Signature from Head of the School

Harbin Institute of Technology, Shenzhen

Fall Semester of 2024

Linear Systems Theory Examination (A)

Question	1	2	3	4	5	6	7	Total
Mark								

P.S.: This paper was memorized and typeset after the examination ended, and there's no cheating behaviour during the examination.

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1. (10 Points) Consider the electrical network shown in Fig. 1.

   (1) Find its state space model when the state are chosen to be $x = [v_c, i_L]^T$. (5 points)

   (2) If R=1, L=1, C=1, is the system controllable? Is it observable? (5 points)

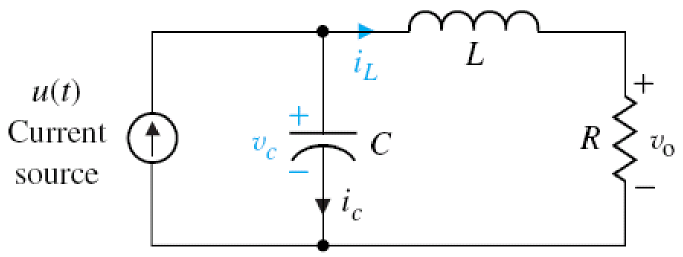

Fig. 1 RLC network

2. (10 Points) Consider the linear algebraic equation

$$\begin{bmatrix} 2 & -1 \\ -3 & 3 \\ -1 & 2 \end{bmatrix} x = y$$

Does a solution x exist and unique in the equation when $y = [-1, 0, -1]^T$? Does a solution exist if $y = [1, 1, 1]^T$? Please give detailed explanation.

3. (10 Points) For matrix $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$, compute A^k and e^{At} .

4. (20 Points) Consider the CT-LTI system $\dot{x} = Ax, x \in R^n$. Suppose there exists a positive constant μ and positive definite matrices P, Q for which the following Lyapunov equation

$$A'P + PA + 2\mu P = -Q$$

holds. Prove that all eigenvalues of A have real parts less than $-\mu$.

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5. (20 Points) Consider the linear system

$$\dot{x} = Ax + Bu,$$

where $x \in \mathbb{R}^n$ is the state and $u \in \mathbb{R}^m$ is the input. Show that this system is controllable if and only if the matrix

$$W_c(t) = \int_0^t e^{A\tau} BB^T e^{A^T \tau} d\tau = \int_0^t e^{A(t-\tau)} BB^T e^{A^T (t-\tau)} d\tau$$

is positive definite for any $t > 0$.

6. (10 points) Consider the LTI system

$$\dot{x} = \begin{bmatrix} -1 & 1 & -1 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u$$

$$y = [1 \quad 0 \quad 0] x$$

Try to design a state feedback control law to shift the eigenvalues to -1, -2, -3 by transforming the above system to controllable form.

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7. (20 Points) Use Dynamic Programming **or** Completing the square to find optimal control sequence u_0, u_1, ..., u_{N-1} to minimize following $J_N(z, u)$

$$J_N(z,u) = x_N^T Q_f x_N + \sum_{k=0}^{N-1} [x_k^T Q x_k + u_k^T R u_k]$$

subject to linear dynamics constraints:

$$x_{k+1} = Ax_k + Bu_k,\ x_0 = z$$

Hint: The recursive Riccati sequence is

$$P_{j+1} \triangleq Q + A^T P_j A - A^T P_j B\left(R + B^T P_j B\right)^{-1} B^T P_j A$$