

电路

第2章 线性直流电路

开课教师：王灿

开课单位：机电学院--电气工程学科



本章导言

本章主要内容包括三部分。第一部分首先介绍电阻的串联与并联化简、星形与三角形联接的等效变换、含源支路的等效变换等；第二部分介绍求解线性直流电路的一般方法，包括支路电流法、回路电流法和节点电压法；第三部分简要介绍运算放大器，含运算放大器电路的分析特点。

1 电阻的串联与并联

2 电源与电阻的串联与并联

3 电阻的星形与三角形联结

4 支路电流法

5 回路电流法

6 节点电压法

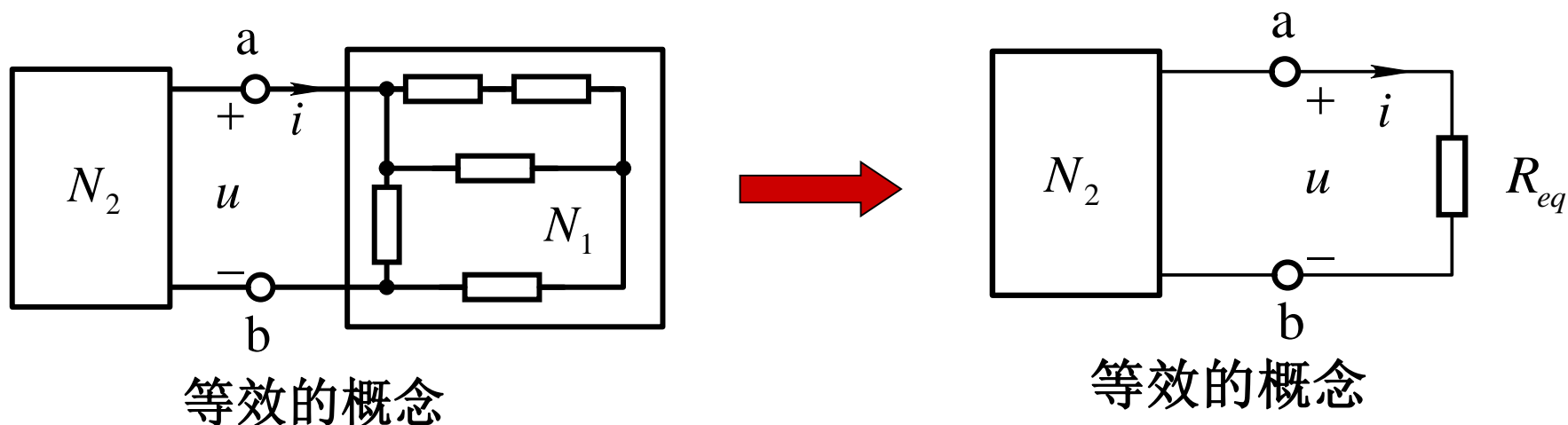
7 运算放大器

8 含运算放大器电路的分析

2.1 电阻网络的等效

基本要求：掌握等效的概念，熟练运用电阻串、并联等效及星三角变换规律计算电路。

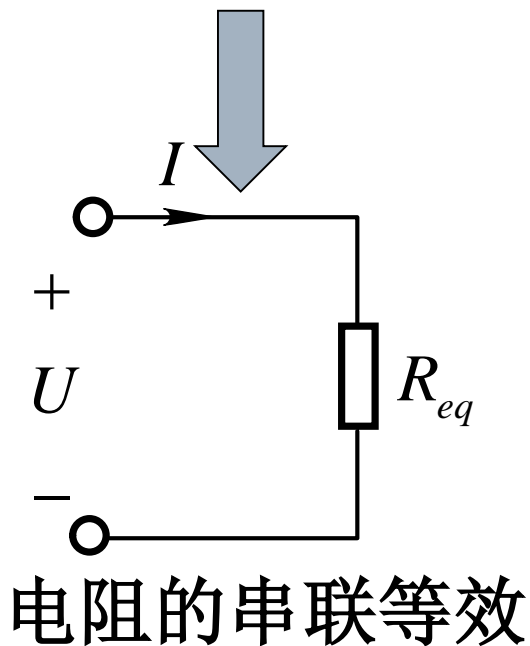
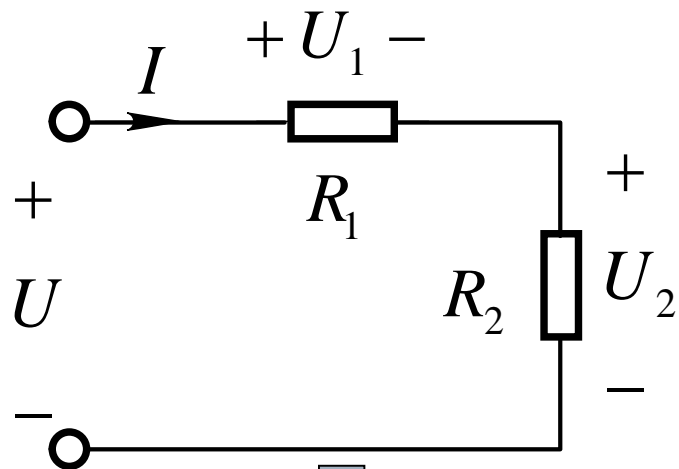
1. 等效：是指被化简的电阻网络 N_1 与等效电阻具有相同的 $u-i$ 关系(即端口方程)，从而用等效电阻 R_{eq} 代替电阻网络 N_1 之后，不改变其余部分的电压和电流。



