数学规划 第4章作业

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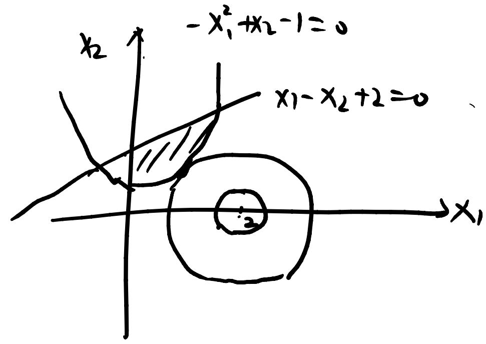
[redacted]

4-2 解 $\max f(X) = 3.6x_1 - 0.4x_1^2 + 1.6x_2 - 0.2x_2^2$ (8)

$x_1 + 0.5x_2 \leq 5$

$X = (x_1, x_2)^T$

4-4 (2) 解 $\min\left((x_1-2)^2 + x_2^2\right)$



$$\text{s.t.}\begin{cases} x_2 \geq x_1^2+1 \\ x_2 \leq x_1+2 \\ x_1, x_2 \geq 0 \end{cases}$$

等高线 $(x_1-2)^2 + x_2^2 = c$.

$$\begin{cases} x_2^2 = x_1^2+1 \\ x_2 \leq x_1+2 \\ x_1, x_2 \geq 0 \end{cases}$$

$\Rightarrow y = x_1^4 + 3x_1^2 - 4x_1 + 5 - c = 0$ 有唯一解

$y' = 4x_1^3 + 6x_1 - 4$.

$y'' = 12x_1 + 6 > 0$

$\therefore y_{min} = y(0.554) = 3.80 - c$

$\therefore c = 3.80$.

$\therefore X^* = (0.554, 1.307)$ $f^* = 3.80$.

4-5 解 (1) 梯度 $\nabla f(X) = \begin{pmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \\ \frac{\partial f}{\partial x_3} \end{pmatrix} = \begin{pmatrix} 2x_1 - 4x_3 \\ 4x_2 \\ 6x_3 - 4x_1 \end{pmatrix}$

Hesse矩阵 $\nabla(\nabla f(X)) = \begin{pmatrix} 2 & 0 & -4 \\ 0 & 4 & 0 \\ -4 & 0 & 6 \end{pmatrix}$

(2) 梯度 $\nabla f(x) = \begin{pmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{pmatrix} = \begin{pmatrix} 3x_2^2 + x_2 e^{x_1x_2} \\ 6x_1x_2 + x_1 e^{x_1x_2} \end{pmatrix}$

Hesse矩阵 $\nabla(\nabla f(x)) = \begin{pmatrix} x_2^2 e^{x_1x_2} & 6x_2 + e^{x_1x_2} + x_1x_2 e^{x_1x_2} \\ 6x_2 + e^{x_1x_2} + x_1x_2 e^{x_1x_2} & 6x_1 + x_1^2 e^{x_1x_2} \end{pmatrix}$

8. 解 $\nabla f(x_1,x_2) = \begin{pmatrix} 8x_1 - 2x_2x_1 \\ 2x_2 - x_1^2 \end{pmatrix}$

$\nabla(\nabla f(x_1,x_2)) = \begin{pmatrix} 8-2x_2 & -2x_1 \\ -2x_1 & 2 \end{pmatrix}$

$x^* = (0,0)^T$ 时 $\nabla(\nabla f(x^*)) = \begin{pmatrix} 8 & 0 \\ 0 & 2 \end{pmatrix}$ 正定 $x^* = (0,0)^T$ 是严格局部极小点

$x^* = (-2\sqrt{2},4)^T$ 时 $\nabla(\nabla f(x^*)) = \begin{pmatrix} 0 & 4\sqrt{2} \\ 4\sqrt{2} & 2 \end{pmatrix}$ 不正定 $\therefore x^* = (-2\sqrt{2},4)$ 不是严格局部最小点 是驻点

$x^* = (2\sqrt{2},4)^T$ 时 $\nabla(\nabla f(x^*)) = \begin{pmatrix} 0 & -4\sqrt{2} \\ -4\sqrt{2} & 2 \end{pmatrix}$ 不正定 $\therefore x^* = (2\sqrt{2},4)$ 是驻点.

4-10 解 (1) $x \leq 4$ 时 $f'(x) = -3(4-x)^2$, $f''(x) = 6(4-x) \geq 0$

$\therefore f(x)$ 为凸函数.

(2) $\nabla f(x) = \begin{pmatrix} 2x_1+2x_2 \\ 2x_1+6x_2 \end{pmatrix}$ $\nabla^2 f(x) = \begin{pmatrix} 2 & 2 \\ 2 & 6 \end{pmatrix} = A$

$|\lambda I - A| = \lambda^2 - 8\lambda + 8$

解得 $\lambda = 4 \pm 2\sqrt{2} > 0$

$\therefore \nabla^2 f(x)$ 正定

$f(x)$ 为严格凸函数.

(3) $\nabla^2 f(x) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = A$

$|\lambda I - A| = \lambda^2 - 1$

解得 $\lambda = \pm 1$, $-1 < 0$

∴ $f(x)$ 不是凸函数.

