数学规划 第5章作业

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5-1 解 $\nabla f(X) = [-1,0]^T$

$\nabla g_1(X) = [-2x_1, -2x_2]^T$，$\nabla g_2(x) = [-3(x_1-1)^2, 1]$

对于 $x^* = (1,0)^T$，$g_1(x^*) = g_2(x^*) = 0$

$\nabla g_1(x^*) = [-2,0]^T$，$\nabla g_2(x^*) = [0,1]^T$

有 $f(x^*) = \frac{1}{2}\nabla g_1(x^*) + 0\,\nabla g_2(x^*)$

∴ x^* 是 K-T 点

对 $\bar{x} = (0,1)^T$，$g_1(x) = g_2(x) = 0$

$\nabla g_1(\bar{x}) = [0,2]^T$，$\nabla g_2(\bar{x}) = [-3,1]^T$

有 $\nabla f(x^*) = -\frac{1}{6}\nabla g_1(x^*) + \frac{1}{3} g_2(x^*)$

$-\frac{1}{6} < 0$

∴ $\bar{x}$ 不是 K-T 点.

5-2(1) 解. $\nabla f(x) = [2(x_1-2), 2(x_2-1)]^T$　　$\nabla g(x) = [-2x_1, -2x_2]^T$

有 $\begin{cases} \nabla f(x^*) = \lambda \nabla g(x^*) \\ \lambda g(x^*) = 0,\ \lambda \geq 0 \end{cases}$

代入数据解得 $x_1 = (2,1)^T$，$\lambda_1 = 0$

$x_2 = (\frac{2}{5}\sqrt{5}, \frac{\sqrt{5}}{5})^T$，$\lambda_2 = -1+\sqrt{5}$

此时 $g(x_1) < 0$，故 x_1 不是最优解

$g(x_2) = 0$

$\nabla^2[f(x_2) - \lambda g(x_2)] = \begin{bmatrix} 2\sqrt{5} & 0 \\ 0 & 2\sqrt{5} \end{bmatrix}$，易知该矩阵正定

∴ 最优解为 $x_2 = (\frac{2}{5}\sqrt{5}, \frac{\sqrt{5}}{5})^T$，

此时 $\min f(x) = 6 - 2\sqrt{5}$.

5-3(2)解 $\varphi(x, M_k) = x_1^2 + 2x_2^2 + M_k[\min(0, x_1+x_2-1)]^2$

$$= (M_k+1)x_1^2 + (M_k+2)x_2^2 + 2M_k x_1 x_2 - 2M_k x_1 - 2M_k x_2 + M_k$$

$$\nabla \varphi(x, M_k) = \begin{bmatrix} (2M_k+2)x_1 + 2M_k x_2 - 2M_k \\ (2M_k+4)x_2 + 2M_k x_1 - 2M_k \end{bmatrix} = 0.$$

$x_1 = \frac{2M_k}{3M_k+2}$, $x_2 = 1 - 2\frac{M_k+1}{3M_k+2}$.

令 $M_k \to +\infty$ 得 $x_1 = \frac{2}{3}$, $x_2 = \frac{1}{3}$.

故最优解 $x^* = (\frac{2}{3}, \frac{1}{3})^T$, 此时 $\min f(x^*) = \frac{2}{3}$.

5-4(2)解 $\varphi(x, r_k) = 5x_1 + 4x_2^2 - r_k \ln(x_1-1) - r_k \ln x_2$

$$\nabla \varphi(x, r_k) = \begin{bmatrix} 5 - \frac{r_k}{x_1-1} \\ 8x_2 - \frac{r_k}{x_2} \end{bmatrix} = 0.$$

解得 $x_1 = 1 + \frac{r_k}{5}$, $x_2 = \sqrt{\frac{r_k}{8}}$ 令 $r_k \to 0$ 得 $x_1 = 1, x_2 = 0$.

最优解 $x^* = (1, 0)^T$ 此时 $\min f(x^*) = 5$.

