1 正逆运动学推导与代码实现

1.1 正运动学

给定机器人机构定义 $\xi_1, \ldots, \xi_6$, 初始位姿 g_0 与各关节角度 $\theta_1, \ldots, \theta_6$, 则机器人末端位姿为 $g_{st} = e^{\hat{\xi_1}\theta_1} \ldots e^{\hat{\xi_6}\theta_6} g_0$。

Matlab 代码如下: 定义函数 Fkine, 为 6R 机器人通用正运动学计算

```
function g_st = Fkine(Xi,theta,g0)

    for i=1:6
        w_hat = [        0    -Xi(6,i)     Xi(5,i); ...
                   Xi(6,i)           0    -Xi(4,i); ...
                  -Xi(5,i)     Xi(4,i)           0];
        ew = eye(3) + w_hat*sin(theta(i)) + w_hat^2*(1-cos(theta(i)));
        e(:,:,i) = [ew (eye(3)-ew)*w_hat*Xi(1:3,i); 0 0 0 1];
    end

    g_st = g0;
    for i=6:-1:1
        g_st = e(:,:,i)*g_st;
    end

end
```

对该机器人, 初始位姿:$g_0 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & L_1+L_2+L_3+L_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

机器人机构:$\begin{bmatrix} \xi_1 \ldots \xi_6 \end{bmatrix} = \begin{bmatrix} 0 & -L_1 & -L_1-L_2 & 0 & -L_1-L_2-L_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$

代入数据:

```
gst0 = [
    -1 0 0 0; ...
    0 -1 0 0; ...
    0 0 1 1475; ...
    0 0 0 1
];

q = [0   0   0   0   0   0; ...
     0   0   0   0   0   0; ...
     0   491   941   941   1391   1391];
w = [0   0   0   0   0   0; ...
     0   1   1   0   1   0; ...
     1   0   0   1   0   1];

```

```
for i=1:6
    v(:,i) = cross(q(1:3,i), w(1:3,i));
end

for i=1:6
    Xi(:,i) = [v(:,i); w(:,i)];
end

theta = [0.3491    -1.0472    0.7854    0.2618    0.4488    0.6283];
g_st = Fkine(Xi, theta, gst0);
```

结果如下:

g_st =

-0.3276 0.9368 0.1229 -465.3276

-0.9242 -0.345 0.1642 -159.3449

0.1962 -0.0597 0.9787 1232.8797

0 0 0 1

使用 Simulink 仿真正运动学结果:

Px = -465.3276, Py = -159.3449, Pz = 1232.8797

R =

-0.3276 0.9368 0.1229

-0.9242 -0.345 0.1642

0.1962 -0.0597 0.9787

1.2 逆运动学

对该机器人, 取 $p_1 = \begin{bmatrix} 0 \\ 0 \\ L_1 \\ 1 \end{bmatrix}, p_2 = \begin{bmatrix} 0 \\ 0 \\ L_1 + L_2 \\ 1 \end{bmatrix}, p_3 = \begin{bmatrix} 0 \\ 0 \\ L_1 + L_2 + L_3 \\ 1 \end{bmatrix}, p_4 = \begin{bmatrix} 0 \\ 0 \\ L_1 + L_2 + L_3 + L_4 \\ 1 \end{bmatrix}, p_5 = \begin{bmatrix} 0 \\ 0.1 \\ L_1 + L_2 + L_3 + L_4 \\ 1 \end{bmatrix}$,, 即图 1中所示。

令 $g_1 = e^{\hat{\xi_1}\theta_1} \dots e^{\hat{\xi_6}\theta_6} = g_{st}g_0^{-1}$, 则 $||g_1 p_3 - p_1|| = ||e^{\hat{\xi_1}\theta_1}e^{\hat{\xi_2}\theta_2}(e^{\hat{\xi_3}\theta_3}p_3 - p_1)|| = ||e^{\hat{\xi_3}\theta_3}p_3 - p_1||$, 由 Sub 3 解出 θ_3。

$e^{\hat{\xi_1}\theta_1}e^{\hat{\xi_2}\theta_2}(e^{\hat{\xi_3}\theta_3}p_3) = g_1 p_3$, 由 Sub 2 解出 θ_1, θ_2。

