HW5 [redacted]

1. (30') Consider the following LTI system

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx + Du\end{aligned} \qquad (1)$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^p$ is the input, $y \in \mathbb{R}^q$ is the output. Show that all eigenvalues of $A + BK$ can be arbitrarily assigned (provided the complex conjugate eigenvalues are assigned in pairs) by selecting a real constant matrix K if and only if (A, B) is controllable.

解：1) 证明：(A,B) 可控 $\Rightarrow$ 特征值可以任意配置

易知 $B\in\mathbb{R}^{n\times p}$，$K\in\mathbb{R}^{p\times n}$

∵系统是能控的，∴存在可逆阵 $P\in\mathbb{R}^{n\times n}$，使系统等价为一个第二能控标准型

即令 $x=P\bar{x}$

$$\dot{\bar{x}} = P^{-1}AP\bar{x}+P^{-1}Bu = \bar{A}\bar{x}+\bar{B}u$$ 其中 $\bar{A}=\begin{bmatrix}0&1&&\\ 0&&\ddots&\\ 0&&&1\\ -\alpha_0&-\alpha_1&\cdots&-\alpha_{n-1}\end{bmatrix}_{n\times n}$ $\bar{B}=\begin{bmatrix}0\\ \vdots\\ 1\end{bmatrix}_{n\times 1}$

令 $\bar{K}=[k_1,\cdots,k_n]_{1\times n}$ $\bar{B}\bar{K}=\begin{bmatrix}0&0&&0\\ \vdots&\vdots&&\vdots\\ k_1&k_2&\cdots&k_n\end{bmatrix}$

∴ $\bar{A}+\bar{B}\bar{K}=\begin{bmatrix}0&1&&\\ &&\ddots&\\ -\alpha_0+k_1&-\alpha_1+k_1&\cdots&-\alpha_{n-1}+k_n\end{bmatrix}$

∴ $\det[\bar{A}+\bar{B}\bar{K}] = s^n+(\alpha_{n-1}-k_n)s^{n-1}+(\alpha_{n-2}-k_{n-1})s^{n-2}+\cdots+(\alpha_0-k_1)$

若期望的特征值为 $\lambda_1,\lambda_2,\cdots\lambda_n$

$$\prod_{i=1}^{n}(s-\lambda_i) = s^n+\beta_{n-1}s^{n-1}+\cdots+\beta_0$$

$$\Rightarrow\begin{cases}\alpha_{n-1}-k_n=\beta_{n-1}\\ \alpha_{n-2}-k_{n-1}=\beta_{n-2}\\ \vdots\end{cases}\Rightarrow\begin{cases}k_n=\alpha_{n-1}-\beta_{n-1}\\ k_{n-1}=\alpha_{n-2}-\beta_{n-2}\\ \vdots\\ k_1=\alpha_0-\beta_0\end{cases}$$ 均有解

∴ $\dot{\bar{x}}=\bar{A}\bar{x}+\bar{B}\bar{K}\bar{x}=(\bar{A}+\bar{B}\bar{K})\bar{x}$

∵ $P^{-1}\dot{x}=(\bar{A}+\bar{B}\bar{K})P^{-1}x = P^{-1}APP^{-1}x+P^{-1}B\bar{K}\cdot P^{-1}x$

$\Rightarrow \dot{x} = Ax+B(\bar{K}\cdot P^{-1})x$

∴ $K=\bar{K}P^{-1}$ 均可求解

∴充分性得证

2) 特征值任意配置 $\Rightarrow$ 系统能控：

可证明系统不能控 $\Rightarrow$ 特征值不能任意配置

证：$P_c=[B\ AB\ A^2B\ \cdots A^{n-1}B]$ $\mathrm{rank}(P_c)=k<n$ 则系统不完全能控

可取 P_c 中线性无关的k个列向量，再补充 $n-k$ 个线性无关列向量组成矩阵 $P=[\underbrace{L_1,L_2,\cdots L_k}_{\{L_1,\cdots L_k\}},\underbrace{\cdots L_n}_{\{L_{k+1},\cdots,L_n\}}]$

