0 0 0 0; ...
    0 1 0 0 0 0 0 0; ...
    1 2 4 8 0 0 0 0; ...
    0 1 4 12 0 0 0 0; ...
    0 0 0 0 1 0 0 0; ...
    0 0 0 0 0 1 0 0; ...
    0 0 0 0 1 2 4 8; ...
    0 0 0 0 0 1 4 12; ...
];
b = [5; 0; 15; 0; 15; 0; -10; 0];
% 计算系数
a = P^(-1)*b;

t_traj = 0:0.01:4;
pos_traj = zeros(1, length(t_traj));
vel_traj = zeros(1, length(t_traj));
```

```
    acc_traj = zeros(1, length(t_traj));
    for i=1:length(t_traj)
        if t_traj(i) <= 2
            pos_traj(i) = a(1) + a(2)*t_traj(i) + a(3)*t_traj(i)^2 + a(4)*t_traj(i
                )^3;
            vel_traj(i) = a(2)*t_traj(i) + 2*a(3)*t_traj(i) + 3*a(4)*t_traj(i)^2;
            acc_traj(i) = 2*a(3) + 6*a(4)*t_traj(i);
        else
            pos_traj(i) = a(5) + a(6)*(t_traj(i)-2) + a(7)*(t_traj(i)-2)^2 + a(8)
                *(t_traj(i)-2)^3;
            vel_traj(i) = a(6)*(t_traj(i)-2) + 2*a(7)*(t_traj(i)-2) + 3*a(8)*(
                t_traj(i)-2)^2;
            acc_traj(i) = 2*a(7) + 6*a(8)*(t_traj(i)-2);
        end
    end

    subplot(2,2,1);
    plot(t_traj, pos_traj);
    title("position");
    subplot(2,2,2);
    plot(t_traj, vel_traj);
    title("velocity");
    subplot(2,2,3);
    plot(t_traj, acc_traj);
    title("acceleration");
```

运行结果如图**??**, 轨迹表达式为 $\dot{\theta}(t) = \begin{cases} \theta = 5 + 7.5(t-t_0)^2 - 2.5(t-t_0)^3, t_0 \le t \le t_v \\ \theta = 15 - 18.75(t-t_v)^2 + 6.25(t-t_v)^3, t_v \le t \le t_g \end{cases}$ 。

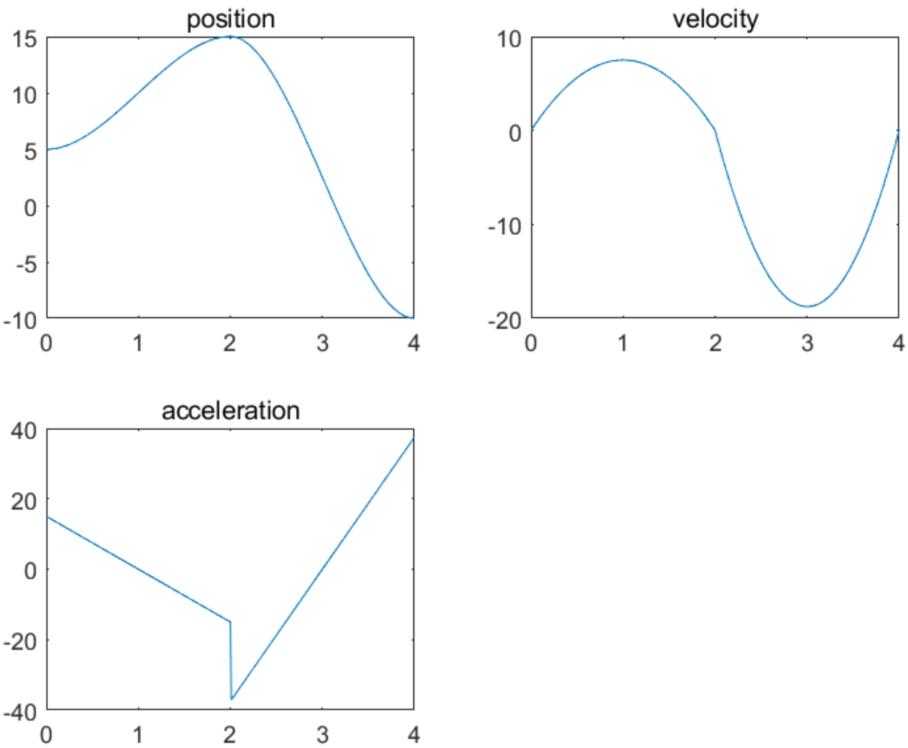

图 3: 轨迹生成结果

3. $\theta(t) = 10 + 90t^2 - 60t^3, \dot{\theta}(t) = 180t - 180t^2, \ddot{\theta}(t) = -360t$

则 $t = 0$ 时,$\theta(0) = 10, \dot{\theta}(0) = 0, \ddot{\theta}(0) = 0$。$t = 1$ 时,$\theta(1) = 40, \dot{\theta}(1) = 0, \ddot{\theta}(1) = -360$。

4. 采用题 (1) 中方法, 在 x, y 方向上分别规划轨迹, 即在题 1 代码中替换输入变量:

