数学规划 第二次作业

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[redacted]

2-1 解 (1) $\max Z = -4y_1 - 5y_2 + y_3$

$$\begin{cases} -2y_1 - y_2 - 2y_3 \le 5 \\ y_1 - 2y_2 - y_3 \le 3 \\ 4y_1 - 2y_2 + y_3 \le 0 \\ y_i \ge 0,\ i = 1,2,3 \end{cases}$$

(2) $\min Z = 10y_1 + 10y_2$

$$\begin{cases} y_1 + 2y_2 \ge 4 \\ 2y_1 + 3y_2 \ge 7 \\ y_1 + 3y_2 \ge 2 \\ y_1, y_2 \ge 0 \end{cases}$$

(3) $\max Z = 3y_1 + 5y_2$

$$\begin{cases} y_1 + 2y_2 \le 2 \\ 2y_1 + y_2 \le 1 \\ 2y_1 + 3y_2 = 4 \\ y_1, y_2 \ge 0 \end{cases}$$

2-2 解 (1) $\min S = x_1 + 2x_2 + 3x_3$

$$\begin{cases} -x_1 + x_2 - x_4 + x_4 = -4 \\ x_1 + x_2 + 2x_3 + x_5 = 7 \\ -x_2 + x_3 + x_6 = -7 \\ x_i \ge 0,\ 1 \le i \le 6,\ i \text{ 取整} \end{cases}$$

		x_1	x_2	x_3	x_4	x_5	x_6
	0	1	2	3	0	0	0
x_4	-4	-1	1	-1	1	0	0
x_5	8	1	1	2	0	1	0
x_6	-2	0	-1	1	0	0	1

		x_1	x_2	x_3	x_4	x_5	x_6
	-4	0	3	2	1	0	0
x_1	4	1	-1	1	-1	0	0
x_5	4	0	2	1	1	1	0
x_6	-2	0	1	1	0	0	1

		x_1	x_2	x_3	x_4	x_5	x_6
	-10	0	0	5	1	0	3
x_1	6	1	0	0	-1	0	1

		x_1	x_2	x_3	x_4	x_5	x_6
	-10	0	0	5	1	0	3
x_1	6	1	0	0	-1	0	1
x_5	0	0	0	3	1	1	2
x_6	2	0	1	-1	0	0	-1

$\therefore x_1=6, x_2=2, x_3=0$ 时

$\min S=10$ 即为所求

(2) $\min S = 3x_1+2x_2+x_3+4x_4$

$$\begin{cases} -2x_1-4x_2-5x_3-x_4+x_5 \le 0 \\ -3x_1+x_2-7x_3+2x_4+x_6 \le -2 \\ -5x_1-2x_2-x_3-6x_4+x_7 \le -15 \\ x_i \ge 0,\ i\in N_+,\ 1\le i\le 7 \end{cases}$$

		x_1	x_2	x_3	x_4	x_5	x_6	x_7
	0	3	2	1	4	0	0	0
x_5	0	-2	-4	-5	-1	1	0	0
x_6	-2	-3	1	-7	2	0	1	0
x_7	-15	-5	-2	-1	6	0	0	1

		x_1	x_2	x_3	x_4	x_5	x_6	x_7
	$-\frac{137}{16}$	0	$\frac{15}{16}$	0	$\frac{3}{4}$	0	$-\frac{1}{16}$	$\frac{9}{16}$
x_5	$\frac{31}{32}$	0	$\frac{15}{32}$	0	$-\frac{21}{8}$	1	$-\frac{23}{32}$	$\frac{1}{32}$
x_3	$-\frac{39}{32}$	0	$-\frac{11}{32}$	1	$-\frac{7}{8}$	0	$-\frac{7}{32}$	$\frac{3}{32}$
x_1	$\frac{103}{32}$	1	$\frac{15}{32}$	0	$\frac{11}{8}$	0	$\frac{1}{32}$	$-\frac{3}{32}$

		x_1	x_2	x_3	x_4	x_5	x_6	x_7
	-9	0	$-\frac{4}{5}$	$\frac{3}{5}$	$\frac{2}{5}$	0	0	$\frac{4}{5}$
x_5	6	0	$-\frac{16}{5}$	$-\frac{23}{5}$	$\frac{7}{5}$	1	0	$-\frac{2}{5}$
x_6	2	0	$\frac{4}{5}$	$-\frac{8}{5}$	$\frac{13}{5}$	0	1	$\frac{4}{5}$
x_1	3	1	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{6}{5}$	0	0	$-\frac{1}{5}$

$\therefore x_1=3, x_2=x_3=x_4=0$ 时

$\min S=9$ 即为所求

2-3.解 $\min S = 5x_1-5x_2-13x_3$

$$\begin{cases} -x_1+x_2+3x_3+x_4=20 \\ 12x_1+4x_2+10x_3+x_5=90 \\ x_i\ge 0,\ 1\le i\le 5,\ i\in N_+ \end{cases}$$

$$\begin{cases} 12x_1+4x_2+10x_3+x_5=90 \\ x_j\geq 0,\ 1\leq j\leq 5,\ j\in N_+ \end{cases}$$

		x_1	x_2	x_3	x_4	x_5
	0	5	-5	-13	0	0
x_4	20	-1	1	3	1	0
x_5	90	12	4	10	0	1

(1) $B^{-1}b=\begin{pmatrix}1&0\\-4&1\end{pmatrix}\begin{pmatrix}30\\90\end{pmatrix}=\begin{pmatrix}30\\-30\end{pmatrix}$ 最优基变量

		x_1	x_2	x_3	x_4	x_5
	0	5	-5	-13	0	0
x_4	30	-1	1	3	1	0
x_5	90	12	4	10	0	1

		x_1	x_2	x_3	x_4	x_5
	$\frac{225}{2}$	20	0	$-\frac{1}{2}$	0	$\frac{5}{4}$
x_4	$\frac{15}{2}$	-4	0	$\frac{1}{2}$	1	$-\frac{1}{4}$
x_2	$\frac{45}{2}$	3	1	$\frac{5}{2}$	0	$\frac{1}{4}$

		x_1	x_2	x_3	x_4	x_5
	117	$\frac{103}{5}$	$\frac{1}{5}$	0	0	$\frac{10}{13}$
x_4	3	$-\frac{23}{5}$	$-\frac{1}{5}$	0	1	$-\frac{3}{10}$
x_3	9	$\frac{6}{5}$	$\frac{2}{5}$	1	0	$\frac{1}{10}$

