[redacted]

1-1 解：设生产A x_1 千克，B x_2 千克.

求产值最高，即求 $\max S = 7x_1 + 12x_2$

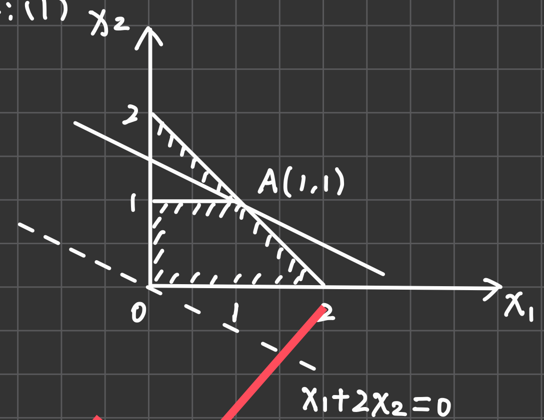
$$\text{s.t.}\begin{cases} 9x_1 + 4x_2 \leq 360 & \text{(煤约束)} \\ 4x_1 + 5x_2 \leq 200 & \text{(电约束)} \\ 3x_1 + 10x_2 \leq 300 & \text{(劳动力约束)} \\ x_1 \geq 0,\ x_2 \geq 0. \end{cases}$$

1-3 解：设第i块地上第j种作物种植面积为 x_{ij}

求总产量最高，即 $\max S = 700x_{11} + 300x_{12} + 900x_{13} + 600x_{21} + 350x_{22} + 800x_{23} + 600x_{31} + 250x_{32} + 700x_{33}$

$$\text{s.t.}\begin{cases} \sum_{j=1}^{3} x_{1j} = 200,\ \sum_{j=1}^{3} x_{2j} = 400,\ \sum_{j=1}^{3} x_{3j} = 600 \\ 700x_{11} + 600x_{21} + 600x_{31} \geq 130000 & \text{(水稻产量约束)} \\ 300x_{12} + 350x_{22} + 250x_{32} \geq 40000 & \text{(大豆产量约束)} \\ 900x_{13} + 800x_{23} + 700x_{33} \geq 250000 & \text{(玉米产量约束)} \\ x_{ij} \geq 0. \end{cases}$$

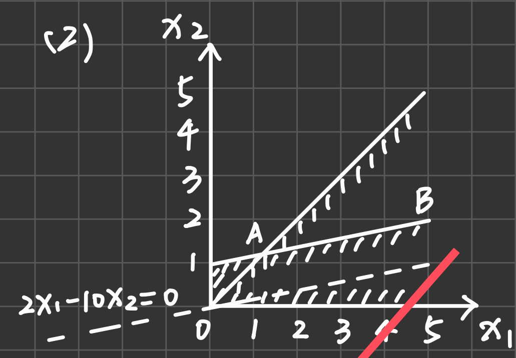
1-5 解：(1)



最优解为 $X = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$,

目标函数取最大值3.

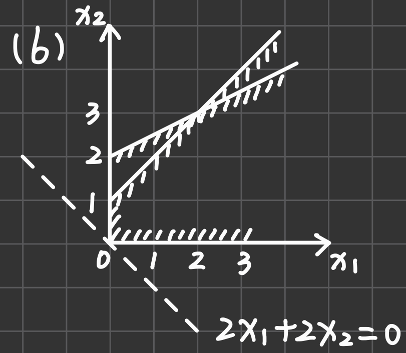
(2)



顶点A及射线AB上的点都是最优解

目标函数取最小值 −10.

(6)



目标函数无有限最优解.

1-6 解:(1) 化为标准形式

$$\min(-S) = -x_1 - x_2 - x_3$$

$$\text{s.t.}\begin{cases} -x_1 \qquad -2x_3 + x_4 = 5 \\ 2x_1 - 3x_2 + x_3 \qquad +x_5 = 3 \\ 2x_1 - 5x_2 + 6x_3 \qquad +x_6 = 5 \\ x_i \geqslant 0,\ i = 1,2,3,4,5,6 \end{cases}$$

			x_1	x_2	x_3	x_4	x_5	x_6
C_B	X_B	0	-1	-1	-1	0	0	0
0	x_4	5	-1	0	-2	1	0	0
0	x_5	3	2	-3	1	0	1	0
0	x_6	5	2	-5	$\boxed{6}$	0	0	1
		$\frac{5}{6}$	$-\frac{2}{3}$	$-\frac{11}{6}$	0	0	0	$\frac{1}{6}$
0	x_4	$\frac{20}{3}$	$-\frac{1}{3}$	$-\frac{5}{3}$	0	1	0	$\frac{1}{3}$
0	x_5	$\frac{13}{6}$	$\boxed{\frac{5}{3}}$	$-\frac{13}{6}$	0	0	1	$-\frac{1}{6}$
-1	x_3	$\frac{5}{6}$	$\frac{1}{3}$	$-\frac{5}{6}$	1	0	0	$\frac{1}{6}$
		$\frac{17}{10}$	0	$-\frac{27}{10}$	0	0	$\frac{2}{5}$	$\frac{1}{10}$
0	x_4	$\frac{71}{10}$	0	$-\frac{21}{10}$	0	1	$\frac{1}{5}$	$\frac{2}{10}$
-1	x_1	$\frac{13}{10}$	1	$-\frac{13}{10}$	0	0	$\frac{3}{5}$	$-\frac{1}{10}$
-1	x_3	$\frac{2}{5}$	0	$-\frac{2}{5}$	1	0	$-\frac{1}{5}$	$\frac{1}{5}$

无有界解.

(2)

			x_1	x_2	x_3	x_4	x_5	x_6
C_B	X_B	15	8	0	3	0	3	0
-3	x_2	5	3	1	2	0	1	0
0	x_6	2	1	0	1	0	2	1
0	x_4	6	1	0	2	1	2	0

$X = (0,5,0,6,0,2)^T$, $S = -15$

(3)

			x_1	x_2	x_3	x_4	x_5	x_6
C_B	X_B	-13	2	0	-6	0	0	-7
1	x_4	9	1	0	0	1	0	6
-1	x_2	2	3	1	-4	0	0	$\boxed{2}$
1	x_5	6	1	0	3	0	1	2
		-6	$\frac{25}{2}$	$\frac{7}{2}$	-20	0	0	0
1	x_4	3	-8	-3	$\boxed{12}$	1	0	0
-1	x_6	1	$\frac{3}{2}$	$\frac{1}{2}$	-2	0	0	1
1	x_5	4	-2	-1	7	0	1	0
		-1	$-\frac{5}{6}$	$-\frac{3}{2}$	0	$\frac{5}{3}$	0	0
1	x_3	$\frac{1}{4}$	$-\frac{2}{3}$	$-\frac{1}{4}$	1	$\frac{1}{12}$	0	0
-1	x_6	$\frac{3}{2}$	$\frac{1}{6}$	0	0	$\frac{1}{6}$	0	1
1	x_5	$\frac{9}{4}$	$\frac{8}{3}$	$\boxed{\frac{3}{4}}$	0	$-\frac{7}{12}$	1	0
		$\frac{7}{2}$	$\frac{9}{2}$	0	0	$\frac{1}{2}$	2	0
1	x_3	1	$\frac{2}{9}$	0	1	$-\frac{1}{9}$	$\frac{1}{3}$	0
-1	x_6	$\frac{3}{2}$	$\frac{1}{6}$	0	0	$\frac{1}{6}$	0	1
-1	x_2	3	$\frac{32}{9}$	1	0	$-\frac{7}{9}$	$\frac{4}{3}$	0

$X=(0,3,1,0,0,\frac{3}{2})^T$, $S=-\frac{7}{2}$.

1-7 解:(1) 辅助线性规划问题

$$\min z = y_1+y_2$$

$$\text{s.t.}\begin{cases} x_1+2x_2+3x_3+y_1=6 \\ 4x_1+5x_2-6x_3+y_2=6 \\ x_j\geqslant 0,\ j=1,2,3,\quad y_i\geqslant 0,\ i=1,2 \end{cases}$$

		x_1	x_2	x_3	y_1	y_2
	-12	-5	-7	3	0	0
y_1	6	1	2	3	1	0
y_2	6	4	$\boxed{5}$	-6	0	1
	$-\frac{18}{5}$	$\frac{3}{5}$	0	$-\frac{27}{5}$	0	$\frac{7}{5}$
y_1	$\frac{18}{5}$	$-\frac{3}{5}$	0	$\boxed{\frac{27}{5}}$	1	$-\frac{2}{5}$
x_2	$\frac{6}{5}$	$\frac{4}{5}$	1	$-\frac{6}{5}$	0	$\frac{1}{5}$
	0	0	0	0	1	1
x_3	$\frac{2}{3}$	$-\frac{1}{9}$	0	1	$\frac{5}{27}$	$-\frac{2}{27}$
x_2	2	$\frac{2}{3}$	1	0	$\frac{2}{9}$	$\frac{1}{9}$

得基本可行解 $X=(0,2,\frac{2}{3})^T$.

		x_1	x_2	x_3
	$\frac{4}{3}$	$\frac{16}{9}$	0	0
x_3	$\frac{2}{3}$	$-\frac{1}{9}$	0	1
x_2	2	$\frac{2}{3}$	1	0

$X=(0,2,\frac{2}{3})^T$, $S=-\frac{4}{3}$

(3) 化为标准形

$\min S = 4x_1+5x_2+6x_3$

$$\text{s.t.}\begin{cases} x_1+x_2+x_3=5 \\ -6x_1+10x_2+5x_3+x_4=20 \\ 5x_1-3x_2+x_3-x_5=15 \\ x_j\geqslant 0,\ j=1,2,3,4,5 \end{cases}$$