```
t = [0, 1, 2];
pos = [0, 2, 3]; % x方向
% pos = [0, 2, 3]; % y方向
```

得 x,y 方向规划结果分别如图??与图??。

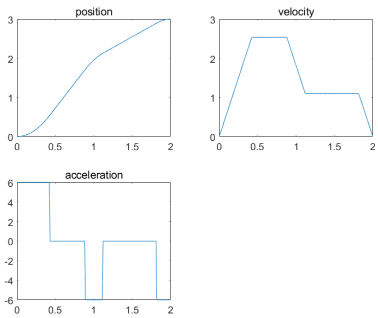

图 4: x 方向规划结果

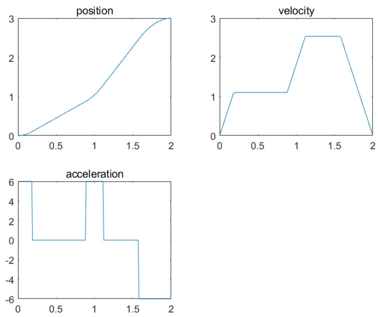

图 5: y 方向规划结果

以 x, y 方向位于为坐标轴, 实际轨迹如图**??**。

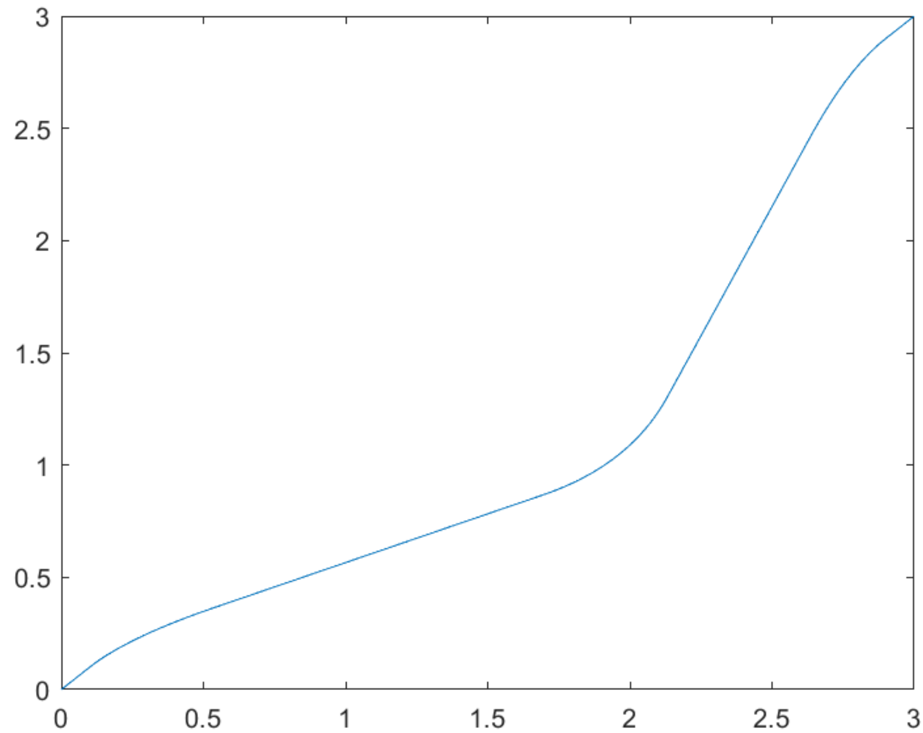

图 6: 规划轨迹