取 $x=P\bar{x}$，则 $\dot{\bar{x}}=\begin{bmatrix}\bar{A}_c&\bar{A}_{12}\\ 0&\bar{A}_{\bar{c}}\end{bmatrix}\bar{x}+\begin{bmatrix}\bar{B}_c\\ 0\end{bmatrix}u$，其中 $\bar{A}_c\in\mathbb{R}^{k\times k}$，$\bar{A}_{\bar{c}}\in\mathbb{R}^{(n-k)\times(n-k)}$，$\bar{B}_c\in\mathbb{R}^{k\times 1}$

取 $u=\bar{K}\bar{x}=[k_1,\cdots k_n]\bar{x}$，$\bar{B}\bar{K}=\begin{bmatrix}b_1k_1&b_1k_2&\cdots&b_1k_n\\ \vdots&&&\\ b_kk_1&b_kk_2&\cdots&b_kk_n\\ &0&&\end{bmatrix}$，

∴ $\bar{A}+\bar{B}\bar{K}$ 只会影响前k行，即无法改变 $[0,\bar{A}_{\bar{c}}]$

∴ $\bar{A}+\bar{B}\bar{K}=\begin{bmatrix}A_c^*&A_{12}^*\\ 0&\bar{A}_{\bar{c}}\end{bmatrix}$ $|sI-\bar{A}+\bar{B}\bar{K}|=\begin{vmatrix}sI-A_c^*&-A_{12}^*\\ 0&sI-\bar{A}_{\bar{c}}\end{vmatrix}=|sI-A_c^*|\cdot|sI-\bar{A}_{\bar{c}}|$ 其中 $|sI-\bar{A}_{\bar{c}}|$ 不随 $\bar{K}$ 而改变

∴可知 $\bar{A}_{\bar{c}}$ 的特征值无法改变

∴得证 系统不能控 $\Rightarrow$ 特征值无法任意配置 $\Rightarrow$ 综上，充要均证毕

2. (20') Consider the LTI system

$$\dot{x}=\begin{bmatrix}-1 & 1 & -1\\ 0 & 2 & 0\\ 0 & 0 & 3\end{bmatrix}x+\begin{bmatrix}0\\1\\1\end{bmatrix}u \qquad (2)$$

$$y=[1\quad 0\quad 0]x$$

3×1 $\quad 1\times 3$

Try to design a state feedback control law to shift the eigenvalues to -1, -2, -3 by transforming the above system to controllable form.

解：$A=\begin{bmatrix}-1 & 1 & -1\\ 0 & 2 & 0\\ 0 & 0 & 3\end{bmatrix}$, $B=\begin{bmatrix}0\\1\\1\end{bmatrix}$,

$$P_c=[B\ \ AB\ \ A^2B]=\begin{bmatrix}0 & 0 & -1\\ 1 & 2 & 4\\ 1 & 3 & 9\end{bmatrix}\rightarrow\begin{bmatrix}0 & 0 & -1\\ 0 & -1 & -5\\ 1 & 3 & 9\end{bmatrix}$$

$\because \mathrm{rank}(P_c)=3$ $\therefore$ 系统是能控的

设状态反馈 $u=Kx$ $\therefore \dot{x}=(A+BK)x$, $K=[k_1\ \ k_2\ \ k_3]$

$$\therefore BK=\begin{bmatrix}0 & 0 & 0\\ k_1 & k_2 & k_3\\ k_1 & k_2 & k_3\end{bmatrix}$$

$$\therefore A+BK=\begin{bmatrix}-1 & 1 & -1\\ k_1 & 2+k_2 & k_3\\ k_1 & k_2 & k_3+3\end{bmatrix}$$

$$|sI-(A+BK)|=\begin{vmatrix}s+1 & -1 & 1\\ -k_1 & s-2-k_2 & -k_3\\ -k_1 & -k_2 & s-k_3-3\end{vmatrix}=1\times(-1)^4\begin{vmatrix}-k_1 & s-2-k_2\\ -k_1 & -k_2\end{vmatrix}+(-1)\times(-1)^3\times\begin{vmatrix}-k_1 & -k_3\\ -k_1 & s-k_3-3\end{vmatrix}+(s+1)\times(-1)^2\times\begin{vmatrix}s-2-k_2 & -k_3\\ -k_2 & s-k_3-3\end{vmatrix}$$

$$=s^3-(k_2+k_3+4)s^2+(2k_2+k_3+1)s+k_1+3k_2+2k_3+6$$

$$(s+1)(s+2)(s+3)=s^3+6s^2+11s+6$$

$$\therefore\begin{cases}6=-k_2-k_3-4\\ 11=2k_2+k_3+1\\ 6=k_1+2k_3+3k_2+6\end{cases}$$

$17=k_2-3$

$k_2=20$ $k_2=20$

$k_3=-30$ $\quad$ $40-30+1$

$26+4=-k_3$ $-k_3=30$

$s-2-20$ $\quad$ $k_1-60+60=0$

$\Rightarrow k_1=0$, $k_2=20$, $k_3=-30$.

