HOMEWORK 3

1. About the Kalman filter

Estimating the position of a car. Figure 1 shows the estimation problem using a nonlinear measurement. The process model has been linearized and discretized as,

$$\begin{aligned}\mathbf{x}_k &= f(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}, \mathbf{w}_{k-1}) \\ &= \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \mathbf{x}_{k-1} + \begin{bmatrix} 0 \\ \Delta t \end{bmatrix} \mathbf{u}_{k-1} + \mathbf{w}_{k-1}\end{aligned}$$

The nonlinear measurement equation is,

$$y_k = \phi_k = h(p_k, v_k) = \arctan\left(\frac{S}{D - p_k}\right) + v_k$$

The process noise and measurement noise are assumed to be white noise, i.e.,

$$v_k \sim \mathcal{N}(0, 0.05), \mathbf{w}_k \sim \mathcal{N}(\mathbf{0}, (0.1) \cdot \mathbf{1}_{2\times 2})$$

The initial state is

$$\mathbf{x}_0 \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 5 \end{bmatrix}, \begin{bmatrix} 0.01 & 0 \\ 0 & 1 \end{bmatrix}\right)$$

The sample instant is $\Delta t = 0.5\text{s}$, the initial input is $u_0 = -2\text{m/s}^2$, the measurements available are

$$y_1 = 20\text{deg}, S = 20\text{m}, D = 60\text{m}$$

(1) Try to derive the Extended Kalman filter and Unscented Kalman filter estimate for $\hat{\boldsymbol{x}}_1, \hat{\boldsymbol{P}}_1$, compare the 2 results.

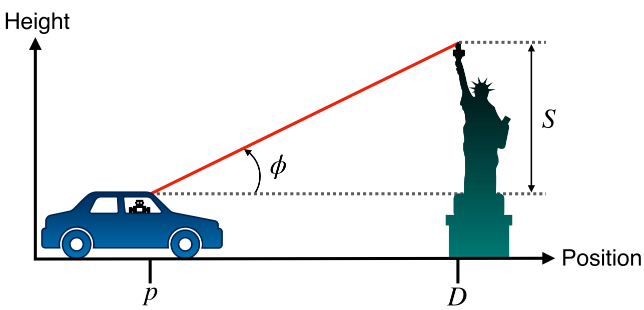

Figure 1. Estimate the position of a car.

(2) If the position observation is given as

$$y_k = [1\ 0]\boldsymbol{x}_k + v_k$$

discuss the existence of a steady-state discrete-time Kalman filter.