辅助线性规划问题

$\min Z = y_1+y_2$

$$\text{s.t.}\begin{cases} x_1+x_2+x_3+y_1=5 \\ -6x_1+10x_2+5x_3+x_4=20 \\ 5x_1-3x_2+x_3-x_5+y_2=15 \\ x_j\geqslant 0,\ j=1,2,3,4,5,\quad y_i\geqslant 0,\ i=1,2 \end{cases}$$

		x_1	x_2	x_3	x_4	x_5	y_1	y_2
	-20	-6	2	-2	0	1	0	0
y_1	5	1	1	1	0	0	1	0
x_4	20	-6	10	5	1	0	0	0
y_2	15	$\boxed{5}$	-3	1	0	-1	0	1
	-2	0	$-\frac{8}{5}$	$-\frac{4}{5}$	0	$-\frac{1}{5}$	0	$\frac{6}{5}$
y_1	2	0	$\boxed{\frac{8}{5}}$	$\frac{4}{5}$	0	$\frac{1}{5}$	1	$-\frac{1}{5}$
x_4	38	0	$\frac{32}{5}$	$\frac{21}{5}$	1	$-\frac{6}{5}$	0	$\frac{6}{5}$
x_1	3	1	$-\frac{3}{5}$	$\frac{1}{5}$	0	$-\frac{1}{5}$	0	$\frac{1}{5}$
	0	0	0	0	0	0	1	1
x_2	$\frac{5}{4}$	0	1	$\frac{1}{2}$	0	$\frac{1}{8}$	$\frac{5}{8}$	$-\frac{1}{8}$
x_4	30	0	0	3	1	-2	-4	2
x_1	$\frac{15}{4}$	1	0	$\frac{1}{2}$	0	$-\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

得基本可行解 $X=(\frac{15}{4},\frac{5}{4},0,30,0)^T$

		4	5	6		
		x_1	x_2	x_3	x_4	x_5
	$-\frac{85}{4}$	0	0	$\frac{3}{2}$	0	$-\frac{1}{8}$
x_2	$\frac{5}{4}$	0	1	$\frac{1}{2}$	0	$\boxed{\frac{1}{8}}$
x_4	30	0	0	3	1	-2
x_1	$\frac{15}{4}$	1	0	$\frac{1}{2}$	0	$-\frac{1}{8}$
	-20	0	1	2	0	0
x_5	10	0	8	4	0	1
x_4	50	0	16	11	1	0
x_1	5	1	1	1	0	0

$X=(5,0,0)^T,\ S=20.$

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2-1 解：(1) $\min S = 5x_1 + 3x_2$

$$\text{s.t.}\begin{cases} -2x_1 + x_2 - 4x_3 \geqslant -4 \\ -x_1 - x_2 - 2x_3 \geqslant -5 \\ 2x_1 - x_2 + x_3 \geqslant 1 \\ x_1 \geqslant 0,\ x_2 \geqslant 0,\ x_3 \geqslant 0. \end{cases}$$

	$x_1 \geqslant 0$	$x_2 \geqslant 0$	$x_3 \geqslant 0$	
$\lambda_1 \geqslant 0$	-2	1	-4	$\geqslant -4$
$\lambda_2 \geqslant 0$	-1	-1	-2	$\geqslant -5$
$\lambda_3 \geqslant 0$	2	-1	1	$\geqslant 1$
	$\leqslant$	$\leqslant$	$\leqslant$	
	5	3	0	

$\Rightarrow$ 对偶规划为 $\max Z = -4\lambda_1 - 5\lambda_2 + \lambda_3$

$$\text{s.t.}\begin{cases} -2\lambda_1 - \lambda_2 + 2\lambda_3 \leqslant 5 \\ \lambda_1 - \lambda_2 - \lambda_3 \leqslant 3 \\ -4\lambda_1 - 2\lambda_2 + \lambda_3 \leqslant 0 \\ \lambda_1 \geqslant 0,\ \lambda_2 \geqslant 0,\ \lambda_3 \geqslant 0. \end{cases}$$

(2) $\min S = -4x_1 - 7x_2 - 2x_3$

$$\text{s.t.}\begin{cases} -x_1 - 2x_2 - x_3 \geqslant -10 \\ -2x_1 - 3x_2 - 3x_3 \geqslant -10 \\ x_1 \geqslant 0,\ x_2 \geqslant 0,\ x_3 \geqslant 0 \end{cases}$$

	$x_1 \geqslant 0$	$x_2 \geqslant 0$	$x_3 \geqslant 0$	
$\lambda_1 \geqslant 0$	-1	-2	-1	$\geqslant -10$
$\lambda_2 \geqslant 0$	-2	-3	-3	$\geqslant -10$
	$\leqslant$	$\leqslant$	$\leqslant$	
	-4	-7	-2	

$\Rightarrow$ 对偶规划为 $\max Z = -10\lambda_1 - 10\lambda_2$

$$\text{s.t.}\begin{cases} -\lambda_1 - 2\lambda_2 \leqslant -4 \\ -2\lambda_1 - 3\lambda_2 \leqslant -7 \\ -\lambda_1 - 3\lambda_2 \leqslant -2 \\ \lambda_1 \geqslant 0,\ \lambda_2 \geqslant 0 \end{cases}$$

(3) $\min S = 2x_1 + x_2 + 4x_3$

$$\text{s.t.}\begin{cases} x_1 + 2x_2 + 2x_3 \geqslant 3 \\ 2x_1 + x_2 + 3x_3 \geqslant 5 \\ x_1 \geqslant 0,\ x_2 \geqslant 0,\ x_3 \text{ 自由} \end{cases}$$

	$x_1 \geqslant 0$	$x_2 \geqslant 0$	x_3	
$\lambda_1 \geqslant 0$	1	2	2	$\geqslant 3$
$\lambda_2 \geqslant 0$	2	1	3	$\geqslant 5$
	$\leqslant$	$\leqslant$	$=$	
	2	1	4	

$\Rightarrow$ 对偶规划为 $\max Z = 3\lambda_1 + 5\lambda_2$

$$\text{s.t.}\begin{cases} \lambda_1 + 2\lambda_2 \leqslant 2 \\ 2\lambda_1 + \lambda_2 \leqslant 1 \\ 2\lambda_1 + 3\lambda_2 = 4 \\ \lambda_1 \geqslant 0,\ \lambda_2 \geqslant 0 \end{cases}$$

2-2 解：(1) $\min S = x_1 + 2x_2 + 3x_3$

$$\text{s.t.}\begin{cases} -x_1 + x_2 - x_3 + x_4 = -4 \\ x_1 + x_2 + 2x_3 + x_5 = 8 \\ -x_1 + x_3 + x_6 = -2 \\ x_j \geqslant 0,\ j = 1,2,3,4,5,6 \end{cases}$$

		x_1	x_2	x_3	x_4	x_5	x_6
	0	1	2	3	0	0	0
x_4	-4	$\boxed{-1}$	1	-1	1	0	0
x_5	8	1	1	2	0	1	0
x_6	-2	-1	0	1	0	0	1
	-4	0	3	2	1	0	0
x_1	4	1	-1	1	-1	0	0
x_5	4	0	2	1	1	1	0
x_6	2	0	-1	2	-1	0	1

得最优解 $X = (4,0,0)^T$, $S = 4$.

对偶问题 $\max Z = -4\lambda_1 + 8\lambda_2 - 2\lambda_3$

$$\text{s.t.}\begin{cases} -\lambda_1 + \lambda_2 - \lambda_3 \leqslant 1 \\ \lambda_1 + \lambda_2 \leqslant 2 \\ -\lambda_1 + 2\lambda_2 + \lambda_3 \leqslant 3 \\ \lambda_1 \geqslant 0,\ \lambda_2 \geqslant 0,\ \lambda_3 \geqslant 0 \end{cases}$$

最优解为 $\lambda = (1\ 0\ 0)\begin{pmatrix} -1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$

$= (-1, 0, 0)$.

