Linear Systems Theory: HW1

（Due: Sep. 19, 2023）

1. (5'+10') Consider the following system

$$\ddot{y} + 0.1\dot{y} + y^5 = 6\sin t \qquad (1)$$

(1) Let $x_1 = y, x_2 = \dot{y}$. Put system into state space model.

(2) Using Matlab to plot the trajectories of y under two initial conditions, i.e., $[x_1(0), x_2(0)]^T = [2,3]^T$ and $[x_1(0), x_2(0)]^T = [2.01, 3.01]^T$. Is there significant difference when $0 \le t < 10s$? How about when $t \ge 40s$?

2. (5'+5'+5')

Consider the electrical network shown in Fig. 2. Find its state space model when the state are chosen to be $x = [v_c, i_L]^T$ and $x = [v_c, v_o]^T$, respectively. What is the relation between these two models?

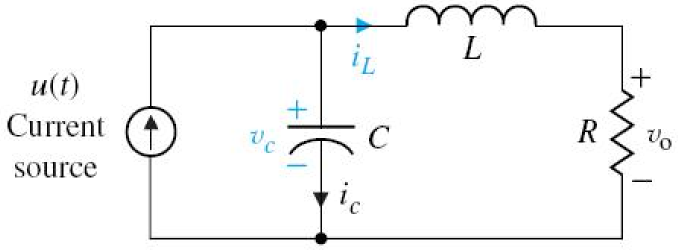

Fig. 2 An electrical network

3. (20')

Let the eigenvalues of A be distinct and let $q_i \in \mathbb{R}^n$ be a right eigenvector of A associated with the eigenvalue λ_i. Define $Q = [q_1, q_2, \cdots, q_n]$ and define $P = Q^{-1} = [p_1^T, p_2^T, \cdots, p_n^T]^T$ where p_i is the i-th row of P. Show that p_i is a left eigenvector of A associated with λ_i.

4. (40')

Given a matrix $A \in \mathbb{R}^{m\times n}$ and vector $y \in \mathbb{R}^m$.

(1). Show that a solution x exists in $Ax = y$ if and only if y lies in the range space of A or, equivalently $rank(A) = rank([A \quad y])$ where $[A \quad y]$ is an augmented matrix.

(2). Show that a solution x exists in $Ax = y$ for every y if and only if A have a rank m (full row rank).

5. (10') (C.T.Chen 1984, Problem 2-13)

Show that similar matrices have the same characteristic polynomials, and consequently the same set of eigenvalues. (Hint: $det(AB)=det(A)det(B)$)