Lecture 1
Introduction

- Course Information
- What is estimation?
- Mathematical description of estimation

Contents

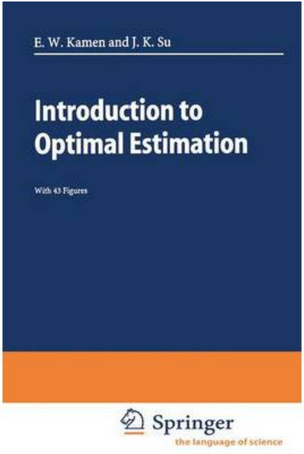
Optimal Estimation

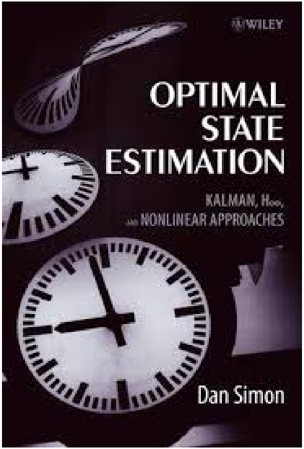
- Credit hour: 32. Credit: 2.
- Final grades=homework$\times$10%+Attendance$\times$10%+Project$\times$ 40%+exam$\times$40%
- Lecturer: Jun Xu, xujunqgy@hit.edu.cn
- Appointments for questions & explanations: send an email
- Teaching assistant: Xinyu Geng, g1649468858@163.com

Reference textbook

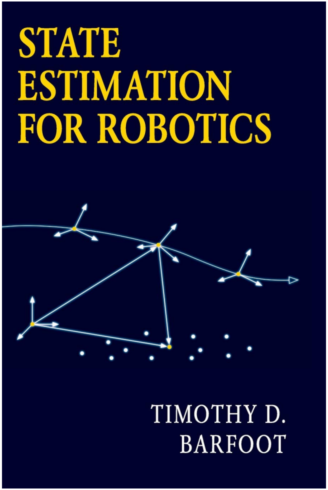
1. E. W. Kamen, J. K. Su. Introduction to Optimal Estimation. Springer-Verlag London, 1999.
2. D. Simon. Optimal state estimation: Kalman, H infinity, and nonlinear approaches. John Wiley & Sons, 2006.
3. T. D. Barfoot. State estimation for robotics. Cambridge University Press, 2017.
4. J. L. Crassidis, J. L. Junkins. Optimal Estimation of Dynamic Systems, Second Edition, CRC Press, 2011.

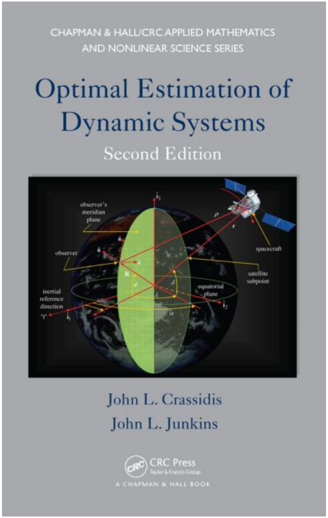
Reference textbook





Reference textbook





Project

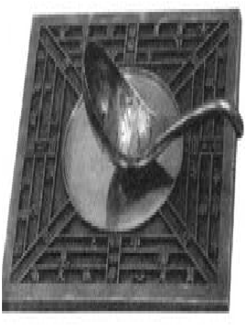
- The project is a team project with two people.
- The project could be an extension of existing methods in the literature or, preferably, involve the original research ideas related to your research interests.

Contents

What is estimation?

Example: Self-Driving Car

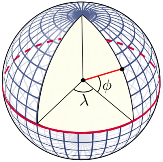
A little history–the magnetic compass



- for divination as early as the Chinese Han Dynasty (since c. 206 BC)
- adopted for navigation by the Song Dynasty Chinese (11th century)
- the first record in Western Europe and the Islamic world is around 1190.