注：等效只是对外电路等效

2.1 电阻网络的等效

2. 电阻的串联



根据KVL及欧姆定律列写电路方程

$$U = U_1 + U_2$$

$$= R_1 I + R_2 I = (R_1 + R_2) I = R_{eq} I$$

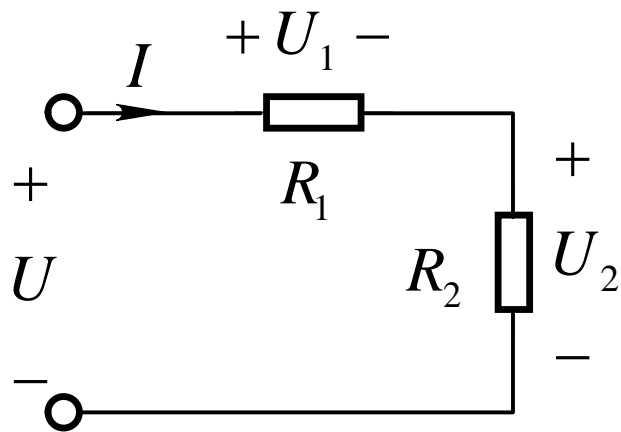
即： $R_{eq} = R_1 + R_2$

推广之

$$R_{eq} = \sum_{k=1}^N R_k$$

2.1 电阻网络的等效

串联分压



$$U_1 = R_1 I = \frac{R_1}{R_1 + R_2} U$$

$$U_2 = R_2 I = \frac{R_2}{R_1 + R_2} U$$

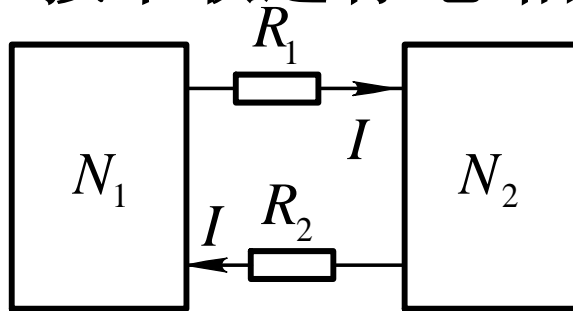
$$U_k = R_k I = \frac{R_k}{R_{eq}} U$$

$$P_1 = U_1 I = R_1 I^2 \quad P_2 = U_2 I = R_2 I^2$$

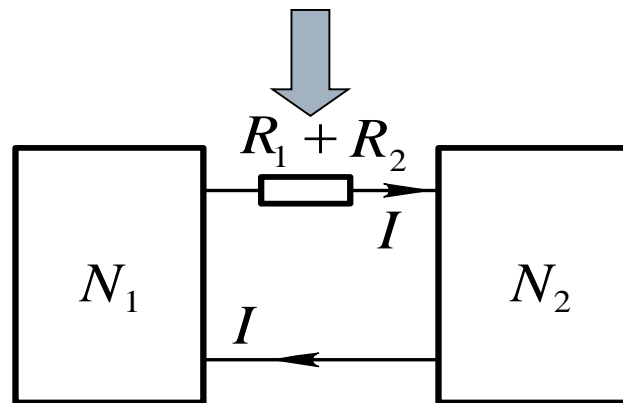


$$\frac{P_1}{P_2} = \frac{U_1}{U_2} = \frac{R_1}{R_2}$$

按串联进行电路的化简:



(a)



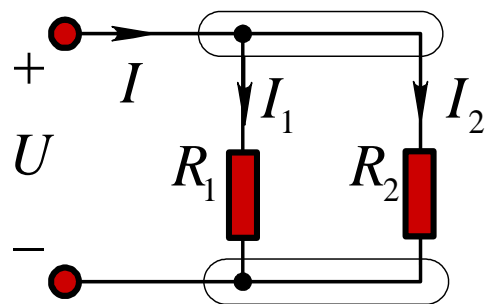
(b)

注: 如此等效值后电路中的哪些量发生了变化?