(2) $\min S = 3x_1 + 2x_2 + x_3 + 4x_4$

$$\text{s.t.}\begin{cases} -2x_1 - 4x_2 - 5x_3 - x_4 + x_5 = 0 \\ -2x_1 + x_2 - 7x_3 + 2x_4 + x_6 = -2 \\ -5x_1 - 2x_2 - x_3 - 6x_4 + x_7 = -15 \\ x_j \geqslant 0,\ j = 1,2,3,4,5,6,7 \end{cases}$$

		x_1	x_2	x_3	x_4	x_5	x_6	x_7
	0	3	2	1	4	0	0	0
x_5	0	-2	-4	-5	-1	1	0	0
x_6	-2	-2	1	$\boxed{-7}$	2	0	1	0
x_7	-15	-5	-2	-1	-6	0	0	1
	$-\frac{2}{7}$	$\frac{19}{7}$	$\frac{15}{7}$	0	$\frac{20}{7}$	0	$\frac{1}{7}$	0
x_5	$\frac{10}{7}$	$-\frac{4}{7}$	$-\frac{33}{7}$	0	$-\frac{17}{7}$	1	$-\frac{5}{7}$	0
x_3	$\frac{2}{7}$	$\frac{2}{7}$	$-\frac{1}{7}$	1	$-\frac{2}{7}$	0	$-\frac{1}{7}$	0
x_7	$-\frac{103}{7}$	$\boxed{-\frac{33}{7}}$	$-\frac{15}{7}$	0	$-\frac{44}{7}$	0	$-\frac{1}{7}$	1
	$-\frac{239}{33}$	0	$\frac{60}{77}$	0	$\frac{2}{3}$	0	$\frac{2}{33}$	$\frac{19}{33}$
x_5	$\frac{106}{33}$	0	$-\frac{44}{11}$	0	$-\frac{5}{3}$	1	$-\frac{23}{33}$	$-\frac{4}{33}$
x_3	$-\frac{20}{33}$	0	$-\frac{1}{7}$	1	$-\frac{2}{3}$	0	$\boxed{-\frac{5}{33}}$	$\frac{2}{33}$
x_1	$\frac{103}{33}$	1	$\frac{5}{11}$	0	$\frac{4}{3}$	0	$\frac{1}{33}$	$-\frac{7}{33}$
	-9	0	$\frac{278}{385}$	$\frac{2}{5}$	$\frac{2}{5}$	0	0	$\frac{3}{5}$
x_5	6	0	$-\frac{1462}{385}$	$-\frac{23}{5}$	$\frac{7}{5}$	1	0	$-\frac{2}{5}$
x_6	4	0	$\frac{33}{35}$	$-\frac{33}{5}$	$\frac{22}{5}$	0	1	$-\frac{2}{5}$
x_1	3	1	$\frac{164}{385}$	$\frac{1}{5}$	$\frac{6}{5}$	0	0	$-\frac{1}{5}$

得最优解 $X=(3,0,0,0)^T$, $S=9$.

对偶问题 $\max z=-2\lambda_2-15\lambda_3$

$$
\text{s.t.}\begin{cases}
-2\lambda_1-2\lambda_2-5\lambda_3\leq 3\\
-4\lambda_1+\lambda_2-2\lambda_3\leq 2\\
-5\lambda_1-7\lambda_2-\lambda_3\leq 1\\
-\lambda_1+2\lambda_3-6\lambda_3\leq 4\\
\lambda_2\geq 0,\lambda_3\geq 0,\lambda_1\text{为自由变量}
\end{cases}
$$

最优解为 $\lambda=(0\ 0\ 3)\begin{pmatrix}1 & 0 & -\frac{2}{5}\\ 0 & 1 & -\frac{2}{5}\\ 0 & 0 & -\frac{1}{5}\end{pmatrix}$

$=(0,0,-\frac{3}{5})$

2-3 解: $\min S=5x_1-5x_2-13x_3$

$$
\text{s.t.}\begin{cases}
-x_1+x_2+3x_3+x_4=20\\
12x_1+4x_2+10x_3+x_5=90\\
x_j\geq 0, j=1,2,3,4,5
\end{cases}
$$

		x_1	x_2	x_3	x_4	x_5
	0	5	-5	-13	0	0
x_4	20	-1	$\boxed{1}$	3	1	0
x_5	90	12	4	10	0	1
	100	0	0	2	5	0
x_2	20	-1	1	3	1	0
x_5	10	16	0	-2	-4	1

得最优解 $X=(0,20,0)^T$, $S=-100$.

(1) $B^{-1}\bar{b}=\begin{pmatrix}1 & 0\\ -4 & 1\end{pmatrix}\begin{pmatrix}30\\ 90\end{pmatrix}=(30,-30)^T$, $S=-150$

		x_1	x_2	x_3	x_4	x_5
	150	0	0	2	5	0
x_2	30	-1	1	3	1	0
x_5	-30	16	0	$\boxed{-2}$	-4	1
	120	16	0	0	51	1
x_2	-15	23	1	0	$\boxed{-5}$	$\frac{3}{2}$
x_3	15	-8	0	1	2	$-\frac{1}{2}$
	117	$\frac{103}{5}$	$\frac{1}{5}$	0	0	$\frac{13}{10}$
x_4	3	$-\frac{23}{5}$	$-\frac{1}{5}$	0	1	$-\frac{3}{10}$
x_3	9	$\frac{6}{5}$	$\frac{2}{5}$	1	0	$\frac{1}{10}$

得最优解 $X=(0,0,9)^T$, $S=-117$

(2) $B^{-1}\bar{b}=\begin{pmatrix}1 & 0\\ -4 & 1\end{pmatrix}\begin{pmatrix}20\\ 70\end{pmatrix}=(20\ -10)^T$, $S=-100$.

		x_1	x_2	x_3	x_4	x_5
	100	0	0	2	5	0
x_2	20	-1	1	3	1	0
x_5	-10	16	0	$\boxed{-2}$	-4	1
	90	16	0	0	51	1
x_2	5	23	1	0	-5	$\frac{3}{2}$
x_3	5	-8	0	1	2	$-\frac{1}{2}$

得最优解 $X=(0,5,5)^T$,

$S=-90$.

(3) $y_{03}=2>0$，$y_{03}'=-8-(-5\ \ 0)\begin{pmatrix}3\\-2\end{pmatrix}=7>0$，最优解不变

(4) $y_{01}=0$，$y_{01}'=5-(-5\ \ 0)\begin{pmatrix}1&0\\-4&1\end{pmatrix}\begin{pmatrix}0\\5\end{pmatrix}=5>0$，最优解不变

(5) 将原最优解 $X=(0,20,0)^T$ 代入新约束，不成立.

引入松弛变量 x_6，即 $2x_1+3x_2+5x_3+x_6=50$

		x_1	x_2	x_3	x_4	x_5	x_6
	0	5	-5	-13	0	0	0
x_4	20	-1	$\boxed{1}$	3	1	0	0
x_5	90	12	4	10	0	1	0
x_6	50	2	3	5	0	0	1
	100	0	0	2	5	0	0
x_2	20	-1	1	3	1	0	0
x_5	10	16	0	-2	-4	1	0
x_6	-10	5	0	$\boxed{-4}$	-3	0	1
	95	$\frac{5}{2}$	0	0	$\frac{7}{2}$	0	$\frac{1}{2}$
x_2	$\frac{25}{2}$	$\frac{11}{4}$	1	0	$-\frac{5}{4}$	0	$\frac{3}{4}$
x_5	15	$\frac{27}{2}$	0	0	$-\frac{5}{2}$	1	$-\frac{1}{2}$
x_3	$\frac{5}{2}$	$-\frac{5}{4}$	0	1	$\frac{3}{4}$	0	$-\frac{1}{4}$

最优解为 $X=(0,\frac{25}{2},\frac{5}{2})^T$，$S=-95$

(6) $y_{04}=-3-(-5\ \ 0)\begin{pmatrix}1&0\\-4&1\end{pmatrix}\begin{pmatrix}2\\6\end{pmatrix}=7>0$，最优解不变.

2-5 解:(1) 初始解:

销地		B_1	B_2	B_3	B_4	产量	
产地	A_1	× 3	× 5	× 9	3* 1	3	$u_1=0$
	A_2	× 4	1* 2	5* 3	1* 8	7	$u_2=7$
	A_3	2* 2	× 7	× 6	2* 4	4	$u_3=3$
销量		2	1	5	6		
		$v_1=-1$	$v_2=-5$	$v_3=-4$	$v_4=1$		

x_{21} 进基，闭回路为 $x_{21},x_{31},x_{34},x_{24}$，调整量为 1，$x_{24}$ 出基.

销地		B_1	B_2	B_3	B_4	产量	
产地	A_1	× 3	× 5	× 9	3* 1	3	$u_1=0$
	A_2	1* 4	1* 2	5* 3	× 8	7	$u_2=5$
	A_3	1* 2	× 7	× 6	3* 4	4	$u_3=3$
销量		2	1	5	6		
		$v_1=-1$	$v_2=-3$	$v_3=-2$	$v_4=1$		

最优解为 $X=(0,0,0,3,1,1,5,0,1,0,0,3)^T$，$S=38$

(3) 产大于销，需增加销地B_6. 初始解:

销地		B_1	B_2	B_3	B_4	B_5	B_6	产量	
产地	A_1	× 10	× 20	4* 5	6* 9	× 10	2* 0	12	$u_1=0$
	A_2	× 2	4* 10	× 8	× 30	× 6	× 0	4	$u_2=-10$
	A_3	3* 1	1* 20	× 7	× 10	3* 4	1* 0	8	$u_3=0$
销量		3	5	4	6	3	3		
		$v_1=1$	$v_2=20$	$v_3=5$	$v_4=9$	$v_5=4$	$v_6=0$		

已为最优解 $X=(0,0,4,6,0,0,4,0,0,0,3,1,0,0,3)^T$, $S=149$.

[redacted]

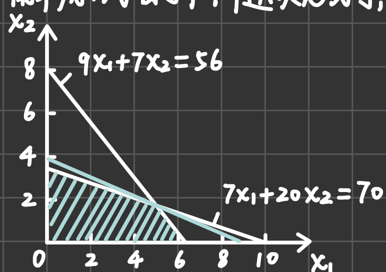
3-1 解：令决策变量 $y=\begin{cases}1, & \text{选择车运}\\ 0, & \text{选择船运}\end{cases}$，设甲、乙货物数量分别为 x_1、x_2，引入一充分大的数 M.