经验证满足

$\therefore K=[0, 20, -30]$

3. (30') Consider system (1). Let C and $\bar{\mathrm{C}}$ be its controllability matrix and observability matrix. If $rank(\mathrm{C}) = n_1 < n$ and $rank(\mathrm{C}) = n_2 < n$. Show that system (1) can be equivalently transformed into the following canonical form:

$$\begin{bmatrix} \dot{\bar{x}}_{co} \\ \dot{\bar{x}}_{c\bar{o}} \\ \dot{\bar{x}}_{\bar{c}o} \\ \dot{\bar{x}}_{\bar{c}\bar{o}} \end{bmatrix} = \begin{bmatrix} \bar{A}_{co} & 0 & \bar{A}_{13} & 0 \\ \bar{A}_{21} & \bar{A}_{c\bar{o}} & \bar{A}_{23} & \bar{A}_{24} \\ 0 & 0 & \bar{A}_{\bar{c}o} & 0 \\ 0 & 0 & \bar{A}_{43} & \bar{A}_{\bar{c}\bar{o}} \end{bmatrix} \begin{bmatrix} \bar{x}_{co} \\ \bar{x}_{c\bar{o}} \\ \bar{x}_{\bar{c}o} \\ \bar{x}_{\bar{c}\bar{o}} \end{bmatrix} + \begin{bmatrix} \bar{B}_{co} \\ \bar{B}_{c\bar{o}} \\ 0 \\ 0 \end{bmatrix} u \tag{3}$$
$$y = \begin{bmatrix} \bar{C}_{co} & 0 & \bar{C}_{\bar{c}o} & 0 \end{bmatrix} \bar{x} + Du$$

Further, the state-space equation (1) is zero-state equivalent to the controllable and observable state-space equation

$$\begin{aligned} \dot{\bar{x}}_{co} &= \bar{A}_{co}\bar{x}_{co} + \bar{B}_{co}u \\ y &= \bar{C}_{co}\bar{x}_{co} + Du \end{aligned} \tag{4}$$

and has the transfer matrix

$$\hat{G}(s) = \bar{C}_{co}(sI - \bar{A}_{co})^{-1}\bar{B}_{co} + D. \tag{5}$$

解：(1) 先证明能控性分解：

$P_c = [B\ AB\ A^2B\ \cdots A^{n-1}B]$

已知 $rank(P) = n_1 < n$ ∴ 在 P_c 中可以找到 n_1 个线性无关的列向量 $\{L_1, \cdots L_{n_1}\}$

再补充 $n-n_1$ 个与 $\{L_1, \cdots L_{n_1}\}$ 均线性无关的列向量组成一个 n 维线性无关组 $\{L_1, \cdots L_{n_1}, L_{n_1+1}, \cdots L_n\}$, $P = [L_1\ L_2, \cdots L_n]$

取 $x = P\bar{x}$

于是有 $\begin{cases} P\dot{\bar{x}} = AP\bar{x} + Bu \\ y = CP\bar{x} + Du \end{cases} \Rightarrow \begin{cases} \dot{\bar{x}} = P^{-1}AP\bar{x} + P^{-1}Bu \\ y = CP\bar{x} + Du \end{cases}$

① 先对 $P^{-1}B = \begin{bmatrix} B_c \\ 0 \end{bmatrix}$ 验证：取 $P^{-1} = \begin{bmatrix} \xi_1^T \\ \xi_2^T \\ \vdots \\ \xi_n^T \end{bmatrix} \Rightarrow P^{-1}P = \begin{bmatrix} \xi_1^T \\ \xi_2^T \\ \vdots \\ \xi_n^T \end{bmatrix} [L_1\ L_2 \cdots L_n] = \begin{bmatrix} 1 & & \\ & 1 & \\ & & \ddots \\ & & & 1 \end{bmatrix}$