2.1 电阻网络的等效

3. 电阻的并联

并联：各电阻都接到同一对节点之间，从而各电阻承受相同电压。



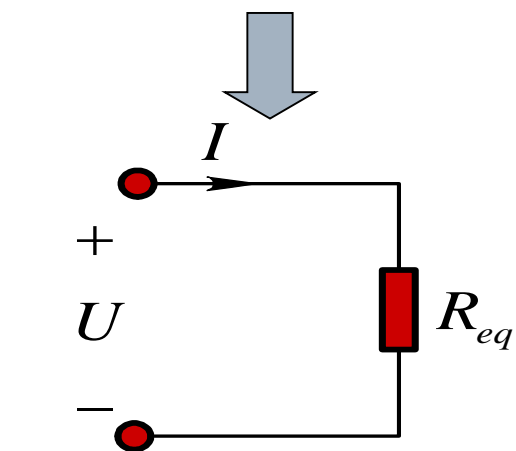
根据KCL及欧姆定律列写电路方程

$$\begin{aligned} I &= I_1 + I_2 \\ &= \frac{U}{R_1} + \frac{U}{R_2} = (G_1 + G_2)U = G_{eq}U \end{aligned}$$

即

$$G_{eq} = G_1 + G_2$$

$$R_{eq} = \frac{1}{G_{eq}} = \frac{R_1 \times R_2}{R_1 + R_2}$$



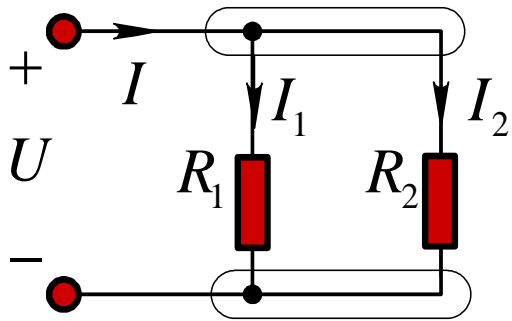
电阻的并联等效

$$G_{eq} = \sum_{k=1}^N G_k$$

$$R_{eq} = \frac{1}{G_{eq}} = \frac{1}{\sum_{k=1}^N G_k} = \frac{1}{\sum_{k=1}^N \frac{1}{R_k}}$$

2.1 电阻网络的等效

并联的应用：电阻的**并联**联接常用于**分流**，其中每个并联电阻只流过总电流的一部分，两个电阻并联时，各个电阻所分担的电流如下：



$$I_1 = G_1 U = \frac{G_1}{G_1 + G_2} I = \frac{R_2}{R_1 + R_2} I$$

$$I_2 = G_2 U = \frac{G_2}{G_1 + G_2} I = \frac{R_1}{R_1 + R_2} I$$

$$I_k = G_k U = G_k (R_{\text{eq}} I) = \frac{G_k}{G_{\text{eq}}} I$$

功率分配 $P_1 = UI_1 = G_1 U^2$ $P_2 = UI_2 = G_2 U^2$

$$\longrightarrow \frac{P_1}{P_2} = \frac{I_1}{I_2} = \frac{G_1}{G_2}$$

2.1 电阻网络的等效

【例题2.1】求图示电路的电压 U_1 及电流 I_2 。

解：先应用并联化简得到图(b)

所示电路

$$R_1 = \frac{12\Omega \times 6\Omega}{12\Omega + 6\Omega} = 4\Omega$$

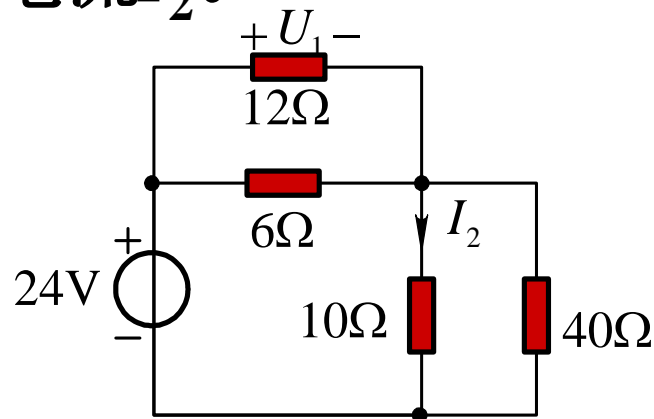
$$R_2 = \frac{10\Omega \times 40\Omega}{10\Omega + 40\Omega} = 8\Omega$$

