Linear System Theory: HW2

(Due: Sept. 26, 2023)

1. (10')
   Consider the linear algebraic equation

$$\begin{bmatrix} 2 & -1 \\ -3 & 3 \\ -1 & 2 \end{bmatrix} x = y$$

   Does a solution x exist and unique in the equation when $y = [-1, 0, -1]^T$? Does a solution exist if $y = [1, 1, 1]^T$. Please give detailed explanation.

2. (20') Find the Jordan canonical form representation of the following matrices:

   (1). $A = \begin{bmatrix} 1 & 4 & 10 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

   (2). $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -4 & -3 \end{bmatrix}$

3. (20') Show that if λ is an eigenvalue of A with eigenvector x , then $f(\lambda)$ is an eigenvalue of $f(A)$ with the same eigenvector A .

4. (20') For matrix $A = \begin{bmatrix} 1 & 4 & 10 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, compute e^{103} and e^{At} .

5. (20') Show that if all eigenvalues of $A \in \mathbb{R}^{n\times n}$ are distinct, then $(sI - A)^{-1}$ can be expressed as

$$(sI - A)^{-1} = \sum_{i=1}^{n} \frac{1}{s - \lambda_i} q_i p_i$$

   where q_i and p_i are the right and left eigenvectors of A associated with eigenvalue λ_i .

6. (10') Find the unit step response of the following system.

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 2 & 3 \end{bmatrix} x$$