5-5(1)解 $\varphi(x, u^{(k)}, c) = x_1^2 + 2x_2^2 + 2x_3^2 + u^{(k)}(x_1+x_2+x_3-4) + \frac{c}{2}(x_1+x_2+x_3-4)^2$

令 $c = 2$

则 $$\nabla(x, u^{(k)}, c) = \begin{bmatrix} 4x_1 + 2x_2 + 2x_3 + u^{(k)} - 8 \\ 6x_2 + 2x_1 + 2x_3 + u^{(k)} - 8 \\ 6x_3 + 2x_1 + 2x_2 + u^{(k)} - 8 \end{bmatrix} = 0.$$

$$x^{(k)} = \begin{pmatrix} x_1^{(k)} \\ x_2^{(k)} \\ x_3^{(k)} \end{pmatrix} = \begin{pmatrix} \frac{8-\mu_1}{16} \\ \frac{8-\mu_1}{12} \\ \frac{8-\mu_1}{12} \end{pmatrix}$$

$$\mu_1^{(k+1)} = \mu_1^{(k)} + ch(x^{(k)}) = \mu_1^{(k)} + 2\left(\frac{8-\mu_1}{6} + \frac{8-\mu_1}{12} + \frac{8-\mu_1}{12} - 4\right) = -\frac{8}{3} + \frac{1}{3}\mu_1^{(k)}$$

有 $\mu_1^* = \frac{1}{3}\mu_1^* - \frac{8}{3}$

$\mu_1^{*} = \mu^{*} + c\,m(x^{*}) = \mu_1 + 2\frac{\ }{6} + \frac{\ }{12} \cdots$

有 $\mu_1^{*} = \frac{1}{3}\mu_1^{*} - \frac{8}{3}$

$\therefore \mu_1^{*} = -4$

$\therefore$ 最优解 $x^{*} = (2,1,1)^T$，此时 $f^{*} = 8$.

数学规划 第6章作业.

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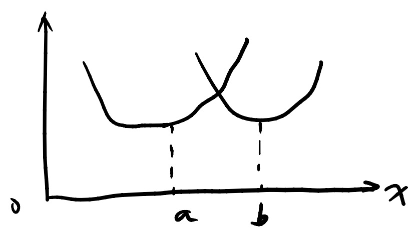
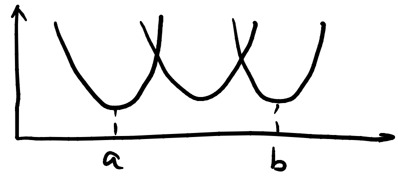
6-3 解



$R_c^{*} \in [a,b]$



$R_c^{*} \in [a,b]$

6-6 解 $\min h(x) = \min[\frac{1}{2}(-x_1 - 8x_2) + \frac{1}{2}(-6x_1 - x_2)] = \min(-\frac{7}{2}x_1 - \frac{9}{2}x_2)$

$x^{*} = (0.8, 1.2)^T$

6-7 解 设 A,B,C 咨询时间分别是 x_1, x_2, x_3 h.

建立数学模型如下

$\min[P_1(d_1^{+} + d_1^{-}) + P_2 d_2^{+}]$

$$\begin{cases} 20x_1 + 22x_2 + 18x_3 + d_1^{+} + d_1^{-} = 500 \\ 0.1x_1 + 0.2x_2 + 0.2x_3 + d_2^{+} + d_2^{-} = 15 \\ x_1 + x_2 + x_3 \le 40 \\ x_3 \le 20 \\ x_i \ge 0,\ i = 1,2,3 \\ d_i^{-}, d_i^{+} \ge 0,\ i = 1,2 \end{cases}$$