由串联分压公式得：

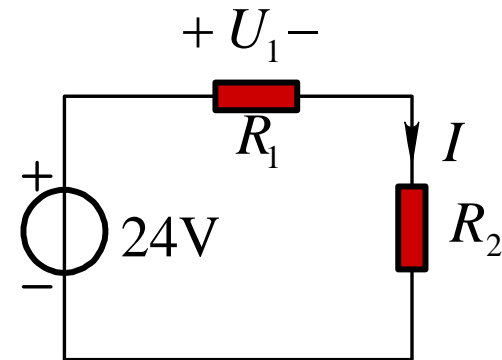
$$U_1 = \frac{R_1}{R_1 + R_2} \times 24\text{V} = 8\text{V}$$

$$I = \frac{24\text{V}}{R_1 + R_2} = 2\text{A}$$

分流公式得
$$I_2 = \frac{40\Omega}{10\Omega + 40\Omega} \times I = 1.6\text{A}$$



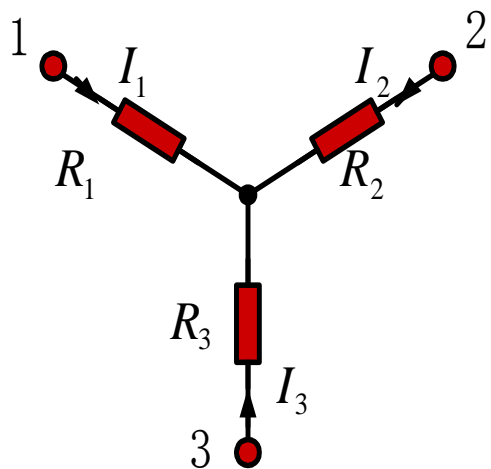
(a)



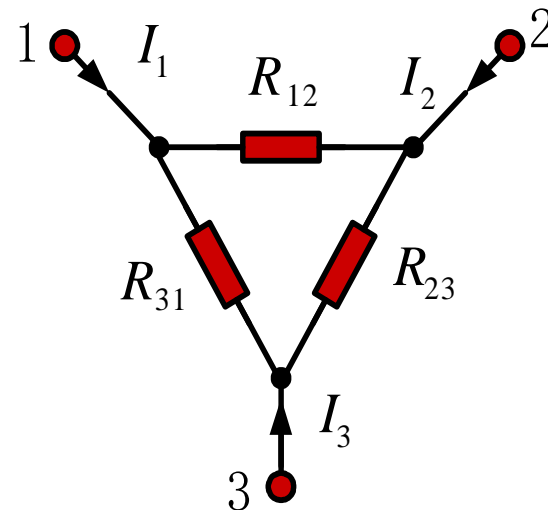
(b)

2.1 电阻网络的等效

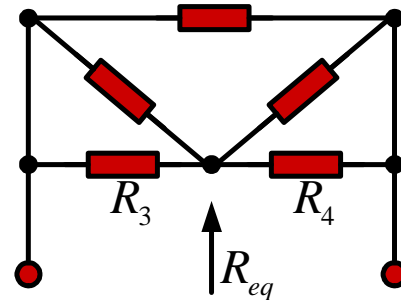
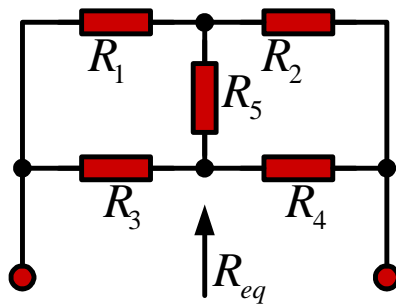
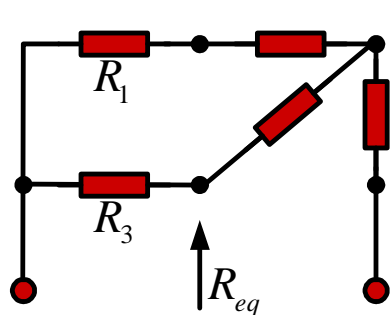
4. 星形联结和三角形连结