易知 B 为 $P_c = [B\ AB \cdots]$ 的一部分，故 B 可由 $\{L_1, \cdots L_{n_1}\}$ 线性表示

由上式可知 $\begin{cases} \xi_k^T \cdot L_k = 1 \\ \xi_{k+1}^T \cdot B = 0 \end{cases}$

∴ $P^{-1}B = \begin{bmatrix} \xi_1^T \\ \xi_2^T \\ \vdots \\ \xi_n^T \end{bmatrix} B = \begin{bmatrix} \xi_1^T B \\ \xi_2^T B \\ \xi_{n_1}^T B \\ \bar{0} \end{bmatrix}$ 即 $P^{-1}B = \begin{bmatrix} B_c \\ 0 \end{bmatrix}$

② 再对 $P^{-1}AP = \begin{bmatrix} \bar{A}_c & * \\ 0 & \bar{A}_{\bar{c}} \end{bmatrix}$ 验证

$AP = A[L_1 \cdots L_{n_1}\ L_{n_1+1} \cdots L_n]$,

对于 AL_j $(j = 1 \cdots n_1)$ 进行研究，

$AP_c = [AB\ A^2B\ A^3B\ \cdots A^nB]$

由凯莱哈密尔顿定理，A^n 是 $I, A, A^2, \cdots A^{n-1}$ 的线性组合

∴ $AP_c = [AB\ A^2B\ A^3B \cdots\ (a_0I + a_1A + a_2A^2 + \cdots a_{n-1}A^{n-1})B]$

∴ AP_c 可由 $\{L_1, \cdots L_{n_1}\}$ 线性表示

即 AL_j $(j = 1 \cdots n_1)$ 可由 $\{L_1, \cdots L_{n_1}\}$ 线性表示

∴ $\xi_{n_1+1}^T AL_j = 0, \cdots \xi_n^T AL_j = 0$

∴ $P^{-1}AP = \begin{bmatrix} \xi_1^T \\ \xi_2^T \\ \vdots \\ \xi_n^T \end{bmatrix} A[L_1\ L_2, \cdots L_n] = \left[\begin{array}{cc|c} \xi_1^T AL_1 & \xi_1^T AL_1 & \xi_1^T AL_{n_1+1} \\ \xi_{n_1}^T AL_1 & & \\ \hline \xi_{n_1+1}^T AL_1 & & \xi_{n_1+1}^T AL_{n_1+1} \end{array}\right] = \left[\begin{array}{c|c} \bar{A}_c & * \\ \hline 0 & \bar{A}_{\bar{c}} \end{array}\right]$

∴ 得证

由上可知：$\begin{bmatrix} \dot{\bar{x}}_c \\ \dot{\bar{x}}_{\bar{c}} \end{bmatrix} = \begin{bmatrix} \bar{A}_c & \bar{A}_{12} \\ 0 & \bar{A}_{\bar{c}} \end{bmatrix} \begin{bmatrix} \bar{x}_c \\ \bar{x}_{\bar{c}} \end{bmatrix} + \begin{bmatrix} \bar{B}_c \\ 0 \end{bmatrix} u$

$\bar{y} = [\bar{C}_c\ \ \bar{C}_{\bar{c}}] \begin{bmatrix} \bar{x}_c \\ \bar{x}_{\bar{c}} \end{bmatrix} + Du$

(2) $O=\begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1}\end{bmatrix}$，$\mathrm{rank}(O)=n_2$，可取出其中$n_2$个线性无关的行向量，再补充$n-n_2$个线性无关的向量组成一个线性无关组
$\{L_1^T, L_2^T, \cdots L_{n_2}^T, L_{n_2+1}^T, \cdots L_n^T\}$

$\therefore P_O=\begin{bmatrix} L_1^T \\ L_2^T \\ \vdots \\ L_{n-1}^T \\ L_n^T\end{bmatrix}$　令 $x=P^{-1}\bar{x}$

设 $P_O^{-1}=[\xi_1, \xi_2 \cdots \xi_{n_2}, \xi_{n_2+1}, \cdots \xi_n]$

$P_O P^{-1}=\begin{bmatrix} L_1^T \\ L_2^T \\ \vdots \\ L_n^T\end{bmatrix}[\xi_1 \cdots \xi_n]=\begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1\end{bmatrix}$