$\therefore x_1=0=x_2 \cdot x_3=9100$ $\min S=-117$

(2) $B^{-1}b=\begin{pmatrix}1&0\\-4&1\end{pmatrix}\begin{pmatrix}20\\70\end{pmatrix}=\begin{pmatrix}20\\10\end{pmatrix}$ $\therefore$ 最优解变量

		x_1	x_2	x_3	x_4	x_5
	100	0	0	2	5	0
x_2	20	-1	1	3	1	0
x_5	-10	16	0	-2	-4	1

		x_1	x_2	x_3	x_4	x_5
	90	16	0	0	1	1
x_2	5	23	1	0	-5	$-\frac{3}{2}$
x_3	5	-8	0	1	2	$-\frac{1}{2}$

$x_1=0, x_2=5, x_3=5$时 $\min s = -90$

(3) $\lambda_1^3 = \lambda_3 + \Delta\lambda_3 = 5 > 0$

最优解不变

(4) $\lambda_1' = c_1 - c_B B^{-1} p' = 5 > 0$

最优解不变

(5) 原最优解不在新约束条件范围内

∴最优解改变

		x_1	x_2	x_3	x_4	x_5	x_6
	0	5	-5	13	0	0	0
x_4	20	-1	1	3	1	0	0
x_5	90	12	4	10	0	1	0
x_6	50	2	3	5	0	0	1

		x_1	x_2	x_3	x_4	x_5	x_6
	$\frac{250}{3}$	$\frac{25}{3}$	0	$-\frac{14}{3}$	0	0	$\frac{5}{3}$
x_4	$\frac{10}{3}$	$-\frac{5}{3}$	0	$\frac{4}{3}$	1	0	$-\frac{1}{3}$
x_5	$\frac{70}{5}$	$\frac{28}{3}$	0	$\frac{10}{3}$	0	1	$-\frac{4}{3}$
x_2	$\frac{50}{3}$	$\frac{2}{3}$	1	$\frac{5}{3}$	0	0	$\frac{1}{3}$

		x_1	x_2	x_3	x_4	x_5	x_6
	95	$\frac{5}{2}$	0	0	$\frac{7}{2}$	0	$-\frac{1}{2}$
x_2	$\frac{5}{2}$	$-\frac{5}{4}$	0	1	$\frac{3}{4}$	0	$-\frac{1}{4}$

	95	$\frac{5}{2}$	0	0	$\frac{5}{2}$	0	$-$
x_3	$\frac{5}{2}$	$-\frac{5}{4}$	0	1	$\frac{3}{4}$	0	$-\frac{3}{4}$
x_5	5	$\frac{27}{2}$	0	0	$-\frac{5}{2}$	1	$-\frac{1}{2}$
x_2	$\frac{25}{2}$	$\frac{11}{4}$	1	0	$-\frac{5}{4}$	0	$\frac{3}{4}$

$x_1=0$, $x_2=\frac{25}{2}$, $x_3=\frac{5}{2}$, $x_4=0$时

$\min s=-95$ 即为所求

(6) $\lambda_4=c_4-c_B B^{-1}p_4=7>0$

∴最优解不变

2-5. 解 (1)

	B_1	B_2	B_3	B_4	
A_1	$3\times$	$5\times$	$9\times$	13^*	3
A_2	$4\times$	2^*	3^*	8^*	7
A_3	2^*	$7\times$	$6\times$	4^*	4
	2	1	5	6	

$v_1=-1$ $v_2=-5$ $v_3=-4$ $v_4=1$

$\lambda_{21}=-2<0$ 进基变量 $x_{21}=1$

	B_1	B_2	B_3	B_4	
A_1	$3\times$	$5\times$	$9\times$	$1\ 3^*$	3
A_2	$4\ 1^*$	$2\ 1^*$	$3\ 5^*$	$8\times$	7
A_3	$2\ 1^*$	$7\times$	$6\times$	$4\ 3^*$	4
	2	1	3	6	

$v_1=-1$ $v_2=-3$ $v_3=-2$ $v_4=1$

$\lambda_{11}=4$ $\lambda_{12}=8$ $\lambda_{13}=11$ $x_{24}=2$ $\lambda_{32}=3$ $\lambda_{33}=5$ 均为正

∴此时即为最优解

求得 $s=38$

(3)

	B_1	B_2	B_3	B_4	B_5	B_6	

(3)

	B_1	B_2	B_3	B_4	B_5	B_6	
A_1	10 ×	20 1*	5 4*	9 6*	10 ×	0 2*	12
A_2	2 ×	10 4*	8 ×	30 ×	6 ×	0 ×	4
A_3	1 3*	20 ×	7 ×	10 ×	4 5*	0 1*	8
	3	5	4	6	3	3	

$V_1=1$ $V_2=20$ $V_3=5$ $V_4=9$ $V_5=4$ $V_6=0$

入行与列成立

∴此时即为最优解

求得 $S=149$