星形(T形)联接

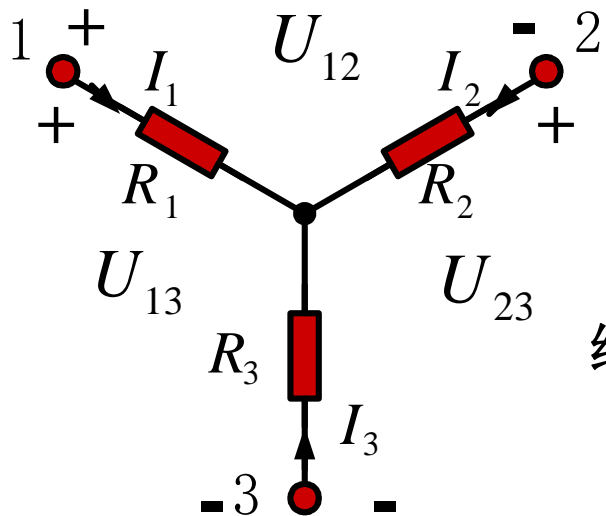


三角形(Δ 形)联接



2.1 电阻网络的等效

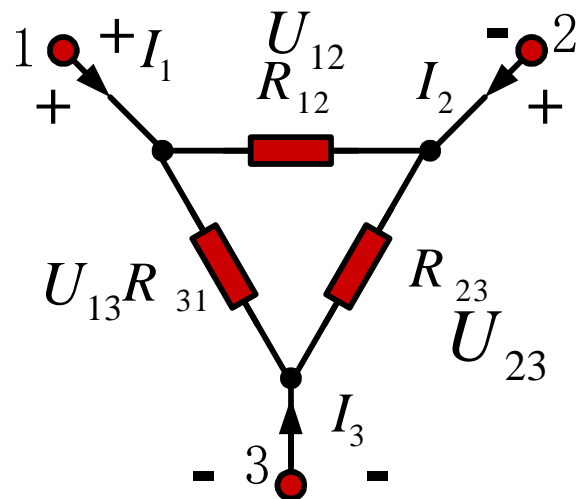
等效条件:



约束条件:

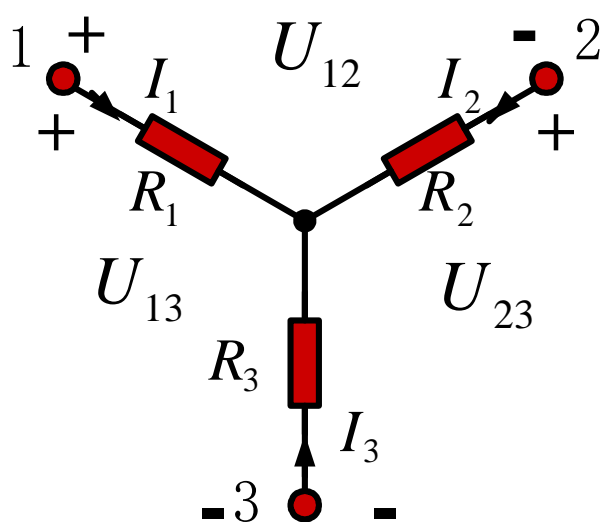
$$I_3 = -(I_1 + I_2)$$

$$U_{13} = U_{12} + U_{23}$$



分析: 将星形连接转换成三角形连接时, 将减少一个节点, 但要增加一个回路; 而将三角形连接转换成星形连接时, 将减少一个回路, 但要增加一个节点。

2.1 电阻网络的等效

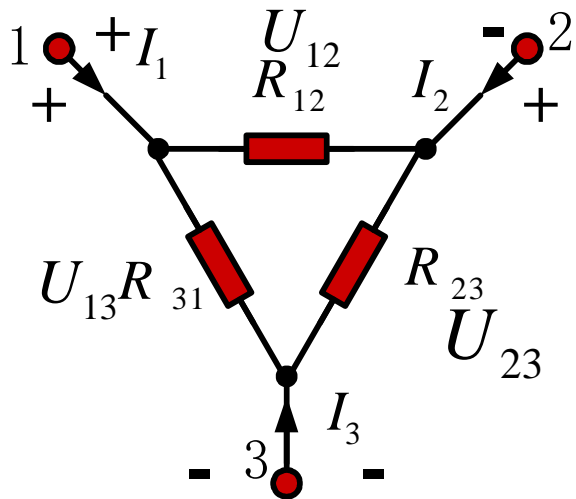


• 星形连接中的电压、电流关系

$$\begin{bmatrix} U_{13} \\ U_{23} \end{bmatrix} = \begin{bmatrix} R_1 + R_3 & R_3 \\ R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$U = RI$$

• 三角形连接中的电压、电流关系

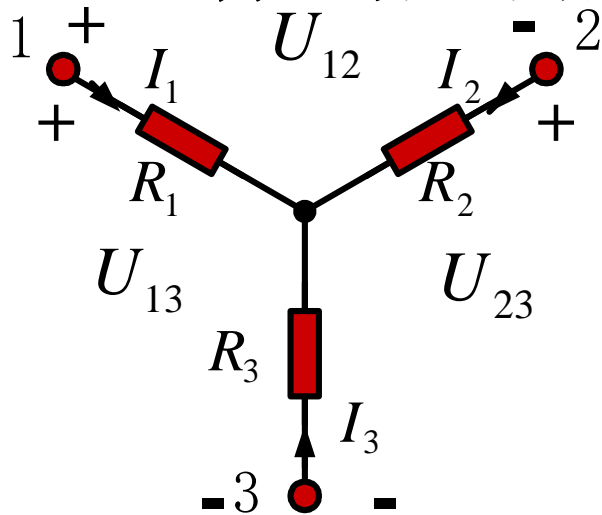


$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} G_{12} + G_{31} & -G_{12} \\ -G_{12} & G_{12} + G_{23} \end{bmatrix} \begin{bmatrix} U_{13} \\ U_{23} \end{bmatrix}$$

