HOMEWORK 1

1. Gaussian RV: uncorrelatedness implies independence

Show that if two random variables X and Y are jointly normal and are uncorrelated, then they are independent.

Hint: Two random variables X, Y are jointly normal if

$$\text{pdf}(X,Y) = f_{X,Y}(x,y) = \frac{1}{2\pi|\det(C_Z)|^{1/2}} \exp\left[-\frac{1}{2}(Z - E(Z))^T C_Z^{-1}(Z - E(Z))\right],$$

in which $Z = [X, Y]^T$.

2. Wide-sense stationary

Consider the signal plus noise $\boldsymbol{z}(n) = \boldsymbol{s} + \boldsymbol{v}(n)$, where $\boldsymbol{s}$ is a RV with $E[\boldsymbol{s}] = 1, E[\boldsymbol{s}^2] = 2$, and for each value of n, $\boldsymbol{v}(n) \sim \mathcal{N}(0,1)$. It is known that $E[\boldsymbol{s}\boldsymbol{v}(i)] = 1$ for all i, and $\boldsymbol{v}(i)$ is independent of $\boldsymbol{v}(j)$ for all $i \neq j$.

(1) Compute the autocorrelation function $R_z(i,j)$ of $\boldsymbol{z}(n)$ for all integers i, j.

(2) Is $\boldsymbol{z}(n)$ WSS(Wide sense stationary)? If so, derive a mathematical expression for $R_z(k)$.