Linear Systems Theory: HW6

（Due: Oct. 31, 2023）

1. (30’) Consider the LTI system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^q$ is the input, and $y \in \mathbb{R}^p$ is the output. Suppose that (A, C) is observable. Show that there exists a suitable matrix $L \in \mathbb{R}^{n \times p}$, such that all eigenvalues of $A - LC$ can be arbitrarily assigned.

2. (20’) Please show that whether the following system can be stabilized by a state feedback control $u = kx$. If it does, find a suitable k.

$$\dot{x} = \begin{bmatrix} 4 & 2 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \tag{2}$$

3. (20’) Consider $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times m}$. For any matrix $K \in \mathbb{R}^{m \times n}$, show that $(A - BK, B)$ is controllable if and only if (A, B) is controllable.

4. (30’) Consider the LTI system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{3}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^q$ is the input, and $y \in \mathbb{R}^p$ is the output. Suppose (A, B, C) is controllable and observable. Let one of its state observers be in the following form

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x}) \tag{4}$$

Show that the observer (4) is a special case of the following observer

$$\begin{aligned} \dot{z} &= Fz + Gy + Hu \\ \hat{x} &= T^{-1} z \end{aligned} \tag{5}$$

i.e., $TA - FT = GC$, $H = TB$, and all eigenvalues of F have negative real parts.