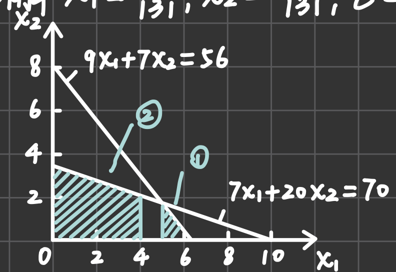
则该问题的整数规划模型为：

$$\min Z = y(100x_1+150x_2)+(1-y)(200x_1+300x_2)$$

$$\text{s.t.}\begin{cases}7x_1+8x_2\le 35+(1-y)M\\ 20x_1+15x_2\le 500+(1-y)M\\ 80x_1+75x_2\le 85+yM\\ 35x_1+42x_2\le 250+yM\\ x_j\ge 0, \text{整数}, j=1,2\\ y=0,1\end{cases}$$

3-2 (1) 解：解原式的伴随规划，得最优解 $x_1=\frac{630}{131}$，$x_2=\frac{238}{131}$，$Z=\frac{46620}{131}$.





则原问题最优解中，$x_1\le 4$ 或 $x_1\ge 5$，原式分解成两枝

$$\max Z = 40x_1+90x_2$$

$$\text{s.t.}\begin{cases}9x_1+7x_2\le 56\\ 7x_1+20x_2\le 70\\ x_1\ge 5, x_2\ge 0, \ x_1、x_2\text{为整数}\end{cases} \quad ①$$

$$\max Z = 40x_1+90x_2$$

$$\text{s.t.}\begin{cases}9x_1+7x_2\le 56\\ 7x_1+20x_2\le 70\\ 0\le x_1\le 4, x_2\ge 0, \ x_1、x_2\text{为整数}\end{cases} \quad ②$$

分别解伴随规划，得① 最优解 $x_1=5$，$x_2=\frac{11}{7}$，$Z=\frac{2390}{7}$

② 最优解 $x_1=4$，$x_2=\frac{21}{10}$，$Z=349$

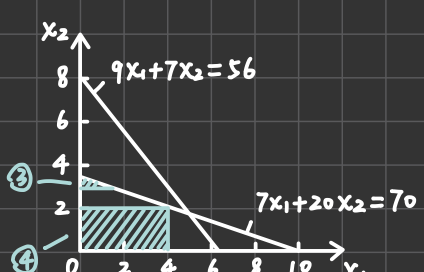
先讨论②，应有 $x_2\le 2$ 或 $x_2\ge 3$，则②分解为

$$\max Z = 40x_1+90x_2$$

$$\text{s.t.}\begin{cases}9x_1+7x_2\le 56\\ 7x_1+20x_2\le 70\\ 0\le x_1\le 4, x_2\ge 3, \ x_1、x_2\text{为整数}\end{cases} \quad ③$$

$$\max Z = 40x_1+90x_2$$

$$\text{s.t.}\begin{cases}9x_1+7x_2\le 56\\ 7x_1+20x_2\le 70\\ 0\le x_1\le 4, 0\le x_2\le 2, x_1、x_2\text{为整数}\end{cases} \quad ④$$

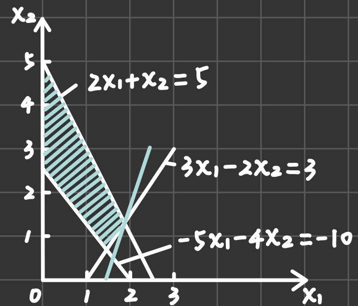


分别解伴随规划，得③最优解 $x_1=\frac{10}{7}$，$x_2=3$，$Z=\frac{2290}{7}$

④最优解 $x_1=4$，$x_2=2$，$Z=340$，即为所求

$\Rightarrow$ 最优解 $X=(4,2)^T$，$Z=340$.

3-3 (2) 解：解原式的伴随规划，得最优解 $x_1=\frac{13}{7}$，$x_2=\frac{9}{7}$，$z=\frac{20}{7}$



在原式中加入松弛变量 u_1、u_2、u_3，得

$\max z = 3x_1 - x_2$

$$\text{s.t.}\begin{cases} 3x_1-2x_2+u_1=3 \\ -5x_1-4x_2+u_2=-10 \\ 2x_1+x_2+u_3=5 \\ x_1\geq 0,\ x_2\geq 0,\text{且都是整数},\ u_j\geq 0,\ j=1,2,3 \end{cases}$$

上式的伴随规划单纯形表如下.

		x_1	x_2	u_1	u_2	u_3
	0	-3	1	0	0	0
u_1	3	$\boxed{3}$	-2	1	0	0
u_2	-10	-5	-4	0	1	0
u_3	5	2	1	0	0	1
	3	0	-1	1	0	0
x_1	1	1	$-\frac{2}{3}$	$\frac{1}{3}$	0	0
u_2	-5	0	$-\frac{22}{3}$	$\frac{5}{3}$	1	0
u_3	3	0	$\boxed{\frac{7}{3}}$	$-\frac{2}{3}$	0	1
	$\frac{20}{7}$	0	0	$\frac{5}{7}$	0	$\frac{3}{7}$
x_1	$\frac{13}{7}$	1	0	$\frac{1}{7}$	0	$\frac{2}{7}$
u_2	$\frac{31}{7}$	0	0	$-\frac{3}{7}$	1	$\frac{22}{7}$
x_2	$\frac{9}{7}$	0	1	$-\frac{2}{7}$	0	$\frac{3}{7}$

由最优表得约束方程：$x_2=\frac{9}{7}+\frac{2}{7}u_1-\frac{3}{7}u_3$

改写得 $x_2+u_3=\frac{9}{7}+\frac{2}{7}u_1+\frac{4}{7}u_3\geq 2$，即 $2u_1+4u_3\geq 5$

解出 u_1、u_3 并代入上式得 $x_1\leq\frac{3}{2}$

将上式化为标准形，加到原最优表上

		x_1	x_2	u_1	u_2	u_3	u_4
	$\frac{20}{7}$	0	0	$\frac{5}{7}$	0	$\frac{3}{7}$	0
x_1	$\frac{13}{7}$	1	0	$\frac{1}{7}$	0	$\frac{2}{7}$	0
u_2	$\frac{31}{7}$	0	0	$-\frac{3}{7}$	1	$\frac{22}{7}$	0
x_2	$\frac{9}{7}$	0	1	$-\frac{2}{7}$	0	$\frac{3}{7}$	0
u_4	$-\frac{5}{7}$	0	0	$-\frac{2}{7}$	0	$\boxed{-\frac{4}{7}}$	1
	$\frac{15}{4}$	0	0	$\frac{1}{2}$	0	0	$\frac{3}{4}$
x_1	$\frac{3}{2}$	1	0	0	0	0	$\frac{1}{2}$
u_2	$\frac{1}{2}$	0	0	-2	1	0	$\frac{22}{4}$
x_2	$\frac{3}{4}$	0	1	$-\frac{1}{2}$	0	0	$\frac{3}{4}$
u_3	$\frac{5}{4}$	0	0	$\frac{1}{2}$	0	1	$-\frac{7}{4}$

由最优表得约束方程：$x_1=\frac{3}{2}-\frac{1}{2}u_4$

改写得 $x_1+u_4=\frac{3}{2}+\frac{1}{2}u_4\geqslant 2$，即 $u_4\geqslant 1$

解出 u_4 并代入上式得 $x_1\leqslant 1$

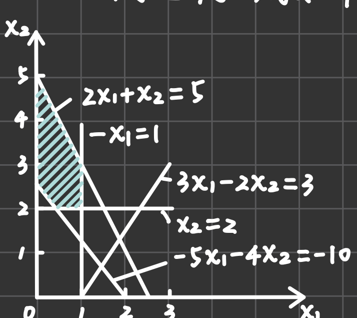
将此约束代替上一新约束，重复步骤

		x_1	x_2	u_1	u_2	u_3	u_4
	$\frac{20}{7}$	0	0	$\frac{5}{7}$	0	$\frac{3}{7}$	0
x_1	$\frac{12}{7}$	1	0	$\frac{1}{7}$	0	$\frac{2}{7}$	0
u_2	$\frac{31}{7}$	0	0	$-\frac{3}{7}$	1	$\frac{22}{7}$	0
x_2	$\frac{9}{7}$	0	1	$-\frac{2}{7}$	0	$\frac{3}{7}$	0
u_4	$-\frac{6}{7}$	0	0	$-\frac{1}{7}$	0	$\boxed{-\frac{2}{7}}$	1
	3	0	0	$\frac{1}{2}$	0	0	$\frac{3}{2}$
x_1	1	1	0	0	0	0	1
u_2	-5	0	0	$\boxed{-2}$	1	0	11
x_2	0	0	1	$-\frac{1}{2}$	0	0	$\frac{3}{2}$
u_3	3	0	0	$\frac{1}{2}$	0	1	$-\frac{7}{2}$
	$\frac{7}{4}$	0	0	0	$\frac{1}{4}$	0	$\frac{17}{4}$
x_1	1	1	0	0	0	0	1
u_1	$\frac{5}{2}$	0	0	1	$-\frac{1}{2}$	0	$-\frac{11}{2}$
x_2	$\frac{5}{4}$	0	1	0	$-\frac{1}{4}$	0	$-\frac{5}{4}$
u_3	$\frac{7}{4}$	0	0	0	$\frac{1}{4}$	1	$-\frac{3}{4}$

由最优表得约束方程 $x_2=\frac{5}{4}+\frac{1}{4}u_2+\frac{5}{4}u_4\geqslant 2$，即 $u_2+5u_4\geqslant 3$

解出 u_2、u_4 并代入上式得 $x_2\geqslant 2$

综上可得可行域如下



得原问题最优解 $X=(1,2)^T$，$z=1$.