易知C是O的一部分，也即$\{L_1^T, L_2^T, \cdots L_{n_2}^T\}$的线性组合

$\therefore CP^{-1}=C[\xi_1\cdots\xi_{n_2}, \cdots \xi_n]=[C_o \;\; 0]$

类似能控性分解

$P^{-1}\dot{\bar{x}}=AP^{-1}\bar{x}+Bu \Rightarrow \dot{\bar{x}}=PAP^{-1}\bar{x}+PBu$

可得：
$$\begin{bmatrix} \dot{\bar{x}}_o \\ \dot{\bar{x}}_{\bar{o}}\end{bmatrix}=\begin{bmatrix} \bar{A}_o & 0 \\ \bar{A}_{21} & \bar{A}_{\bar{o}}\end{bmatrix}\begin{bmatrix} \bar{x}_o \\ \bar{x}_{\bar{o}}\end{bmatrix}+\begin{bmatrix} \bar{B}_o \\ \bar{B}_{\bar{o}}\end{bmatrix}u$$
$$\bar{y}=[\bar{C}_o \;\; 0]\bar{x}+Du$$

由(1)(2)易知先对系统作能控性分解，再作能观性分解，便可得到$c,\bar{c},o,\bar{o}$四个部分

如下
$$\begin{bmatrix} \dot{\bar{x}}_{co} \\ \dot{\bar{x}}_{c\bar{o}} \\ \dot{\bar{x}}_{\bar{c}o} \\ \dot{\bar{x}}_{\bar{c}\bar{o}}\end{bmatrix}=\begin{bmatrix} \bar{A}_{co} & 0 & \bar{A}_{13} & 0 \\ \bar{A}_{21} & \bar{A}_{c\bar{o}} & \bar{A}_{23} & \bar{A}_{24} \\ 0 & 0 & \bar{A}_{\bar{c}o} & 0 \\ 0 & 0 & \bar{A}_{43} & \bar{A}_{\bar{c}\bar{o}}\end{bmatrix}\begin{bmatrix} \bar{x}_{co} \\ \bar{x}_{c\bar{o}} \\ \bar{x}_{\bar{c}o} \\ \bar{x}_{\bar{c}\bar{o}}\end{bmatrix}+\begin{bmatrix} \bar{B}_{co} \\ \bar{B}_{c\bar{o}} \\ 0 \\ 0\end{bmatrix}u$$
$$y=[\bar{C}_{co} \;\; 0 \;\; \bar{C}_{\bar{c}o} \;\; 0]\bar{x}+Du$$

(3) 证明系统的传递函数

$$\bar{G}(s)=\bar{C}(sI-\bar{A})^{-1}\bar{B}+D$$
$$=\bar{C}\begin{bmatrix} sI-\bar{A}_{co} & 0 & -\bar{A}_{13} & 0 \\ -\bar{A}_{21} & sI-\bar{A}_{c\bar{o}} & -\bar{A}_{23} & -\bar{A}_{24} \\ 0 & 0 & sI-\bar{A}_{\bar{c}o} & 0 \\ 0 & 0 & -\bar{A}_{43} & sI-\bar{A}_{\bar{c}\bar{o}}\end{bmatrix}^{-1}\bar{B}+D$$

由分块矩阵求逆：$A^{-1}=\begin{bmatrix} A_1 & A_2 \\ 0 & A_4\end{bmatrix}^{-1}=\begin{bmatrix} A_1^{-1} & -A_1^{-1}A_2A_4^{-1} \\ 0 & A_4^{-1}\end{bmatrix}$

$$A^{-1}=\begin{bmatrix} A_1 & 0 \\ A_3 & A_4\end{bmatrix}^{-1}=\begin{bmatrix} A_1^{-1} & 0 \\ -A_4^{-1}A_3A_1^{-1} & A_4^{-1}\end{bmatrix}$$

可知 $\bar{G}(s)=[\bar{C}_{co} \;\; 0 \;\; \bar{C}_{\bar{c}o} \;\; 0]\begin{bmatrix} (sI-\bar{A}_{co})^{-1} & 0 & * & 0 \\ * & (sI-\bar{A}_{c\bar{o}})^{-1} & * & * \\ 0 & 0 & * & 0 \\ 0 & 0 & * & *\end{bmatrix}\begin{bmatrix} \bar{B}_{co} \\ \bar{B}_{c\bar{o}} \\ 0 \\ 0\end{bmatrix}$

$$=\bar{C}_{co}(sI-\bar{A}_{co})^{-1}\bar{B}_{co}+D$$

$\therefore$ 得证

4. (20') Consider the system

$$\dot{x} = Ax + bu,\ y = cx \qquad (6)$$

where $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -2 & 1 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$, c=[1 0 1].