6-8 解 设 A_1 运往 B_1, B_2, B_3 的量分别为 x_{11}, x_{12}, x_{13}

A_2 运往 B_1, B_2, B_3 的量分别为 x_{21} x $x_{2?}$

6-8 解 设A_1运往B_1,B_2,B_3的量分别为x_{11},x_{12},x_{13}

A_2运往B_1,B_2,B_3的量分别为x_{21},x_{22},x_{23}

$$\min\left[P_1(d_1^+ + d_1^-) + P_2(d_2^+ + d_3^-) + P_3(d_4^+ + d_4^-) + P_4 d_5^- + P_5(d_6^+ - d_6^- + d_7^+ - d_7^-) + P_6(d_8^- + d_8^+)\right]$$

$$\begin{cases} x_{13}+x_{23}+d_1^- - d_1^+ = 5000 \\ x_{11}+x_{21}+d_2^- - d_2^+ = 1500 \\ x_{21}+x_{22}+d_3^- - d_3^+ = 1125 \\ 10x_{11}+4x_{12}+12x_{13}+8x_{21}+10x_{22}+3x_{23}+d_4^- - d_4^+ = 0 \\ x_{21}+d_5^- - d_5^+ = 1000 \\ x_{13}+d_6^- - d_6^+ = 3000 \\ x_{22}+d_7^- - d_7^+ = 1500 \\ x_{11}+x_{12}+x_{13} \le 3000 \\ x_{21}+x_{22}+x_{23} \le 4000 \\ \dfrac{x_{11}+x_{21}}{2000} - \dfrac{x_{21}+x_{22}}{1500} + d_8^- - d_8^+ = 0 \end{cases}$$

6-9 解 ① 序列法

P_1级单目标线性规划

$\min z = d^{-1}$

$$\begin{cases} x_1+x_2+d_1^- - d_1^+ = 80 \\ x_i \ge 0 \quad i=1,2 \\ d_i^+, d_i^- \ge 0 \end{cases}$$

P_2级单目标线性规划

$\min z = d_{11}^+$

$$\begin{cases} x_1+x_2-d_1^+ = 80 \\ d_1^+ + d_{11}^- - d_{11}^+ = 10 \\ x_1, x_2 \ge 0 \\ d_i^+, d_i^- \ge 0 \end{cases}$$

$d_{11}^+ = 0$ 可成立，此时P_2级达到最优 $\min d_{11}^+ = 0$

P_3级单目标线性规划

$\min z = 5d_2^- + 3d_3^-$

$$\begin{cases} x_1+x_2-d_1^+ = 80 \end{cases}$$

$$\begin{cases} x_1+x_2-d_1^+=80 \\ x_1+d_2^- =70 \\ x_2+d_3^- =45 \\ d_1^+ + d_{11}^- =10 \\ x_1, x_2 \geq 0, \\ d_i^+, d_i^- \geq 0 \end{cases}$$

取 $d_2^- = d_3^- = 0$，此时 P_3 级达到最优

则 $x_1=70$，$x_2=45$，$d_1^+=35$，$d_1^- = -25<0$，不成立

		x_1	x_2	d_1^-	d_1^+	d_2^-	d_3^-	d_{11}^-
	-485	-5	-3	0	0	0	0	0
d_1^-	80	1	1	1	-1	0	0	0
d_2^-	70	1	0	0	0	1	0	0
d_3^-	45	0	1	0	0	0	1	0
d_{11}^-	10	0	0	0	1	0	0	1

		x_1	x_2	d_1^-	d_1^+	d_2^-	d_3^-	d_{11}^-
	-135	0	-3	0	0	5	0	0
d_1^-	10	0	1	1	-1	-1	0	0
d_2^-	70	1	0	0	0	1	0	0
d_3^-	45	0	1	0	0	0	1	0
d_{11}^-	10	0	0	0	1	0	0	1

		x_1	x_2	d_1^-	d_1^+	d_2^-	d_3^-	d_{11}^-
	-105	0	0	3	-3	2	0	0
x_2	10	0	1	1	-1	-1	0	0
x_1	70	1	0	0	0	1	0	0
d_3^-	35	0	0	-1	1	0	1	0
d^-[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1	0	0	1

d_3	35	0	0	-1	1	0	1	0
d_{11}^-	10	0	0	0	1	0	0	1

		x_1	x_2	d_1^-	↓ d_1^+	d_2^-	d_3^-	d_4^-
	-75	0	0	3	0	2	0	3
x_2	20	0	1	1	0	-1	0	1
x_1	70	1	0	0	0	1	0	0
d_3^-	25	0	0	-1	0	1	1	-1
d_1^+	10	0	0	0	1	0	0	1