3-5 解：
$$B=(M-c_{ij})=\begin{pmatrix}0&10&8&10&8\\9&8&11&11&11\\10&0&5&3&5\\2&3&11&11&7\\13&7&10&7&11\end{pmatrix}\begin{matrix}\\-8\\\\-2\\-7\end{matrix}\Rightarrow\begin{pmatrix}0&10&8&10&8\\1&0&3&3&3\\10&0&5&3&5\\0&1&9&9&5\\6&0&3&0&4\end{pmatrix}\Rightarrow\begin{pmatrix}0&10&5&10&5\\1&0&0&3&0\\10&0&2&3&2\\0&1&6&9&2\\6&0&0&0&1\end{pmatrix}$$
$$\begin{matrix}&&-3&&-3\end{matrix}$$

$b'_{11}=0$, $b'_{41}=\varphi$; $b'_{32}=0$, $b'_{22}=b'_{52}=\varphi$; $b'_{54}=0$, $b'_{53}=\varphi$; $b'_{25}=0$, $b'_{23}=\varphi$

$$B'=\begin{pmatrix}0^*&10&5&10&5\\1&\varphi&\varphi&3&0^*\\10&0^*&2&3&2\\\varphi&1&6&9&2\\6&\varphi&\varphi&0^*&1\end{pmatrix}\begin{matrix}\checkmark\ -1\\\\\\\checkmark\ -1\\\\\end{matrix}\Rightarrow\begin{pmatrix}0&9&4&9&4\\2&0&0&3&0\\11&0&2&3&2\\0&0&5&8&1\\7&0&0&0&1\end{pmatrix}\Rightarrow\begin{pmatrix}0^*&9&4&9&4\\2&\varphi&\varphi&3&0^*\\11&0^*&2&3&2\\\varphi&\varphi&5&8&1\\7&\varphi&\varphi&0^*&1\end{pmatrix}\begin{matrix}\checkmark\ -1\\\\\checkmark\ -1\\\checkmark\ -1\\\\\end{matrix}$$
$$\begin{matrix}\checkmark\\+1\end{matrix}\qquad\qquad\begin{matrix}\checkmark&\checkmark\\+1&+1\end{matrix}$$

$$\Rightarrow \begin{pmatrix} 0 & 9 & 3 & 8 & 3 \\ 3 & 1 & 0 & 3 & 0 \\ 11 & 0 & 1 & 2 & 1 \\ 0 & 0 & 4 & 7 & 0 \\ 8 & 1 & 0 & 0 & 1 \end{pmatrix} \quad 得 B'' = \begin{pmatrix} 0^* & 9 & 3 & 8 & 3 \\ 3 & 1 & 0^* & 3 & \varphi \\ 11 & 0^* & 1 & 2 & 1 \\ \varphi & \varphi & 4 & 7 & 0^* \\ 8 & 1 & \varphi & 0^* & 1 \end{pmatrix}$$

即最优解为 $x_{11}=x_{23}=x_{32}=x_{45}=x_{54}=1$，其余 $x_{ij}=0$

3-6 解：

(x_1,x_2,x_3,x_4)	z值	(a)	(b)	(c)	过滤条件
(0,0,0,0)	0	√	×		
(0,0,0,1)	4	√	√	√	$z \le 4$
(0,0,1,0)	3	√	×		
(0,1,0,0)	5				
(1,0,0,0)	2	×			
(0,0,1,1)	7				
(0,1,0,1)	9				
(1,0,0,1)	6				
(0,1,1,0)	8				
(1,0,1,0)	5				
(1,1,0,0)	7				
(0,1,1,1)	12				
(1,0,1,1)	9				
(1,1,0,1)	11				
(1,1,1,0)	10				
(1,1,1,1)	14				

得最优解 $X=(0,0,0,1)^T$，$z=4$

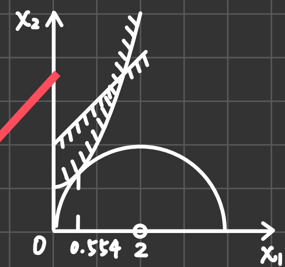
[redacted]

4-2 解: $\max 3.6x_1-0.4x_1^2+1.6x_2-0.2x_2^2$ (略)

$$\text{s.t.}\begin{cases} x_1+0.5x_2\leq 5 \\ x_1\geq 0,\ x_2\geq 0. \end{cases}$$

4-4 (2) 解: $\min f(X)=\min[(x_1-2)^2+x_2^2]$

$$\text{s.t.}\begin{cases} g_1(X)=-x_1^2+x_2-1\geq 0 \\ g_2(X)=x_1-x_2+2\geq 0 \\ x_1,x_2\geq 0 \end{cases}$$



$\Rightarrow$ 最优解为 $x=(0.554,\ 1.306)^T$

4-5 解: (1) $\frac{\partial f}{\partial x_1}=2x_1-4x_3,\ \frac{\partial f}{\partial x_2}=4x_2,\ \frac{\partial f}{\partial x_3}=6x_3-4x_1$

$$\frac{\partial^2 f}{\partial x_1^2}=2,\ \frac{\partial^2 f}{\partial x_1\partial x_2}=0,\ \frac{\partial^2 f}{\partial x_1\partial x_3}=-4,\ \frac{\partial^2 f}{\partial x_2^2}=4,\ \frac{\partial^2 f}{\partial x_2\partial x_3}=0,\ \frac{\partial^2 f}{\partial x_3^2}=6.$$

$$\nabla f(X)=(2x_1-4x_3,\ 4x_2,\ 6x_3-4x_1)^T.\quad \nabla^2 f(X)=\begin{pmatrix} 2 & 0 & -4 \\ 0 & 4 & 0 \\ -4 & 0 & 6 \end{pmatrix}$$

(2) $\frac{\partial f}{\partial x_1}=3x_2^2+x_2e^{x_1x_2},\ \frac{\partial f}{\partial x_2}=6x_1x_2+x_1e^{x_1x_2}$

$$\frac{\partial^2 f}{\partial x_1^2}=x_2^2e^{x_1x_2},\ \frac{\partial^2 f}{\partial x_1\partial x_2}=6x_2+e^{x_1x_2}+x_1x_2e^{x_1x_2},\ \frac{\partial^2 f}{\partial x_2^2}=6x_1+x_1^2e^{x_1x_2}.$$

$$\nabla f(X)=(3x_2^2+x_2e^{x_1x_2},\ 6x_1x_2+x_1e^{x_1x_2})$$

$$\nabla^2 f(X)=\begin{pmatrix} x_2^2e^{x_1x_2} & 6x_2+(1+x_1x_2)e^{x_1x_2} \\ 6x_2+(1+x_1x_2)e^{x_1x_2} & 6x_1+x_1^2e^{x_1x_2} \end{pmatrix}$$

4-8 证: $\frac{\partial f}{\partial x_1}=8x_1-2x_1x_2,\ \frac{\partial f}{\partial x_2}=2x_2-x_1^2,\ \frac{\partial^2 f}{\partial x_1^2}=8-2x_2,\ \frac{\partial^2 f}{\partial x_1\partial x_2}=-2x_1,\ \frac{\partial^2 f}{\partial x_2^2}=2$

$\nabla f(X^*)=0,\ \nabla^2 f(X^*)=\begin{pmatrix} 8 & 0 \\ 0 & 2 \end{pmatrix}$ 正定, 故 $X^*=(0,0)^T$ 是 $f(X)$ 的严格局部极小点.

$\nabla f(X_1)=(-16\sqrt{2}+16\sqrt{2},\ 8-8)^T=0,\ \nabla f(X_2)=(16\sqrt{2}-16\sqrt{2},\ 8-8)^T=0,$

$\nabla^2 f(X_1)=\begin{pmatrix} 8-8 & 4\sqrt{2} \\ 4\sqrt{2} & 2 \end{pmatrix}=\begin{pmatrix} 0 & 4\sqrt{2} \\ 4\sqrt{2} & 2 \end{pmatrix}$ 非正定, $\nabla^2 f(X_2)=\begin{pmatrix} 8-8 & -4\sqrt{2} \\ -4\sqrt{2} & 2 \end{pmatrix}=\begin{pmatrix} 0 & -4\sqrt{2} \\ -4\sqrt{2} & 2 \end{pmatrix}$ 非正定

故 X_1、X_2 是驻点而不是极值点.

4-10 解: (1) $\nabla f(X)=\frac{\partial f}{\partial x}=-3(4-x)^2,\ \nabla^2 f(X)=\frac{\partial^2 f}{\partial x^2}=6(4-x)$

$\forall x\leq 4,\ y\neq 0$, 则 $4-x\geq 0,\ y^2>0$, 即 $6(4-x)y^2\geq 0$, 故 $\nabla^2 f(X)$ 半正定.

故 $f(X)$ 为凸函数.

(2) $\frac{\partial f}{\partial x_1}=2x_1+2x_2,\ \frac{\partial f}{\partial x_2}=2x_1+6x_2,\ \frac{\partial^2 f}{\partial x_1^2}=2,\ \frac{\partial^2 f}{\partial x_1\partial x_2}=2,\ \frac{\partial^2 f}{\partial x_2^2}=6,\ \nabla^2 f(X)=\begin{pmatrix} 2 & 2 \\ 2 & 6 \end{pmatrix}$ 正定.

故 $f(X)$ 为凸函数.