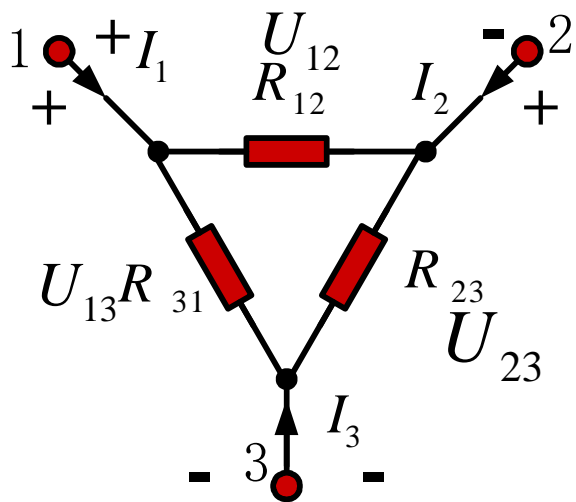
$$I = GU \quad U = G^{-1}I$$

2.1 电阻网络的等效

• 由此得二者之间的等效条件是



Y形—Δ形



Δ形—Y形

$$\left. \begin{aligned} R_{12} &= \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} \\ R_{23} &= \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} \\ R_{31} &= \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} \end{aligned} \right\}$$

$$\left. \begin{aligned} R_1 &= \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}} \\ R_2 &= \frac{R_{23} R_{12}}{R_{12} + R_{23} + R_{31}} \\ R_3 &= \frac{R_{31} R_{23}}{R_{12} + R_{23} + R_{31}} \end{aligned} \right\}$$

2.1 电阻网络的等效

三个相等的电阻接成Y形或 Δ 形时的等效变换是：

$$\begin{aligned} R_1 = R_2 = R_3 = R_Y \\ R_{12} = R_{23} = R_{31} = R_{\Delta} = 3R_Y \end{aligned} \quad \longrightarrow \quad R_Y = \frac{1}{3} R_{\Delta}$$

Y形— Δ 形

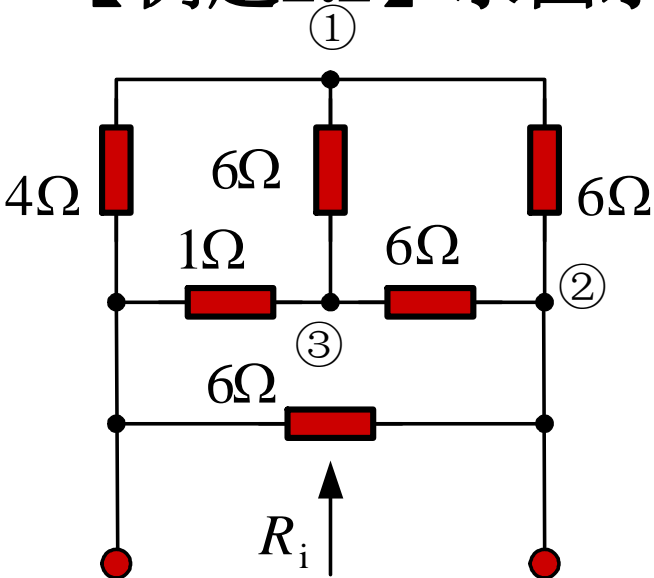
$$\left. \begin{aligned} R_{12} &= \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} \\ R_{23} &= \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} \\ R_{31} &= \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} \end{aligned} \right\}$$

Δ 形—Y形

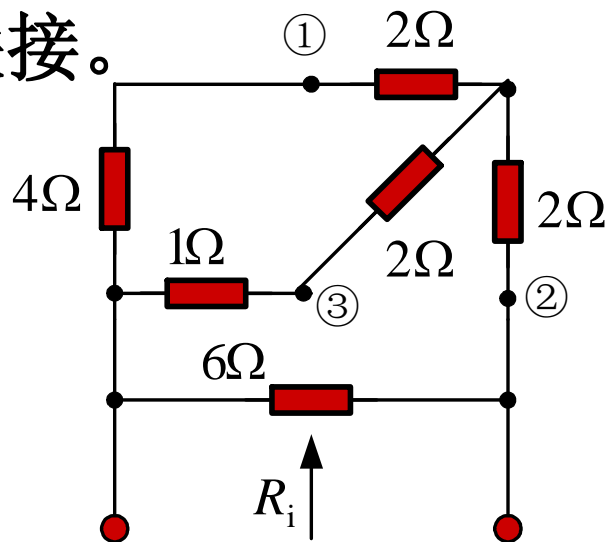
$$\left. \begin{aligned} R_1 &= \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}} \\ R_2 &= \frac{R_{23} R_{12}}{R_{12} + R_{23} + R_{31}} \\ R_3 &= \frac{R_{31} R_{23}}{R_{12} + R_{23} + R_{31}} \end{aligned} \right\}$$