P_4级 $\min z = d_1^+ = 10$ 无法到达

$d_1^+ - d_{11}^+ = 10$

$\therefore d_{11}^+ = 0$

故条件 P_1, P_2 可达到 P_3, P_4 不可达到

$\therefore x_1 = 70,\ x_2 = 20$

$d_1^+ = 10 \quad d_3^- = 25 \quad d_1^- = d_2^- = d_{11}^- = d_{11}^+ = 0$.

② 单纯形法.

		↓ x_1	x_2	d_1^-	d_1^+	d_2^-	d_3^-	d_{11}^-	d_{11}^+
d_1^-	80	1	1	1	-1	0	0	0	0
← d_2^-	70	1	0	0	0	1	0	0	0
d_3^-	45	0	1	0	0	0	1	0	0
d_{11}^-	10	0	0	0	1	0	0	1	-1
P_1	-80	-1	-1	0	1	0	0	0	0
P_2	0	0	0	0	0	0	0	0	1

P_3	-485	5	-3	0	0	0	0	0	0
P_4	0	0	0	1	0	0	0	0	0.

		x_1	$\overset{\downarrow}{x_2}$	d_1^-	d_1^+	d_2^-	d_3^-	d_{11}^-	d_{11}^+
$\leftarrow d_1^-$	10	0	1	1	-1	-1	0	0	0
x_1	70	1	0	0	0	1	0	0	0
d_3^-	45	0	1	0	0	0	1	0	0
d_{11}^-	10	0	0	0	1	0	0	1	-1
P_1	-10	0	-1	0	1	1	0	0	0
P_2	0	0	0	0	0	0	0	0	1
P_3	-135	0	-3	0	0	5	0	0	0
P_4	0	0	0	0	1	0	0	0	0.

		x_1	x_2	d_1^-	$\overset{\downarrow}{d_1^+}$	d_2^-	d_3^-	d_{11}^-	d_{11}^+.
x_2	10	0	1	1	-1	-1	0	0	0
x_1	70	1	0	0	0	1	0	0	0
d_3^-	35	0	0	-1	1	1	1	0	0
$\leftarrow d_{11}^-$	10	0	0	0	1	0	0	-1	1
P_1	0	0	0	1	0	0	0	0	0
P_2	0	0	0	0	0	0	0	0	1
P_3	-105	0	0	3	-3	2	0	0	0
P_4	0	0	0	0	1	0	0	0	0.

x_1	x_2	d_1^-	d_1^+	d_2^-	d_3^-	d_{11}^-	d_{11}^+

		x_1	x_2	d_1^-	d_1^+	d_2^-	d_3^-	d_4^-	d_4^+
x_2	20	0	1	1	0	-1	0	1	-1
x_1	75	1	0	0	0	1	0	0	0
d_3^-	25	0	0	-1	0	1	1	-1	1
d_1^+	10	0	0	0	1	0	0	1	-1
P_1	0	0	0	1	0	0	0	0	0
P_2	0	0	0	0	0	0	0	0	1
P_3	-35	0	0	3	0	2	0	3	-3
P_4	-10	0	0	0	0	0	0	-1	1

再进行行换基元法以使负检验数数量

∴ P_1, P_2 可达到，P_3, P_4 不可达到

$x_1=70$, $x_2=20$.

$d_1^+=10$ $d_3^-=25$ $d_1^-=d_2^-=d_4^-=d_4^+=0$