(a) Is the system observable?

(b) Compute a matrix K such that $A + Kc$ has three eigenvalue at -1.

(c) Given that $A + bf$ is asymptotically stable for $f = [-9 \quad -74 \quad -24]$, compute an output feedback controller that stabilizes the system (6).

解：(a) $P_o = \begin{bmatrix} c \\ cA \\ cA^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ -2 & 2 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix}$

$\mathrm{rank}(P_o) = 3$

∴可观

(b) 假设 $K = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix}$ ∴ $KC = \begin{bmatrix} k_1 & 0 & k_1 \\ k_2 & 0 & k_2 \\ k_3 & 0 & k_3 \end{bmatrix}$

∴ $A+KC = \begin{bmatrix} k_1+1 & 0 & k_1 \\ k_2+1 & 1 & k_2 \\ k_3-2 & 1 & k_3+1 \end{bmatrix}$

∴ $|sI-(A+KC)| = \begin{vmatrix} s-k_1-1 & 0 & -k_1 \\ -k_2-1 & s-1 & -k_2 \\ -k_3+2 & -1 & s-k_3-1 \end{vmatrix} = (s-k_1-1)\times(-1)^2\times\begin{vmatrix} s-1 & -k_2 \\ -1 & s-k_3-1 \end{vmatrix} + (-k_1)\times(-1)^4\times\begin{vmatrix} -k_2-1 & s-1 \\ -k_3+2 & -1 \end{vmatrix}$

$= s^3 - (k_1+k_3+3)s^2 + (4k_1 - k_2 + 2k_3 + 3)s - 4k_1 + k_2 - k_3 - 1$

$\Rightarrow \begin{cases} -k_1-k_3-3=3 \\ 4k_1-k_2+2k_3+3=3 \\ -4k_1+k_2-k_3-1=1 \end{cases} \Rightarrow \begin{cases} k_1=-8 \\ k_2=-28 \\ k_3=2 \end{cases}$

∴ $K=[-8,-28,2]$

(c)

$u = Fy = FCx$

$b = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}_{3\times1}$, $C = [1\ 0\ 1]_{1\times3}$

∴ $\dot{x} = (A+bFC)x$

令 $F = f$

∴ $bFC = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} F [1\ 0\ 1]$

$= \begin{bmatrix} 2f & 0 & 2f \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

∴ $A+bFC = \begin{bmatrix} 2f+1 & 0 & 2f \\ 1 & 1 & 0 \\ -2 & 1 & 1 \end{bmatrix}$

∴ $|sI-(A+bFC)| = \begin{vmatrix} s-2f-1 & 0 & -2f \\ -1 & s-1 & 0 \\ 2 & -1 & s-1 \end{vmatrix} = (s-2f-1)\times(s-1)^2 + (-2f)\times(1-2s+2)$

$= (s-2f-1)\times(s^2-2s+1) + 4fs - 6f$

$= s^3 - 2s^2 + s - (2f+1)s^2 + 2(2f+1)s - (2f+1) + 4fs - 6f$

$= s^3 - (2f+3)s^2 + (4f+2+1+4f)s - 2f - 1 - 6f$

$= s^3 - (2f+3)s^2 + (8f+3)s - 1 - 8f$

由劳斯判据：

$s^3 \quad 1 \quad 8f+13$

$s^2 \quad -2f-3 \quad -1-8f$

$s \quad \dfrac{-16f^2-22f-8}{-2f-3}$

可知 $\begin{cases} -2f-3>0 \\ \dfrac{-16f^2-22f-8}{-2f-3} > 0 \end{cases} \Rightarrow$ 无解，即找不合适的输出反馈增益