数学规划 第一次作业

2022年3月14日 16:54

[redacted]

1. 解 设生产产品A、B的质量分别为 x_1, x_2

利润最大 $\max S = 7x_1 + 12x_2$

$$\text{s.t.}\begin{cases} 9x_1 + 4x_2 \le 360 \\ 4x_1 + 5x_2 \le 200 \\ 3x_1 + 10x_2 \le 300 \\ x_1, x_2 \ge 0 \end{cases}$$

3. 解 设每块地种植数量为

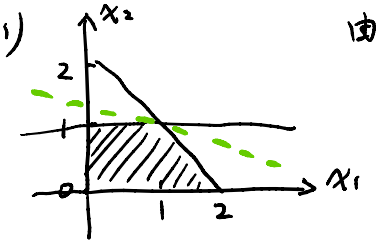
	甲	乙	丙
水稻	x_1	x_2	x_3
大豆	x_4	x_5	x_6
玉米	x_7	x_8	x_9

利润最大 $\max S = 700x_1 + 600x_2 + 600x_3 + 300x_4 + 350x_5 + 250x_6 + 900x_7 + 800x_8 + 700x_9$

$$\begin{cases} 700x_1 + 600x_2 + 600x_3 \ge 130000 \\ 300x_4 + 350x_5 + 250x_6 \ge 40000 \\ 900x_7 + 800x_8 + 700x_9 \ge 250000 \\ x_i \ge 0,\ i \in N_+,\ 1 \le i \le 9 \\ x_1 + x_4 + x_7 \le 200 \\ x_2 + x_5 + x_8 \le 400 \\ x_3 + x_6 + x_9 \le 600 \end{cases}$$

以上即为所求模型

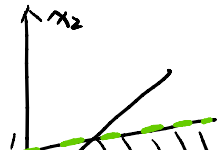
5. (1)



由图可知 $(x_1, x_2) = (1, 1)$ 时

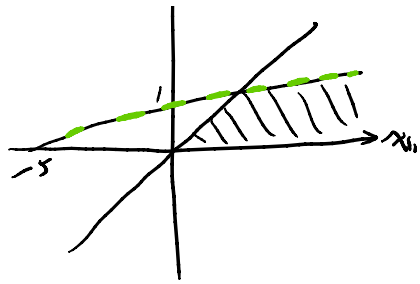
$\max S = 3$

(2)



由图知 (x_1, x_2) 在

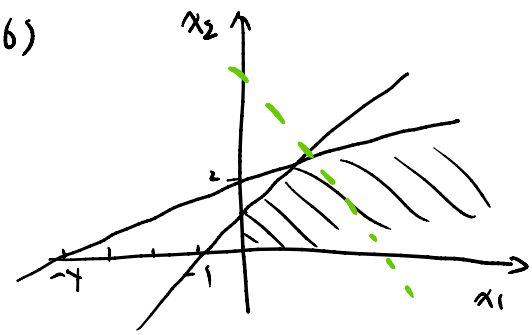
直线 $x_1 - 5x_2 = -5$ 上时



直线 $x_1-5x_2=-5$ 上时

$\max S=-10$

(6)



由图知 $x_1\to+\infty$ 时

$\max S\to+\infty$

6 (1) $\min(-S)=-x_1-x_2-x_3$

$$\begin{cases} -x_1 \quad -2x_3+x_4=5 \\ 2x_1-3x_2+x_3+x_5=3 \\ 3x_1-5x_2+6x_3+x_6=5 \\ x_i\geq 0,\quad i\in N_+ \text{且} 1\leq i\leq 6 \end{cases}$$

选取基变量 x_4 x_5 x_6

			x_1	x_2	x_3	x_4	x_5	x_6
			-1	-1	-1	0	0	0
C_B	x_B	0	-1	-1	-1	0	0	0
0	x_4	5	-1	0	-2	1	0	0
0	x_5	3	3	2	-3	0	1	0
0	x_6	5	2	-5	6	0	0	1

不存在有限最优解

(2) 选取基变量 x_1, x_4, x_6

			x_1	x_2	x_3	x_4	x_5	x_6
			-1	-3	-3	0	0	0
C_B	x_B	15	8	0	3	0	3	0
-3	x_2	5	3	1	2	0	1	0
0	x_4	2	1	0	1	0	2	1
0	x_6	6	1	0	2	1	2	0

$x^*=(0,5,0,2,0,6)^T$, $S=-15$

(3) 选取基变量 x_2 x_4 x_5

			x_1	x_2	x_3	x_4	x_5	x_6
			1	-1	1	1	1	-1
C_B	x_B	-13	2	0	-6	0	0	-7
1	x_4	9	1	1	0	1	0	6
-1	x_2	2	3	1	-4	0	0	2
1	x_5	6	1	0	3	0	1	2