2.1 电阻网络的等效

【例题2.2】求图示电路的等效电阻 R_i ？

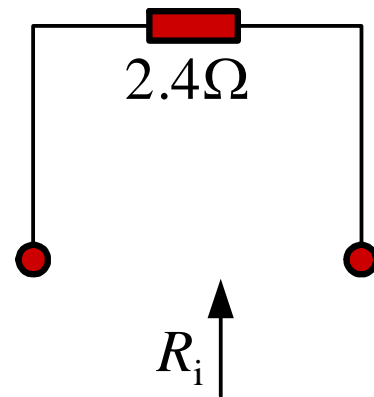


解：将节点①、②、③之间的对称 Δ 形联接电阻化为等效对称的Y形联接。



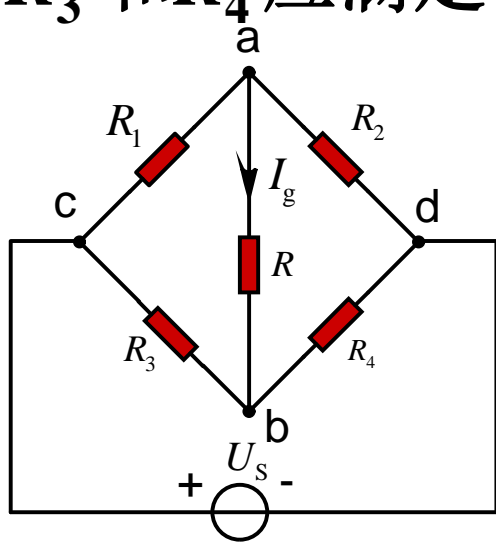
用串并联化简等效后的电路求出等效电阻

$$R_i = 6\Omega \parallel [(4\Omega + 2\Omega) \parallel (1\Omega + 2\Omega) + 2\Omega] = 2.4\Omega$$

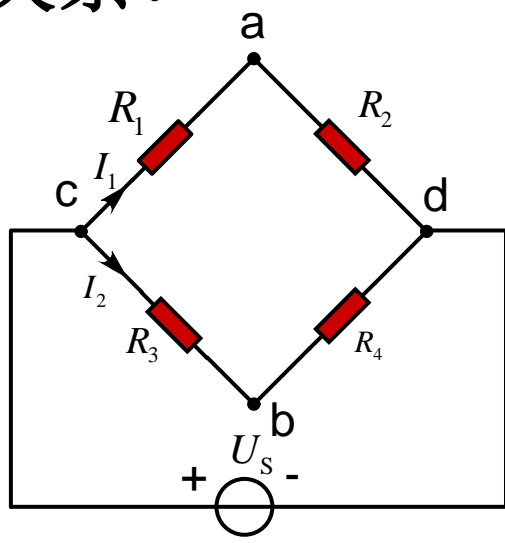


2.1 电阻网络的等效

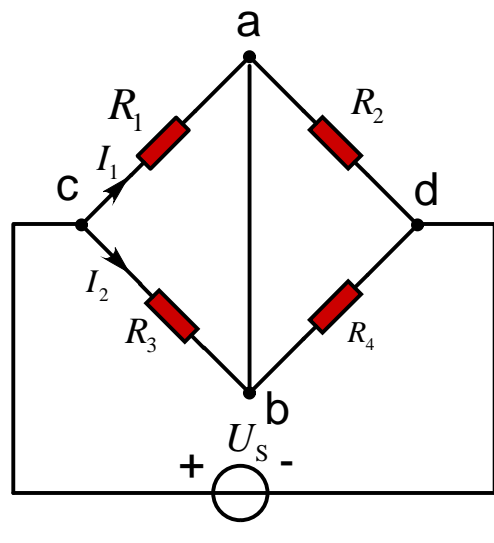
【例题2.3】图示电路是桥形电阻电路，当 $I_g=0$ ， $U_{ab}=0$ 时称电桥是平衡的，试说明电桥平衡时的电阻 R_1 ， R_2 ， R_3 和 R_4 应满足什么关系？



(a)



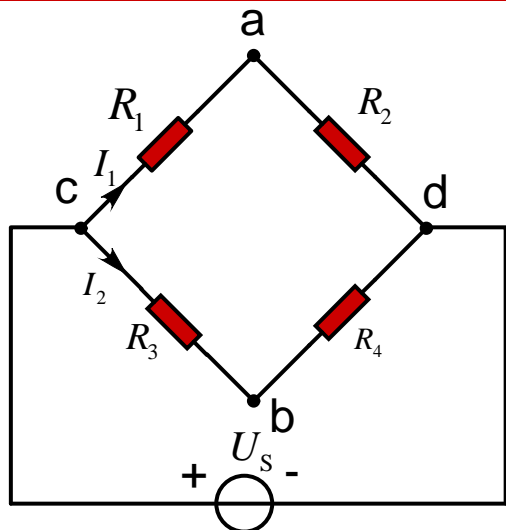
(b)



