Linear Systems Theory: HW5

（Due: Oct. 24, 2023）

1. (30') Consider the following LTI system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^p$ is the input, $y \in \mathbb{R}^q$ is the output. Show that all eigenvalues of $A+BK$ can be arbitrarily assigned (provided the complex conjugate eigenvalues are assigned in pairs) by selecting a real constant matrix K if and only if (A,B) is controllable.

2. (20') Consider the LTI system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} -1 & 1 & -1 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u \\ y &= [1 \quad 0 \quad 0] x \end{aligned} \tag{2}$$

Try to design a state feedback control law to shift the eigenvalues to -1, -2, -3 by transforming the above system to controllable form.

3. (30') Consider system (1). Let C and $\bar{\mathrm{C}}$ be its controllability matrix and observability matrix. If $rank(\mathrm{C}) = n_1 < n$ and $rank(\mathrm{C}) = n_2 < n$. Show that system (1) can be equivalently transformed into the following canonical form:

$$\begin{aligned} \begin{bmatrix} \dot{\bar{x}}_{co} \\ \dot{\bar{x}}_{c\bar{o}} \\ \dot{\bar{x}}_{\bar{c}o} \\ \dot{\bar{x}}_{\bar{c}\bar{o}} \end{bmatrix} &= \begin{bmatrix} \bar{A}_{co} & 0 & \bar{A}_{13} & 0 \\ \bar{A}_{21} & \bar{A}_{c\bar{o}} & \bar{A}_{23} & \bar{A}_{24} \\ 0 & 0 & \bar{A}_{\bar{c}o} & 0 \\ 0 & 0 & \bar{A}_{43} & \bar{A}_{\bar{c}\bar{o}} \end{bmatrix} \begin{bmatrix} \bar{x}_{co} \\ \bar{x}_{c\bar{o}} \\ \bar{x}_{\bar{c}o} \\ \bar{x}_{\bar{c}\bar{o}} \end{bmatrix} + \begin{bmatrix} \bar{B}_{co} \\ \bar{B}_{c\bar{o}} \\ 0 \\ 0 \end{bmatrix} u \\ y &= \begin{bmatrix} \bar{C}_{co} & 0 & \bar{C}_{\bar{c}o} & 0 \end{bmatrix} \bar{x} + Du \end{aligned} \tag{3}$$

Further, the state-space equation (1) is zero-state equivalent to the controllable and observable state-space equation

$$\begin{aligned} \dot{\bar{x}}_{co} &= \bar{A}_{co} \bar{x}_{co} + \bar{B}_{co} u \\ y &= \bar{C}_{co} \bar{x}_{co} + Du \end{aligned} \tag{4}$$

and has the transfer matrix

$$\hat{G}(s) = \bar{C}_{co} (sI - \bar{A}_{co})^{-1} \bar{B}_{co} + D\,. \tag{5}$$

4. (20') Consider the system

$$\dot{x} = Ax + bu, \; y = cx \tag{6}$$

where $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -2 & 1 & 1 \end{bmatrix}, b = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$, c=$[1 \quad 0 \quad 1]$.

(a) Is the system observable?

(b) Compute a matrix K such that $A + Kc$ has three eigenvalue at -1.

(c) Given that $A + bf$ is asymptotically stable for $f = [-9 \quad -74 \quad -24]$, compute an output feedback controller that stabilizes the system (6).