(3) $\frac{\partial f}{\partial x_1}=x_2,\ \frac{\partial f}{\partial x_2}=x_1,\ \nabla f(X)=(x_2\ \ x_1)^T$

$\forall Y=(y_1\ \ y_2)^T\in R,\ f(X)+\nabla f(X)^T(Y-X)-f(Y)=x_2y_1-x_1x_2+x_1y_2-y_1y_2=(y_1-x_1)(x_2-y_2)$

$f(X)$ 既不是凸函数也不是凹函数.

4-13 解：$a_1=0,\ b_1=10,\ \varepsilon=0.03,\ \lambda_1=a_1+0.382(b_1-a_1)=3.82,\ \lambda_2=a_1+0.618(b_1-a_1)=6.18$

$f_1=f(\lambda_1)=-6.3276,\ f_2=f(\lambda_2)=3.1124,\ \lambda_2-\lambda_1=2.36>\varepsilon,\ f_2>f_1$

$a_2=0,\ b_2=6.18,\ \lambda_2=3.82,\ \lambda_1=2.36,\ f_2=-6.3276,\ f_1=f(2.36)=-6.5904$

$\lambda_2-\lambda_1=1.46>\varepsilon,\ f_2>f_1,$

$a_3=0,\ b_3=3.82,\ \lambda_2=2.36,\ \lambda_1=1.46,\ f_2=-6.5904,\ f_1=f(1.46)=-4.6284$

$\lambda_2-\lambda_1=0.9>\varepsilon,\ f_2<f_1,$

$a_4=1.46,\ b_4=3.82,\ \lambda_1=2.36,\ \lambda_2=2.92,\ f_1=-6.5904,\ f_2=f(2.92)=-6.9936$

$\lambda_2-\lambda_1=0.56>\varepsilon,\ f_2<f_1,$

$a_5=2.36,\ b_5=3.82,\ \lambda_1=2.92,\ \lambda_2=3.26,\ f_1=-6.9936,\ f_2=f(3.26)=-6.9324$

$\lambda_2-\lambda_1=0.34>\varepsilon,\ f_2>f_1,$

$a_6=2.36,\ b_6=3.26,\ \lambda_2=2.92,\ \lambda_1=2.7,\ f_2=-6.9936,\ f_1=f(2.7)=-6.91$

$\lambda_2-\lambda_1=0.22>\varepsilon,\ f_2<f_1,$

$a_7=2.7,\ b_7=3.26,\ \lambda_1=2.92,\ \lambda_2=3.04,\ f_1=-6.9936,\ f_2=f(3.04)=-6.9984$

$\lambda_2-\lambda_1=0.12>\varepsilon,\ f_2<f_1$

$a_8=2.92,\ b_8=3.26,\ \lambda_1=3.04,\ \lambda_2=3.14,\ f_1=-6.9984,\ f_2=f(3.14)=-6.9804$

$\lambda_2-\lambda_1=0.1>\varepsilon,\ f_2>f_1,$

$a_9=2.92,\ b_9=3.14,\ \lambda_2=3.04,\ \lambda_1=3.02,\ f_2=-6.9984,\ f_1=f(3.02)=-6.9996$

$\lambda_2-\lambda_1=0.02<\varepsilon$，故 $\lambda^*=\dfrac{\lambda_1+\lambda_2}{2}=3.03$

$f_2>f_1$，故最后区间为 $[2.92,3.04]$，最优解 $X^*=2.98,\ f(X^*)=-6.9996.$

4-14 解：$f(X)=x_1^2+2x_2^2,\ g(X)=(2x_1,4x_2)^T,\ Q=\begin{pmatrix}2&0\\0&4\end{pmatrix}$

$X^{(0)}=(4,4)^T,\ P^{(0)}=-\nabla f(X^{(0)})=(-8,-16)^T,\ \|P^{(0)}\|=17.89,$

$X^{(1)}=X^{(0)}-[g^{(0)T}g^{(0)}/g^{(0)T}Qg^{(0)}]g^{(0)}=(4,4)^T-\frac{5}{18}(8,16)^T=(1.78,-0.44)^T$

$P^{(1)}=-\nabla f(X^{(1)})=(-3.56,1.76)^T,\ \|P^{(1)}\|=3.97$

$X^{(2)}=X^{(1)}-[g^{(1)T}g^{(1)}/g^{(1)T}Qg^{(1)}]g^{(1)}=(1.78,-0.44)^T-\frac{15.7712}{37.7376}(3.56,-1.76)^T=(0.29,0.30)^T$

$P^{(2)}=-\nabla f(X^{(2)})=(-0.58,-1.2)^T,\ \|P^{(2)}\|=1.33$

$X^{(3)}=X^{(2)}-[g^{(2)T}g^{(2)}/g^{(2)T}Qg^{(2)}]g^{(2)}=(0.29,0.30)^T-\frac{1.7764}{6.4328}(0.58,1.2)^T=(0.13,-0.03)^T$

4-15 (1) 解：取 $X^{(0)}=(0,0,0)^T,\ g(X)=(2x_1-2,8x_2,18x_3+18)^T,\ G(X)=\begin{pmatrix}2&0&0\\0&8&0\\0&0&18\end{pmatrix},\ G(X)^{-1}=\begin{pmatrix}\frac{1}{2}&0&0\\0&\frac{1}{8}&0\\0&0&\frac{1}{18}\end{pmatrix}$

$X^{(1)}=X^{(0)}-G(X^{(0)})^{-1}g(X^{(0)})=(0,0,0)^T-\begin{pmatrix}\frac{1}{2}&0&0\\0&\frac{1}{8}&0\\0&0&\frac{1}{18}\end{pmatrix}(-2,0,18)^T=(1,0,-1)^T$ 即为极小点.

4-16 解：$Q=\begin{pmatrix}2&-1\\-1&2\end{pmatrix},\ g(X)=(2x_1-x_2+2,-x_1+2x_2-4)^T,$

$P^{(0)}=-g^{(0)}=(-4,2)^T,\ \lambda_0=g^{(0)T}g^{(0)}/P^{(0)T}QP^{(0)}=\frac{5}{14},$

$X^{(1)}=X^{(0)}+\lambda_0P^{(0)}=(2,2)^T+\frac{5}{14}(-4,2)^T=(0.57,2.71)^T$

$g^{(1)}=(0.43,0.85)^T,\ \beta_0=g^{(1)T}QP^{(0)}/P^{(0)T}QP^{(0)}=\dfrac{2.5}{56}=0.045$

$P^{(1)}=-g^{(1)}+\beta_0P^{(0)}=(-0.43,-0.85)^T+0.045(-4,2)^T=(-0.61,-0.76)^T$

$\lambda_1 = g^{(1)T} g^{(1)} / p^{(1)T} Q p^{(1)} = \frac{0.9074}{0.9722} = 0.933,$

$X^{(2)} = X^{(1)} + \lambda_1 p^{(1)} = (0.57, 2.71)^T + 0.933(-0.61, -0.76)^T = (0, 2)^T$

$g^{(2)} = (0,0)^T$, 迭代终止, $X^* = (0,2)^T$.

[redacted]

5-1 证：$f(X)$、$g_1(X)$、$g_2(X)$在R^2上连续可微，由K-T条件：

$$\begin{cases}(-1,0)^T-\mu_1(-2x_1,-2x_2)^T-\mu_2(-3(x_1-1)^2,1)^T=0\\ \mu_1\geqslant 0,\ \mu_2\geqslant 0\\ \mu_1(1-x_1^2-x_2^2)=0,\ \mu_2[x_2-(x_1-1)^3]=0\end{cases}$$

代入$X^*=(1,0)^T$，符合K-T条件，此时$\mu_1^*=\frac{1}{2}$，$\mu_2^*=0$；代入$\bar{X}=(0,-1)^T$，不符合K-T条件.

故$X^*=(1,0)^T$是K-T点，$\bar{X}=(0,-1)^T$不是K-T点.

5-2 解：(1) $f(X)$、$g(X)$在R^2上连续可微，由K-T条件：

$$\begin{cases}(2x_1-4,2x_2-2)^T-\mu(-2x_1,-2x_2)^T=0\\ \mu\geqslant 0\\ \mu(1-x_1^2-x_2^2)=0\end{cases}$$

解得 $\begin{cases}x_1=2\\ x_2=1\\ \mu=0\end{cases}$ $\begin{cases}x_1=\frac{2\sqrt{5}}{5}\\ x_2=\frac{\sqrt{5}}{5}\\ \mu=\sqrt{5}-1\end{cases}$

$g(X^{(1)})=1-4-1=-4<0$，舍去；

在$X^{(2)}=(\frac{2\sqrt{5}}{5},\frac{\sqrt{5}}{5})$处，$\nabla^2 f(X^{(2)})-\mu\nabla^2 g(X^{(2)})=\begin{pmatrix}2\sqrt{5}&0\\0&2\sqrt{5}\end{pmatrix}$正定.

故原问题最优解为$X^*=(\frac{2\sqrt{5}}{5},\frac{\sqrt{5}}{5})^T$，$f(X^*)=6-2\sqrt{5}$

5-3 解：(2) 构造罚函数：$\varphi(X,M)=x_1^2+2x_2^2+M[\min(0,x_1+x_2-1)]^2$

$$\frac{\partial\varphi}{\partial x_1}=2x_1+2M[\min(0,x_1+x_2-1)],\ \frac{\partial\varphi}{\partial x_2}=4x_2+2M[\min(0,x_1+x_2-1)]$$

对不满足约束条件的点$X=(x_1,x_2)^T$，有：$x_1+x_2<1$.

