Signature from Head of the School

Harbin Institute of Technology, Shenzhen

Fall Semester of 2024

Convex Optimization and Optimal Control Examination (A)

Question	One	Two	Three	Four	Five	Six	Seven	Eight	Nine	Ten	Total
Mark											

P.S.: This paper was memorized and typeset after the examination ended, and there's no cheating behaviour during the examination.

NAME: ______ STUDENT ID: ______ CLASS: ______ DEPARTMENT: ______

密 封 线

1. (16 Points) Prove that the following two sets are convex sets. (8 points each)

(1) $\{a=(a_0,a_1,\ldots,a_{k-1})\in R^k \mid p(0)=1, |p(t)|\le 1, p(t)=a_0+a_1t+\cdots+a_{k-1}t^{k-1}\}$

(2) $\{(x,y,z) \mid z\ge x^2+y^2\}\subset R^3$

2. (16 Points) Prove that the following two functions are convex (or concave). (8 points each)

(1) The perspective of a function $f: R^n \to R$ is the function $g: R^n \times R \to R$

$$g(x,t) = tf(x/t), \quad \textbf{dom}\ g = \{(x,t) \mid x/t \in \textbf{dom}\ f, t > 0\}$$

Prove that *g* is convex if *f* is convex.

(2) $L(x,\lambda,v) = \inf_{x \in D}(\sum_{i=1}^{m} \lambda_i x_i + \sum_{i=1}^{m} v_i x_i^2), \lambda, v \in R^m$

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3. (10 Points) Formulate the following problem into a convex optimization problem:

$$\min \|x\|_1$$
$$\text{s.t. } Ax = b$$

4. (10 Points) Formulate the KKT conditions for the following convex optimization problem:

$$\min \frac{1}{2} x^T P x + q^T x + r$$
$$\text{s.t. } Ax = b$$

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密 封 线

5. (20 Points) Formulate the dual problems for the following convex optimization problems. (10 points each)

(1)

$$\begin{aligned} &\min x^T x \\ &\text{s.t. } Ax = b \end{aligned}$$

(2)

$$\begin{aligned} &\min c^T x \\ &\text{s.t. } Ax = b \\ &\quad\ x \geq 0 \end{aligned}$$

6. (15 Points) Suppose the state equations of the system is

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = u \end{cases}$$

The initial conditions are

$$\begin{cases} x_1(0) = 1 \\ x_2(0) = 1 \end{cases}$$

and the terminal conditions are

$$\begin{cases} x_1(1) = 0 \\ x_2(1) \text{ is free} \end{cases}$$

Derive the optimal control rule to minimize

$$J(u) = \int_0^1 u^2(t)\mathrm{d}t \ .$$

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密 封 线

7. (13 Points) Formulate the following robust linear program problem into convex optimization problem.

$$\min c^T x$$

$$\text{s.t. } a_i^T x \le b_i, \text{where } a_i \in \mathcal{E}_i = \{\overline{a}_i + P_i u \mid \|u\|_2 \le 1\}, \overline{a}_i \in R^n, P_i \in R^{n\times n}$$

$$i = 1, \ldots, n$$