令 $g_2 = e^{\hat{\xi_4}\theta_4}e^{\hat{\xi_5}\theta_5}e^{\hat{\xi_6}\theta_6} = (e^{\hat{\xi_1}\theta_1}e^{\hat{\xi_2}\theta_2}e^{\hat{\xi_3}\theta_3})^{-1}g_1$, 则 $e^{\hat{\xi_4}\theta_4}e^{\hat{\xi_5}\theta_5}p_4 = g_2 p_4$, 由 Sub 2 解出 θ_4, θ_5。

此时 $g_3 = e^{\hat{\xi_6}\theta_6} = (e^{\hat{\xi_4}\theta_4}e^{\hat{\xi_5}\theta_5})^{-1}g_2$, $e^{\hat{\xi_6}\theta_6}p_5 = g_3 p_5$, 由 Sub 1 解出 θ_6。

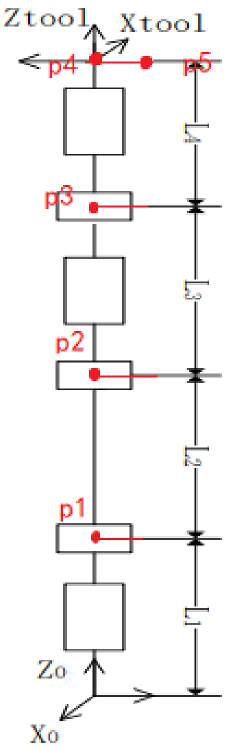

图 1: 逆运动学求解使用到的点

Matlab 代码如下:

```
function theta = Ikine6s(gst,L)
    subs = subquestions;

    for i=0:7
        % 初始姿态
        g0 = [-1 0 0 0; 0 -1 0 0; 0 0 1 sum(L); 0 0 0 1];
        % 取机械臂上点
        p1 = [0; 0; L(1); 1];
        p2 = [0; 0; sum(L(1:2)); 1];
        p3 = [0; 0; sum(L(1:3)); 1];
        p4 = [0; 0; sum(L(1:4)); 1];
        p5 = [0; 0.1; sum(L(1:4)); 1];

        g1 = gst*inv(g0);
        theta(i+1,3) = subs.Sub3(p3, p1, p2, [-sum(L(1:2));0;0;0;1;0], norm(g1*p3-p1), bitand(i,4));

        % 求解theta3
        ew3 = [cos(theta(i+1,3)) 0 sin(theta(i+1,3)); 0 1 0; -sin(theta(i+1,3)) 0 cos(theta(i+1,3))];
        e3 = [ew3 (eye(3)-ew3)*[0 0 1; 0 0 0; -1 0 0]*[-sum(L(1:2));0;0]; 0 0 0 1];
```

```
        [theta(i+1,1),theta(i+1,2)] = subs.Sub2(e3*p3, g1*p3, p1, [0;0;0;0;0;1], [-L(1);0;0;0;1;0], bitand(i,2));

        % 求解theta1,2
        ew1 = [cos(theta(i+1,1)) -sin(theta(i+1,1)) 0; sin(theta(i+1,1)) cos(theta(i+1,1)) 0; 0 0 1];
        e1 = [ew1 [0;0;0]; 0 0 0 1];
        ew2 = [cos(theta(i+1,2)) 0 sin(theta(i+1,2)); 0 1 0; -sin(theta(i+1,2)) 0 cos(theta(i+1,2))];
        e2 = [ew2 (eye(3)-ew2)*[0 0 1; 0 0 0; -1 0 0]*[-L(1);0;0]; 0 0 0 1];
        g2 = inv(e1*e2*e3)*g1;
        [theta(i+1,4),theta(i+1,5)] = subs.Sub2(p4, g2*p4, p3, [0;0;0;0;0;1], [-sum(L(1:3));0;0;0;1;0], bitand(i,1));

        % 求解theta4,5
        ew4 = [cos(theta(i+1,4)) -sin(theta(i+1,4)) 0; sin(theta(i+1,4)) cos(theta(i+1,4)) 0; 0 0 1];
        e4 = [ew4 [0;0;0]; 0 0 0 1];
        ew5 = [cos(theta(i+1,5)) 0 sin(theta(i+1,5)); 0 1 0; -sin(theta(i+1,5)) 0 cos(theta(i+1,5))];
        e5 = [ew5 (eye(3)-ew5)*[0 0 1; 0 0 0; -1 0 0]*[-sum(L(1:3));0;0]; 0 0 0 1];

        % 求解theta6
        g3 = inv(e4*e5)*g2;
        theta(i+1,6) = subs.Sub1(p5, g3*p5, p3, [0;0;0;0;0;1]);
    end
end
```