令$\frac{\partial\varphi}{\partial x_1}=\frac{\partial\varphi}{\partial x_2}=0$，得$2x_1+2M(x_1+x_2-1)=0$，$4x_2+2M(x_1+x_2-1)=0$

得$\min\varphi(X,M_k)$的解为$X(M_k)=(\frac{2M_k}{3M_k+2},\frac{M_k}{3M_k+2})^T$.

当$M_k\to\infty$时，$X(M_k)$趋于原问题最优解$X^*=(\frac{2}{3},\frac{1}{3})^T$.

5-4 解：(2) 构造障碍函数：$\varphi(X,r)=5x_1+4x_2^2-r\ln(x_1-1)-r\ln x_2$.

$$\frac{\partial\varphi}{\partial x_1}=5-\frac{r}{x_1-1}=0,\ \frac{\partial\varphi}{\partial x_2}=8x_2-\frac{r}{x_2}=0$$

解得$X(r_k)=(1+\frac{r_k}{5},\frac{\sqrt{2r_k}}{4})^T$.

当$r_k\to 0$时，$X(r_k)$趋于原问题最优解$X^*=(1,0)^T$.

5-5 解：(1) 增广Lagrange函数为：

$$\varphi(X,\lambda)=x_1^2+2x_2^2+2x_3^2+\lambda(x_1+x_2+x_3-4)+\frac{c}{2}(x_1+x_2+x_3-4)^2$$

$$\frac{\partial\varphi}{\partial x_1}=2x_1+\lambda+c(x_1+x_2+x_3-4),\ \frac{\partial\varphi}{\partial x_2}=4x_2+\lambda+c(x_1+x_2+x_3-4),$$

$$\frac{\partial\varphi}{\partial x_3}=4x_3+\lambda+c(x_1+x_2+x_3-4).$$

令 $\nabla\varphi(X,\lambda)=0$，得 $\varphi(X,\lambda)$ 极小点为 $x_1=\frac{4c-\lambda}{2c+2}$，$x_2=\frac{4c-\lambda}{4c+4}$，$x_3=\frac{4c-\lambda}{4c+4}$

取 $c=5$，$\lambda^{(1)}=0$，则 $x_1^{(1)}=\frac{5}{3}$，$x_2^{(1)}=\frac{5}{6}$，$x_3^{(1)}=\frac{5}{6}$.

$\lambda^{(2)}=\lambda^{(1)}+c\left(\frac{5}{3}+\frac{5}{6}+\frac{5}{6}-4\right)=-\frac{10}{3}$，$x_1^{(2)}=\frac{35}{18}$，$x_2^{(2)}=x_3^{(2)}=\frac{35}{36}$.

$\lambda^{(3)}=\lambda^{(2)}+c\left(\frac{35}{18}+\frac{35}{36}+\frac{35}{36}-4\right)=-\frac{35}{9}$，$x_1^{(3)}=\frac{215}{108}$，$x_2^{(3)}=x_3^{(3)}=\frac{215}{216}$.

$\lambda^{(4)}=\lambda^{(3)}+c\left(\frac{215}{108}+\frac{215}{216}+\frac{215}{216}-4\right)=-\frac{215}{54}$，$x_1^{(4)}=\frac{1295}{648}$，$x_2^{(4)}=x_3^{(4)}=\frac{1295}{1296}$

该问题的精确值为 $X^*=(2,1,1)^T$，相应乘子 $\lambda^*=-4$.

6-3 解:

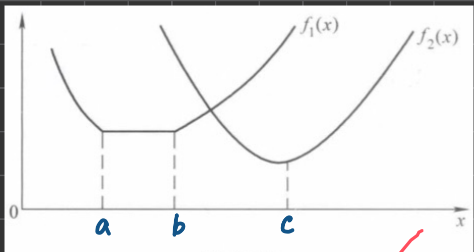


图 6-10

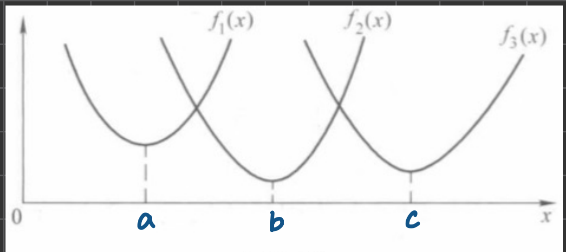


图 6-11

$Re^*=[b,c]$

$Re^*=[a,b]\cup[b,c]$

6-6 解: 构造 $\min h[F(X)]=\frac{1}{2}f_1(X)+\frac{1}{2}f_2(X)=-\frac{7}{2}x_1-\frac{9}{2}x_2$

$$\text{s.t.}\begin{cases}-3x_1-8x_2+12\geq 0\\ -x_1-x_2+2\geq 0\\ 0\leq x_1\leq 1.5,\ x_2\geq 0,\ x_3\geq 0,\ x_4\geq 0.\end{cases}$$

$h[F(X)]$、$g_1(X)$、$g_2(X)$ 在 R^2 上连续可微. 由K-T条件:

$$\begin{cases}\left(-\frac{7}{2},-\frac{9}{2}\right)^T-\mu_1(-3,-8)^T-\mu_2(-1,-1)^T=0\\ \mu_1\geq 0,\ \mu_2\geq 0\\ \mu_1(-3x_1-8x_2+12)=0,\ \mu_2(-x_1-x_2+2)=0.\end{cases}$$

解得 $\begin{cases}x_1=\frac{4}{5}\\ x_2=\frac{6}{5}\end{cases}$

故原问题最优解为 $X^*=\left(\frac{4}{5},\frac{6}{5}\right)^T$.

6-7 解: 设该工程师每月为A、B、C企业分别提供 x_1、x_2、x_3 小时咨询服务.

$\min Z=p_1d_1^-+p_2d_2^+$

$$\text{s.t.}\begin{cases}20x_1+22x_2+18x_3+d_1^--d_1^+=500\\ 0.1x_1+0.2x_2+0.2x_3+d_2^+-d_2^-=15\\ x_1+x_2+x_3\leq 40\\ x_1\leq 80\\ x_2\leq 60\\ x_3\leq 20\\ x_1,x_2,x_3\geq 0,\ d_i^-\geq 0,\ d_i^+\geq 0,\ i=1,2\end{cases}$$

6-8 解: 设从 A_i 运至 B_j 的数量为 x_{ij}.

$\min z=P_1d_1^-+P_2(d_2^-+d_3^-+d_4^-)+P_3d_5^++P_4d_6^-+P_5(d_7^++d_8^+)+P_6(d_9^-+d_9^+)$

$$\begin{cases}x_{13}+x_{23}+d_1^-=5000\\ x_{11}+x_{21}+d_2^--d_2^+=1500\end{cases}$$

$$
\text{s.t.}\begin{cases}
x_{12}+x_{22}+d_3^- - d_3^+ = 1125\\
x_{13}+x_{23}+d_4^- - d_4^+ = 3750\\
10x_{11}+4x_{12}+12x_{13}+8x_{21}+10x_{22}+3x_{23}-d_5^+=0\\
x_{21}+d_6^- - d_6^+ = 1000\\
x_{13}-d_7^+=0\\
x_{22}-d_8^+=0\\
3x_{11}+3x_{21}-4x_{12}-4x_{22}+d_9^- - d_9^+=0\\
x_{11}+x_{12}+x_{13}=3000\\
x_{21}+x_{22}+x_{23}=4000\\
x_{11}+x_{21}\leq 2000\\
x_{12}+x_{22}\leq 1500\\
x_{ij},d_i^-,d_i^+\geqslant 0.
\end{cases}
$$

6-9 解：(2) SLGP：

$$\min z_1 = d_1^-$$

$$\text{s.t.}\begin{cases} x_1+x_2+d_1^- - d_1^+ = 80\\ x_1,x_2,d_1^-,d_1^+\geqslant 0\end{cases}$$

P_1级目标最优解为 $d_1^-=0$, $\min z_1 = z_1^* = 0$.

$$\min z_2 = d_{11}^+$$

$$\text{s.t.}\begin{cases} x_1+x_2-d_1^+ = 80\\ d_1^+ + d_{11}^- - d_{11}^+ = 10\\ x_1,x_2,d_i^-,d_i^+\geqslant 0,\ i=1,11.\end{cases}$$

P_2级目标最优解为 $d_{11}^+=0$, $\min z_2 = z_2^* = 0$.

$$\min z_3 = 5d_2^- + 3d_3^-$$

$$\text{s.t.}\begin{cases} x_1+x_2-d_1^+ = 80\\ d_1^+ + d_{11}^- = 10,\\ x_1+d_2^- = 70\\ x_2+d_3^- = 45\\ x_1,x_2,d_i^-,d_i^+\geqslant 0,\ i=1,2,3,11.\end{cases}$$

			0	0	-3	0	2	0
		-105	x_1	x_2	d_1^+	d_{11}^-	d_2^-	d_3^-
0	x_2	10	0	1	-1	0	-1	0
0	d_{11}^-	10	0	0	$\boxed{1}$	1	0	0
0	x_1	70	1	0	0	0	1	0
3	d_3^-	35	0	0	1	0	1	1
		-75	0	0	0	3	2	0
0	x_2	20	0	1	0	1	-1	0
0	d_1^+	10	0	0	1	1	0	0
0	x_1	70	1	0	0	0	1	0
3	d_3^-	25	0	0	0	-1	1	1

P_3级目标最优解为 $d_2^-=0$, $d_3^-=25$, $\min z_3 = z_3^* = 75$.