13. 解迭代

① $\lambda_1 = 3.82$, $\lambda_2 = 6.18$ $b-a=10 > \varepsilon = 0.3$

$f(\lambda_1) > f(\lambda_2)$

② $a=0$, $b=6.18$ $\lambda_1 = 2.36$, $\lambda_2 = 3.82$

$b-a > \varepsilon$ $f(\lambda_1) < f(\lambda_2)$

③ $a=0$, $b=3.82$ $\lambda_1 = 1.46$, $\lambda_2 = 2.36$

$b-a > \varepsilon$, $f(\lambda_1) < f(\lambda_2)$

④ $a=1.46$, $b=3.82$ $\lambda_1 = 2.36$, $\lambda_2 = 2.92$

$b-a > \varepsilon$ $f(\lambda_1) > f(\lambda_2)$

⑤ $a=2.36$ $b=3.82$ $\lambda_1 = 2.92$ $\lambda_2 = 3.26$

$b-a > \varepsilon$ $f(\lambda_1) > f(\lambda_2)$

⑥ $a=2.36$ $b=3.26$ $\lambda_1 = 2.70$ $\lambda_2 = 2.92$

$b-a > \varepsilon$ $f(\lambda_1) < f(\lambda_2)$

⑦ $a=2.70$ $b=3.26$ $\lambda_1 = 2.92$ $\lambda_2 = 3.04$.

$b-a > \varepsilon$ $f(\lambda_1) > f(\lambda_2)$

⑧ $a=2.92$ $b=3.26$ $\lambda_1 = 3.04$ $\lambda_2 = 3.14$.

$b-a > \varepsilon$ $f(\lambda_1) < f(\lambda_2)$

⑨ $a=2.92$ $b=3.14$ $\lambda_1 = 3.02$ $\lambda_2 = 3.04$

$b-a = 0.22 < \varepsilon$ $\lambda^* = \frac{a+b}{2} = 3.03$

4-14 解 $\nabla f(x) = \begin{pmatrix} 2x_1 \\ 4x_2 \end{pmatrix}$ $Q = \begin{pmatrix} 2 & 0 \\ 0 & 4 \end{pmatrix}$ $g^{(k)} = \nabla f(x^{(k)})$

$x^{(0)} = (4,4)^T$ $f(x^{(0)}) = 48$ $g^{(0)} = \nabla f(x^{(0)})$ $\|g^{(0)}\| = 17.89$ $\lambda_0 = \frac{g^{(0)T} \cdot g^{(0)}}{g^{(0)T} Q g^{(0)}} = \frac{5}{18}$.

$x^{(1)} = x^{(0)} - \lambda_0 g^{(0)} = (\frac{16}{9}; -\frac{4}{9})^T$ $f(x^{(1)}) = \frac{32}{9}$ $\|g^{(1)}\| = 3.98$ $\lambda_1 = \frac{5}{12}$

$x^{(2)} = (\frac{8}{27}, \frac{8}{27})^T$. $f(x^{(2)}) = \frac{64}{243}$ $g^{(2)} = (\frac{16}{27}, -\frac{9}{16})^T$, $\|g^{(2)}\| = 1.23$ $\lambda_2 = \frac{5}{18}$

$x^{(3)} = (\frac{32}{243}, \frac{-8}{243})^T$ $f(x^{(3)}) = \frac{128}{6561}$ $g^{(3)} = (\frac{64}{243}, \frac{-32}{243})^T$ $\|g^{(3)}\| = 0.29$

4、15 解. $\nabla f(x) = \begin{pmatrix} 2x_1-2 \\ 8x_2 \\ 18x_3+18 \end{pmatrix}$ $\nabla^2 f(x) = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 18 \end{pmatrix}$

取 $X^{(0)} = (0, 0, 0)^T$.

$X^{(1)} = X^{(0)} - \nabla^2 f(X^{(0)})^{-1} \nabla f(X^{(0)}) = (1 \quad 0 \quad -1)^T$

$\nabla^2 f(x)$ 正定 极小点为 $X^{(1)} = (1 \quad 0 \quad -1)^T$

4-16 解 $f(x) = \frac{1}{2} X^T Q X$. $Q = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$ $b = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$ $X_0 = (2, 2)^T$

$g(x) = \begin{pmatrix} 2x_1 - x_2 + 2 \\ -x_1 + 2x_2 - 4 \end{pmatrix}$ $p^{(0)} = -g^{(0)} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$ $\lambda_0 = 0.36$

$X^{(1)} = (0.56 \quad 0.72)^T$ $g(1) = (0.4 \quad 0.88)^T$ $\|g^{(1)}\| = 0.97$

$\beta_0 = \frac{g^{(1)T} Q p^{(0)}}{p^{(0)T} Q p^{(0)}} = 0.054$ $p^{(1)} = -g^{(1)} + \beta_0 p^{(0)} = \begin{pmatrix} -0.616 \\ -0.772 \end{pmatrix}$

$\lambda_1 = \frac{g^{(1)T} g^{(1)}}{p^{(1)T} Q p^{(1)}} = 0.93$

$X^{(2)} = X^{(1)} + \lambda_1 p' = \begin{pmatrix} -0.01 \\ 2.00 \end{pmatrix}$

$g(2) = \begin{pmatrix} 0.02 \\ -0.01 \end{pmatrix}$ $\|g(2)\| = 0.0155 \approx 0$

$\therefore X^* = X(2)$ 即为所求.