A little history–Navigation at sea

- In the 15th century, global navigation on the open sea became possible
- following a compass bearing
- Latitude determination (using quadrant)
- Longitude determination (knowing the time of the day)



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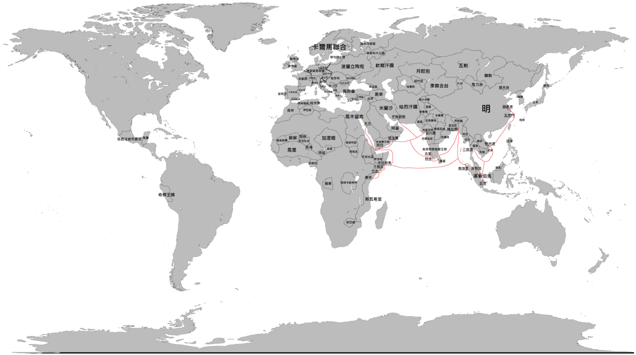

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欽察汗國
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順天府
明
劉家港
五虎門
奧斯曼
黑羊
白羊
米蘭沙
哈烈汗國
馬木留克
天方
阿曼
阿丹
加涅姆
蘇蘇
馬里
古里
三寶港
新州港
滿者伯夷
斯瓦希里
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A little history–Estimation in astronomy

- the method of least squares was pioneered by Gauss
  1. developed the technique to minimize the impact of measurement error in the prediction of orbits
  2. used least squares to predict the position of the dwarf planet when he was 23!
  3. proved that the least-squares method is optimal under the assumption of normally distributed errors when he was 31!

A little history–Kalman filter

- minimize the impact of measurement error
- Kalman published two landmark papers in 1960
  - introduced the notion of observability (a state can be inferred from a set of measurements in a dynamic system)
  - introduced an optimal framework for estimating a systems's state in the presence of measurement noise
  - developed the famous Kalman filter

A little history–Kalman filter

Application in Apollo 11 lunar module (estimate the module's position above the lunar surface based on noisy radar measurements)

An sketch of Kalman filter

Early estimation milestones

1654	Pascal and Fermat lay foundations of probability theory
1764	Bayes' rule
1801	Gauss uses least-squares to estimate the orbit of the planetoid Cares
1805	Legendre publishes "least squares"
1913	Markov chains
1933	(Chapman)-Kolmogorov equations
1949	Wiener filter
1960	Kalman(Bucy) filter
1965	Rauch-Tung-Striebel smoother
1970	Jazwinski coins "Bayes filter"

A little history–new technologies

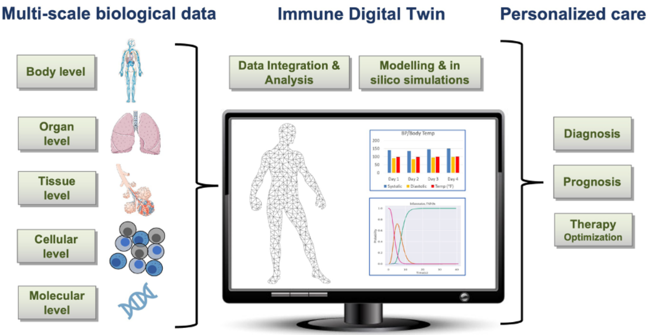
- faster and cheaper computers
- digital cameras
- laser imaging
- the Global Positioning System (GPS)

Application of estimation

Applicable to virtually all areas of engineering and science

- aerospace systems: the estimation of an aircraft's or spacecraft's position and velocity based on radar measurements of position
- communications: the estimation of congestion in a computer communications network
- biomedical engineering: the estimation of the health of a person's heart based on an electrocardiogram
- manufactoring, chemical engineering, robotics, economics, ecology, and many others

Medical application



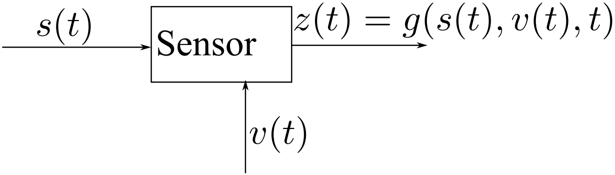
Contents

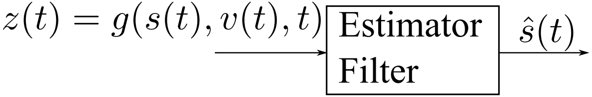
What is estimation?

- A collection of measurements is provided;
- The statistical properties of the noise is known (or assumed to be known);
- Estimation is the problem of reconstructing the underlying state (signal, parameter) of a system given a sequence of measurements as well as a priori model (information) of the systems.*

*Leo Breiman. "Statistical Modeling: The Two Cultures (with comments and a rejoinder by the author)". In: *Statistical Science* 16.3 (Aug. 2001). Publisher: Institute of Mathematical Statistics, pp. 199–231. ISSN: 0883-4237, 2168-8745. DOI: 10.1214/ss/1009213726.