$\min z_4 = d_1^+ = 10$

综上，原问题最优解为 $X^*=(70,20)^T$, $z^*=(0,0,75,10)$, $d_1^+=10$, $d_3^-=25$.

单纯形算法：

	c_j		0	0	P_1	$5P_3$	$3P_3$	0	P_4	P_2
C_B	X_B	b	x_1	x_2	d_1^-	d_2^-	d_3^-	d_{11}^-	d_1^+	d_{11}^+
P_1	d_1^-	80	1	1	1	0	0	0	-1	0
$5P_3$	d_2^-	70	$\boxed{1}$	0	0	1	0	0	0	0
$3P_3$	d_3^-	45	0	1	0	0	1	0	0	0
0	d_{11}^-	10	0	0	0	0	0	1	1	-1
	P_1	-80	-1	-1	0	0	0	0	1	0
	P_2	0	0	0	0	0	0	0	0	1
	P_3	-485	-5	-3	0	0	0	0	0	0
	P_4	0	0	0	0	0	0	0	1	0
P_1	d_1^-	10	0	$\boxed{1}$	1	-1	0	0	-1	0
0	x_1	70	1	0	0	1	0	0	0	0
$3P_3$	d_3^-	45	0	1	0	0	1	0	0	0
0	d_{11}^-	10	0	0	0	0	0	1	1	-1
	P_1	-10	0	-1	0	1	0	0	1	0
	P_2	0	0	0	0	0	0	0	0	1
	P_3	-135	0	-3	0	5	0	0	0	0
	P_4	0	0	0	0	0	0	0	1	0
0	x_2	10	0	1	1	-1	0	0	-1	0
0	x_1	70	1	0	0	1	0	0	0	0
$3P_3$	d_3^-	35	0	0	-1	1	1	0	1	0
0	d_{11}^-	10	0	0	0	0	0	1	$\boxed{1}$	-1
	P_1	0	0	0	1	0	0	0	0	0
	P_2	0	0	0	0	0	0	0	0	1
	P_3	-105	0	0	3	2	0	0	-3	0
	P_4	0	0	0	0	0	0	0	1	0
0	x_2	20	0	1	1	-1	0	1	0	-1
0	x_1	70	1	0	0	1	0	0	0	0
$3P_3$	d_3^-	25	0	0	-1	1	1	-1	0	1
P_4	d_1^+	10	0	0	0	0	0	1	1	-1
	P_1	0	0	0	1	0	0	0	0	0
	P_2	0	0	0	0	0	0	0	0	1
	P_3	-75	0	0	3	2	0	3	0	-3
	P_4	-10	0	0	0	0	0	-1	0	1

无改进余地.

故最优解为 $X^*=(70,20)^T$, $Z^*=(0,0,75,10)$, $d_1^+=10$, $d_3^-=25$.

[redacted]

7-1 解: $f_4(D_1)=d(D_1,E)=3,\ f_4(D_2)=d(D_2,E)=1,\ f_4(D_3)=d(D_3,E)=5$

$$f_3(C_i)=\min_j[d(C_i,D_j)+f_4(D_j)]$$

$$\Rightarrow f_3(C_1)=\min\begin{Bmatrix}2+3\\5+1\\3+5\end{Bmatrix}=\min\begin{Bmatrix}5\\6\\8\end{Bmatrix}=5.\ 即\ C_1\to D_1\to E$$

$$f_3(C_2)=\min\begin{Bmatrix}1+3\\4+1\\2+5\end{Bmatrix}=\min\begin{Bmatrix}4\\5\\7\end{Bmatrix}=4.\ 即\ C_2\to D_1\to E$$

同理,

$$f_2(B_1)=\min\begin{Bmatrix}4+3\\4+5\\3+4\end{Bmatrix}=7,\ 即\ B_1\to D_1\to E\ 或\ B_1\to C_2\to D_1\to E$$

$$f_2(B_2)=\min\begin{Bmatrix}1+5\\3+4\end{Bmatrix}=6,\ 即\ B_2\to C_1\to D_1\to E$$

$$f_2(B_3)=\min\begin{Bmatrix}3+5\\5+4\\3+5\end{Bmatrix}=8,\ 即\ B_3\to C_1\to D_1\to E\ 或\ B_3\to D_3\to E$$

$$f_1(A)=\min\begin{Bmatrix}3+7\\2+6\\1+8\end{Bmatrix}=8,\ 即\ A\to B_2\to C_1\to D_1\to E,\ 长度为\ 8.$$

7-2 解: $f_6(O)=2,\ f_6(P)=1$

$$f_5(L)=5+2=7,\ f_5(M)=\min\begin{Bmatrix}2+2\\8+1\end{Bmatrix}=4,\ f_5(N)=4+1=5$$

$$f_4(H)=3+7=10,\ f_4(I)=\min\begin{Bmatrix}3+7\\4+4\end{Bmatrix}=8,\ f_4(J)=\min\begin{Bmatrix}2+4\\5+5\end{Bmatrix}=6,\ f_4(K)=2+5=7$$

$$f_3(E)=\min\begin{Bmatrix}2+10\\1+8\end{Bmatrix}=9,\ f_3(F)=\min\begin{Bmatrix}1+8\\2+6\end{Bmatrix}=8,\ f_3(G)=\min\begin{Bmatrix}7+6\\4+7\end{Bmatrix}=11$$

$$f_2(C)=\min\begin{Bmatrix}5+9\\4+8\end{Bmatrix}=12,\ f_2(D)=\min\begin{Bmatrix}7+8\\3+11\end{Bmatrix}=14$$

$$f_1(A)=\min\begin{Bmatrix}4+12\\3+14\end{Bmatrix}=16$$

最短路线为 $A\to C\to F\to J\to M\to O\to B$, 距离为16.

7-5 解: 设 $k=1,2,3$, x_k 为第k阶段初未分配人员数, u_k 为分配为第k地区人员数, $g_k(u_k)$ 为收益, $f_k(x_k)$ 为将 x_k 名人员分配到第k个地区至第3个地区所获最大收益.

$$\begin{cases}f_k(x_k)=\max\limits_{u_k=0,1,2,3,4,5,6}[g_k(u_k)+f_{k+1}(x_k-u_k)],\ k=1,2,3\\ f_4(x_4)=0.\end{cases}$$

① $k=3$

$$f_3(x_3)=\max_{u_3=0,\cdots,6}[g_3(u_3)]$$

u_3		0	1	2	3	4	5	6	u_3^*	$f_3(x_3,u_3^*)$
x_3	0	0							0	0
	1	0	75						1	75
	2	0	75	100					2	100
	3	0	75	100	120				3	120
	4	0	75	100	120	135			4	135
	5	0	75	100	120	135	150		5	150
	6	0	75	100	120	135	150	180	6	180

② $k=2$

$$f_2(x_2)=\max_{u_2}[g_2(u_2)+f_3(x_2-u_2)]$$

u_2		0	1	2	3	4	5	6	u_2^*	$f_2(x_2,u_2^*)$
x_2	0	0							0	0
	1	75	65						0	75
	2	100	140	85					1	140
	3	120	165	160	110				1	165
	4	135	185	185	185	140			1	185
	5	150	200	205	210	215	160		4	215
	6	180	245	220	230	240	235	175	2	245

③ $k=1$

$$f_1(x_1)=\max_{u_1}[g_1(u_1)+f_2(6-u_1)]$$

u_1	0	1	2	3	4	5	6	u_1^*	$f_1(x_1,u_1^*)$
$x_1=6$	245	275	265	270	255	205	150	1	275

故分配方案如下:

地区	销售人员	利润
甲	1	60
乙	4	140
丙	1	75
总利润		275

7-9 解: 当$k=1$时,

S	0~6	7~13
$f_1(S)$	0	9
x_1^*	0	1

当$k=2$时,

S	0~4	5~6	7~9	10~11	12~13
x_2	0	1	0, 1	0, 1, 2	0, 1, 2
c_2+f_1	0	4	9, 4	9, 4, 8	9, 13, 8
$f_2(S)$	0	4	9	9	13
x_2^*	0	1	0	0	1

当k=3时，

S	0~3	4	5~6	7	8	9~10	11	12	13
x_3	0	1	0,1	0,1	0,1,2	0,1,2	0,1,2	0,1,2,3	0,1,2,3
c_3+f_2	0	3	4,3	9,3	9,3,6	9,7,6	9,12,6	13,12,6,9	13,12,10,9
$f_3(S)$	0	3	4	9	9	9	12	13	13
x_3^*	0	1	0	0	0	0	1	0	0

当k=4时，

S	0~2	3	4	5	6	7	8	9	10	11	12~13
x_4	0	1	0,1	0,1	0,1,2	0,1,2	0,1,2	0,1,2,3	0,1,2,3	0,1,2,3	0,1,2,3,4
c_4+f_3	0	2	3,2	4,2	4,2,4	9,5,4	9,6,4	9,6,4,6	9,11,7,6	12,11,8,6	13,11,8,6,8
$f_4(S)$	0	2	3	4	4	9	9	9	11	12	13
x_4^*	0	1	0	0	0,2	0	0	0	1	0	0

当k=5时，

$$f_5(13)=\max_{0\le x_5\le 13}\{0.5x_5+f_4(13-x_5)\}=\max\{f_4(13),\,0.5+f_4(12),\,\cdots,\,6.5+f_4(0)\}$$

$$=13.5$$

$\Rightarrow x_1^*=1,\ x_2^*=1,\ x_3^*=0,\ x_4^*=0,\ x_5^*=1$，最大价值为13.5元.