Student Name:

Student ID:

答题内容写在边线外视为无效

Major:

# 哈尔滨工业大学深圳研究生院

## 2016 年 春 季学期期末考试试卷

## HIT Shenzhen Graduate School Examination Paper

**Course Name:** 自适应控制 **Lecturer:** 梅 杰

| Question | One | Two | Three | Four | Five | Six | Seven | Eight | Nine | Ten | Total |
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| Mark | | | | | | | | | | | |

1. (20 points) Answer the following questions:

   (a) What are the definitions of indirect and direct adaptive control?

   (b) What are the four methods for robust adaptive control mentioned in our class?

2. (20 points) Consider the first-order plant

$$\dot{x} = -ax + b[u + \theta_1\phi_1(x)] - \theta_2\phi_2(x),$$

where $a, b, \theta_1$ and $\theta_2$ are unknown constants with $b > 0$, while $\phi_1(x)$ and $\phi_2(x)$ are Lipschitz-continuous in $x$.

Design $u$, such that all signals in the closed-loop system are bounded and $x$ tracks the state $x_{ref}$ of the following reference model given by

$$\dot{x}_{ref} = a_{ref}x_{ref} + b_{ref}u_c(t),$$

where $a_{ref} < 0$ and $b_{ref}$ are known, $u_c(t)$ is the input command which is bounded and piecewise continuous.

3. (30 points) Consider a linear system with nonlinear matched uncertainties in the form

$$\dot{x} = Ax + B\Lambda[u + \Theta^T\Phi(x)] + \varepsilon(t),$$

where $x \in \mathbb{R}^{n\times n}$ is the state, $u \in \mathbb{R}^m$ is the control input, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $\Lambda^{m\times m}$, $\Theta \in \mathbb{R}^{N\times m}$ are constant matrices, and $\varepsilon(t) \in \mathbb{R}^n$ is the disturbance. Assume that the pair $(A, B\Lambda)$ is controllable. $\Phi(x) = (\phi_1(x), \ldots \phi_n(x))^T \in \mathbb{R}^N$ denotes the known regressor vector, whose components $\phi_i(x)$ are assumed to be Lipschitz-continuous in $x$.

   (a) Assume that $\varepsilon(t) = 0$, $B$ is known, while $A$ and $\Lambda$ are unknown. In addition, it is assumed that $\Lambda$ is diagonal with $m$ nonzero diagonal elements $\lambda_1, \lambda_2, \ldots, \lambda_m$, and the signs of all $\lambda_i$ are known. Design and analyze a directed MRAC scheme that can stabilize the system and regulate $x$ towards zero.

   (b) Assume that $\|\varepsilon(t)\| \leq \varepsilon_f$, $\forall t > 0$, where $\varepsilon_f > 0$, and $A$, $B$, $\Lambda$ are all known, while $\Lambda$ is a positive diagonal matrix. Design a $\sigma$-modification robust control algorithm that can stabilize the system and regulate $x$ towards the neighborhood of zero.

   (*Hint:* For positive definite matrixes $P$ and $\Gamma$, we have

$$\lambda_{\min}(P)x^Tx \leq x^TPx \leq \lambda_{\max}(P)x^Tx,$$
$$x^TP\varepsilon(t) \leq \lambda_{\max}(P)\|x\|\|\varepsilon(t)\|$$
$$\frac{1}{\lambda_{\max}(\Gamma)}\text{tr}(\widetilde{\Theta}\widetilde{\Theta}^T) \leq \text{tr}(\widetilde{\Theta}\Gamma^{-1}\widetilde{\Theta}^T) \leq \frac{1}{\lambda_{\min}(\Gamma)}\text{tr}(\widetilde{\Theta}\widetilde{\Theta}^T),$$
$$2\text{tr}(\Theta\widetilde{\Theta}^T) \leq \text{tr}(\Theta\Theta^T) + \text{tr}(\widetilde{\Theta}\widetilde{\Theta}^T),$$

   where $\lambda_{\min}(\cdot)$ and $\lambda_{\min}(\cdot)$ denote, respectively, the minimum and maximum eigenvalues of a positive definite matrix. )

4. (30 points) We denote by $\|x\|$ the absolute value of $x$ if $x$ is a scalar and the Euclidean norm of $x$ if $x$ is a vector. For functions of time, the $L_p$ norm is given by

$$\|x\|_p = \left(\int_0^\infty \|x(\tau)\|^p \mathrm{d}\tau\right)^{\frac{1}{p}},$$

for $p \in [1, \infty)$, while

$$\|x\|_\infty = \sup_{t\geq 0}\|x(t)\|.$$

We say that $x \in \mathbb{L}_p$ when $\|x\|_p < \infty$.

   (a) Write down Barbalat's lemma and use it to prove the following corollary.

   **Corollary 0.1** *If $x \in \mathbb{L}_2 \bigcap \mathbb{L}_\infty$ and $\dot{x} \in \mathbb{L}_\infty$, then $\lim_{t\to\infty} x(t) = 0$.*

   (b) Consider the following first-order system

$$\dot{x} = u + d_1(t) + d_2(t)x + d_3(t)x^2,$$

   where $d_1(t), d_2(t)$, and $d_3(t)$ are time-varying continuous functions satisfying

$$\max_{t\geq 0}\{\|d_1(t)\|, \|d_2(t)\|, \|d_3(t)\|\} \leq d_{\max},$$

   for some unknown positive constant $d_{\max}$. Design a control algorithm combing the adaptive and sliding control to stabilize the system and regulate $x$ towards to zero. (Use the corollary in (a) to prove the result.)