代消得

			x_1	x_2	x_3	x_4	x_5	x_6
C_B	x_B	$\frac{7}{2}$	$\frac{5}{2}$	0	0	$\frac{1}{2}$	2	0
	x_6	$\frac{3}{2}$	$\frac{1}{6}$	0	0	$\frac{1}{6}$	0	1
	x_2	3	$\frac{32}{9}$	1	0	$-\frac{2}{9}$	$\frac{4}{3}$	0
	x_3	1	$\frac{2}{9}$	0	1	$-\frac{1}{9}$	$\frac{1}{3}$	0

可得 $x^* = (0, 3, 1, 0, 0, \frac{3}{2})^T$, $s = -\frac{7}{2}$

7. 解(1) $\min Z = y_1 + y_2$

$$\text{s.t.} \begin{cases} x_1 + 2x_2 + 3x_3 + y_1 = 6 \\ 4x_1 + 5x_2 - 6x_3 + y_2 = 6 \\ x_1, x_2, x_3, y_1, y_2 \geq 0 \end{cases}$$

			0	0	0	1	1
			x_1	x_2	x_3	y_1	y_2
C_B		-12	-5	-7	3	0	0
	y_1	6	1	2	3	1	0
	y_2	6	4	5	-6	0	1

∴

			x_1	x_2	x_3	y_1	y_2
C_B		$-\frac{9}{2}$	0	$-\frac{3}{4}$	$-\frac{9}{2}$	0	$-\frac{5}{4}$
	y_1	$\frac{9}{2}$	0	$\frac{3}{4}$	$\frac{9}{2}$	1	$-\frac{1}{4}$
	x_1	$\frac{3}{2}$	1	$\frac{3}{4}$	$-\frac{3}{2}$	0	$\frac{1}{4}$

∴

			x_1	x_2	x_3	y_1	y_2
C_B		$-\frac{18}{5}$	$\frac{3}{5}$	0	$-\frac{27}{5}$	0	$\frac{7}{5}$
	y_1	$\frac{18}{5}$	$-\frac{3}{5}$	0	$\frac{27}{5}$	1	$-\frac{7}{5}$
	x_2	$\frac{6}{5}$	$\frac{4}{5}$	1	$-\frac{6}{5}$	0	$\frac{1}{5}$

∴

			x_1	x_2	x_3	y_1	y_2
C_B		0	0	0	0	1	1
	x_3	$\frac{2}{3}$	$-\frac{1}{9}$	0	1	$\frac{1}{27}$	$-\frac{1}{27}$
	x_2	2	$\frac{1}{3}$	1	0	$\frac{2}{9}$	$\frac{1}{9}$

∴

			x_1	x_2	x_3
c_B	x_3	$\frac{4}{3}$	2	0	0
	x_3	$\frac{2}{3}$	$-\frac{1}{9}$	0	1
	x_2	2	$\frac{2}{3}$	1	0

$x^* = (0, 2, \frac{2}{3})$ $\quad S = -\frac{4}{3}$

(2) $\min S = y_1 + y_2 + y_3$

$$\begin{cases} x_1 + x_2 + x_3 = 5 \\ -6x_1 + 10x_2 + 5x_3 + x_4 = 20 \\ 5x_1 - 3x_2 + x_3 - x_5 = 15 \\ x_1, x_2, x_3, y_1, y_2, y_3 \geq 0 \end{cases}$$

		x_1	x_2	x_3	x_4	x_5	y_1	y_2
	-20	-6	2	-2	0	1	0	0
y_1	5	1	1	1	0	0	1	0
x_4	20	-6	10	5	1	0	0	0
x_2	15	5	-3	1	0	-1	0	1

∴

		x_1	x_2	x_3	x_4	x_5	y_1	y_2
	-2	0	$-\frac{8}{5}$	$-\frac{4}{5}$	0	$-\frac{1}{5}$	0	$\frac{6}{5}$
y_1	2	0	$\frac{8}{5}$	$\frac{4}{5}$	0	$\frac{1}{5}$	1	$-\frac{1}{5}$
x_4	38	0	$\frac{52}{5}$	$\frac{31}{5}$	1	$-\frac{6}{5}$	0	$\frac{6}{5}$
x_1	3	1	$-\frac{3}{5}$	$-\frac{3}{5}$	0	$-\frac{1}{5}$	0	$\frac{1}{5}$

∴

			x_2	x_3	x_4	x_5	y_1	y_2

∴

		x_1	x_2	x_3	x_4	x_5	y_1	y_2
	0	0	0	0	0	0	1	1
x_2	$\frac{5}{4}$	0	1	$\frac{1}{2}$	0	$-\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
x_4	80	0	0	3	1	2	-4	2
x_1	$\frac{15}{4}$	1	0	$\frac{1}{2}$	0	$-\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

∴ $x^* = (5, 0, 0)$ $s = 20$