结果如下:

theta =

-2.7925 0.2618 0.7854 -3.0219 1.2231 0.8242

-2.7925 0.2618 0.7854 0.1197 -1.2231 -2.3174

0.3491 -1.0472 0.7854 0.2618 0.4488 0.6283

0.3491 -1.0472 0.7854 -2.8798 -0.4488 -2.5133

-2.7925 1.0472 5.4978 -2.8798 0.4488 0.6283

-2.7925 1.0472 5.4978 0.2618 -0.4488 -2.5133

0.3491 -0.2618 5.4978 0.1197 1.2231 0.8242

0.3491 -0.2618 5.4978 -3.0219 -1.2231 -2.3174

2 机器人仿真

对以上 8 组解进行 Simulink 仿真, 结果如下。

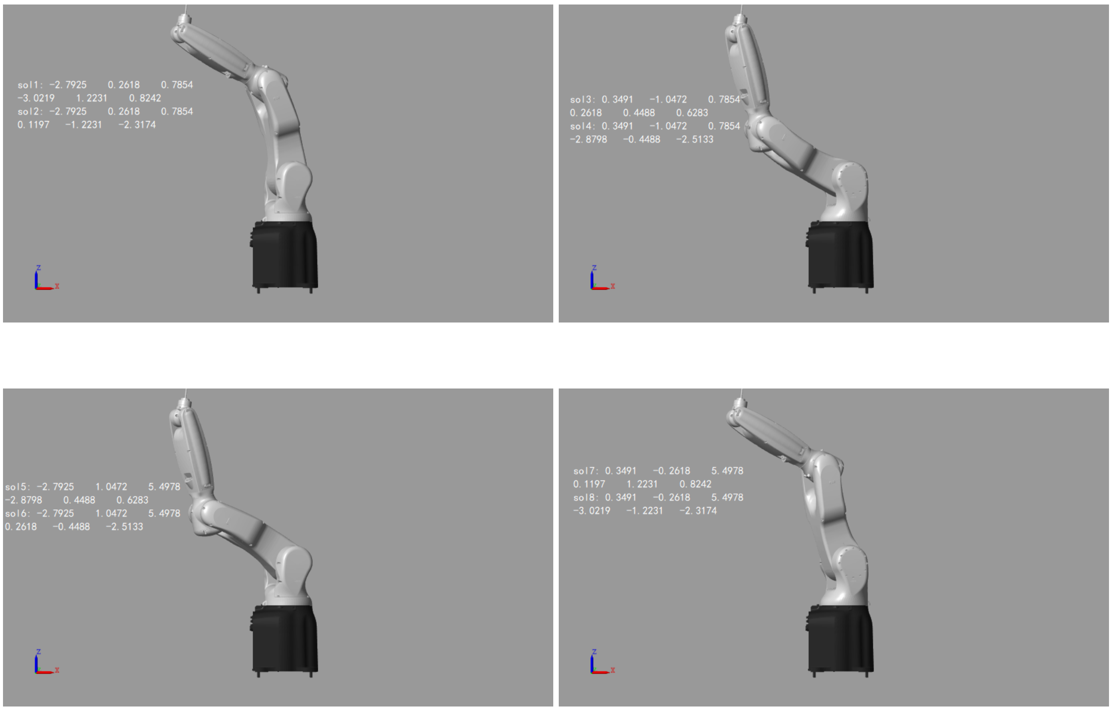

图 2: Simulink 仿真结果

3 选取逆解的方法

得到 8 组解后, 根据机器人各关节的限制条件, 选取其中一个可行解。在轨迹规划或伺服系统中, 可选取一个与当前机器人姿态最”相近”的可行解, 例如令 $F = \sum_{i=1}^{6}(\theta_{isol} - \theta_{inow})$, 选取 F 最小的一组解为关节空间目标。