Signal Estimation





Signal Estimation

- $s(t)$ is a real-valued function of the continuous-time t
- $z(t)$ is generated from $s(t)$, $v(t)$ is a noise or disturbance term
- explanation 1: in a communications system $s(t)$ may be a transmitted signal and $z(t)$ is the received signal (a distorted version of $s(t)$)
- explanation 2: $z(t)$ may be a measurement of the signal $s(t)$ obtained from a sensor ($s(t)$ may be the output of a process or system)
- objective: estimate the true signal $s(t)$ from $z(t)$, provided that the distribution of the noise $v(t)$ is known.

Signal Estimation

Target tracking

$$z(t) = s(t) + v(t)$$

$z(t)$ is a noisy measurement of a target's position provided by a radar

Signal Estimation

- Reconstruction of $s(t)$ from $z(t)$: filtering or estimation
- The estimator (filter) is a **dynamical system**: the estimate $\hat{s}(t)$ at time t generated by the estimator is not simply a function of $z(t)$, rather,

$$\hat{s}(t) = f(\{z(\tau) | -\infty < \tau \leq t\}, t)$$

- Objective: let $\hat{s}(t)$ be as close to $s(t)$ as possible

State Estimation

A real life example for state estimation:

State Estimation

When I drive into a tunnel, my GPS continues to show me moving forward, even though it isn't getting any new position sensing data.

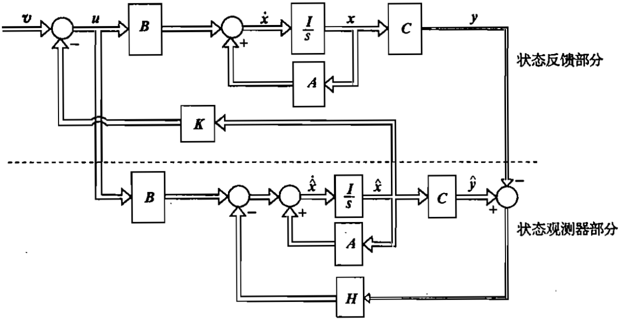
In our case, given

- a car with (approximately known) dynamics
- noisy sensor data (position and velocity)
- control commands (from the driver)
- our current estimate of the car's state

How can we predict the car's next state? How can we control the car?

State Estimation

State observer: a special kind of state estimator



$$\lim_{t\to\infty}(\hat{x}(t)-x(t))=0$$

State Estimation

Linear time-invariant continuous-time system:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t)$$

Corresponding state observer:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + H(y(t) - \hat{y}(t))$$
$$\hat{y}(t) = C\hat{x}(t)$$

Parameter Estimation

Least squares (LS) estimation of signal parameters: Assume

$$s(n) = \sum_{j=1}^{q} \theta_j \gamma_j(n)$$

$$z(n) = s(n) + v(n)$$

where $\theta_1, \theta_2, \ldots, \theta_q$ are unknown parameters and

$\gamma_1(n), \gamma_2(n), \ldots, \gamma_q(n)$ are known functions of n.

How to estimate the parameter?

$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_q \end{bmatrix}$$

Parameter Estimation

$$\min[z(1) - \hat{s}(1)]^2 + [z(2) - \hat{s}(2)]^2 + \ldots + [z(n) - \hat{s}(n)]^2$$

where

$$\hat{s}(n) = \sum_{j=1}^{q} \hat{\theta}_j \gamma_j(n)$$

Parameter Estimation

$$\min(Z_n - \Gamma_n\hat{\theta})^T(Z_n - \Gamma_n\hat{\theta})$$

where $Z_n = \begin{bmatrix} z(1) \\ z(2) \\ \vdots \\ z(n) \end{bmatrix}$, $\Gamma_n = \begin{bmatrix} \gamma(1) \\ \gamma(2) \\ \vdots \\ \gamma(n) \end{bmatrix}$,
$\gamma(n) = [\gamma_1(n), \gamma_2(n), \ldots, \gamma_q(n)]$.

Parameter Estimation

LS estimate of θ,

$$\hat{\theta} = \left[\Gamma_n^T \Gamma_n\right]^{-1} \Gamma_n^T Z_n$$

provided that Γ_n is full column rank.

What we are going to do

- Discuss mathematical approaches to the best possible way of estimating signal, state or parameters
  - more in the field of engineering, or applied mathematics
- The approaches that we present for estimation are given with the goal of eventual implementation in software
  - mostly geared toward discrete-time systems

Questions?