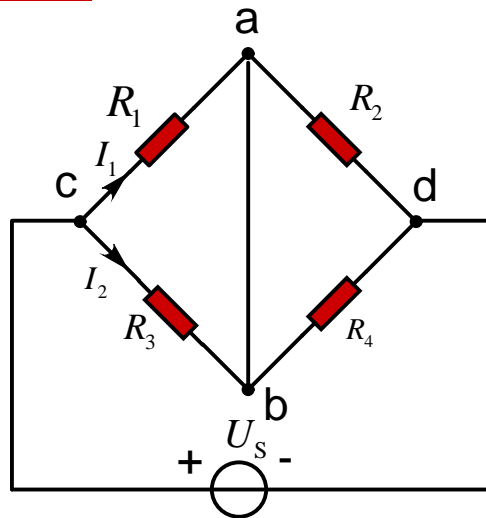
(c)

解：电桥平衡时 $I_g=0$ ， $U_{ab}=0$ 。从 $I_g=0$ 角度看， a ， b 两点间是开路的，如图(b)所示；从 $U_{ab}=0$ 角度看， a ， b 两点间是短路的，如图(c)所示。

2.1 电阻网络的等效



(b)



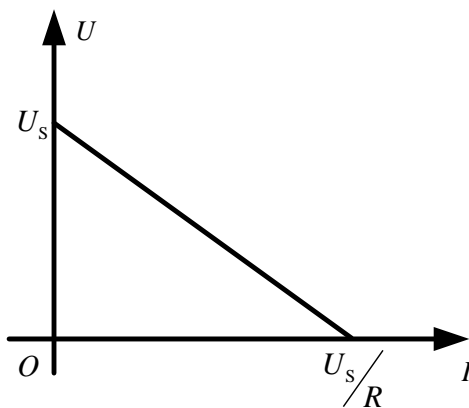
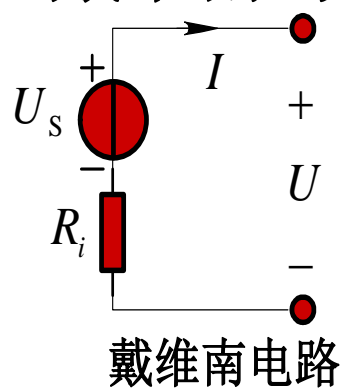
(c)

$$\begin{aligned}
 &\text{由图b有 } I_1 = \frac{U_s}{R_1 + R_2} \quad I_2 = \frac{U_s}{R_3 + R_4} \\
 &\text{由图c有 } U_{ca} = U_{cb} \quad \text{即 } R_1 I_1 = R_3 I_2
 \end{aligned}
 \left. \vphantom{\begin{aligned} I_1 = \frac{U_s}{R_1 + R_2} \\ I_2 = \frac{U_s}{R_3 + R_4} \end{aligned}} \right\} \begin{aligned}
 &\frac{R_1 U_s}{R_1 + R_2} = \frac{R_3 U_s}{R_3 + R_4} \\
 &\text{即 } \frac{R_1}{R_2} = \frac{R_3}{R_4}
 \end{aligned}$$

2.2 含源支路的等效变换

基本要求：掌握各种含源支路的等效化简方法和戴维南、诺顿两种典型电路之间的等效变换规律，能熟练运用这些等效规律化简电路。

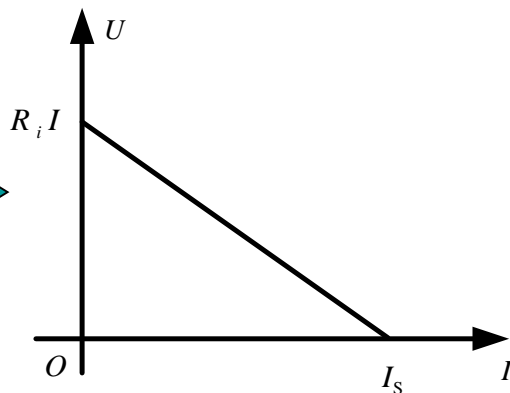
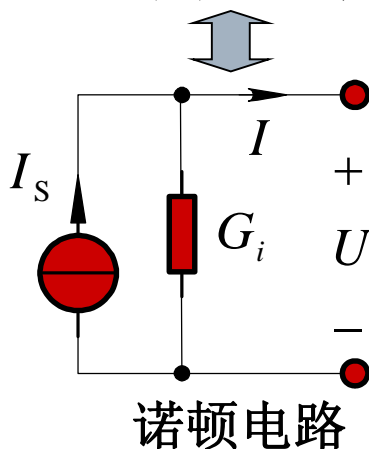
1 戴维南与诺顿电路



端口特性方程

$$U = U_S - R_i I$$

$$I = I_S - G_i U$$

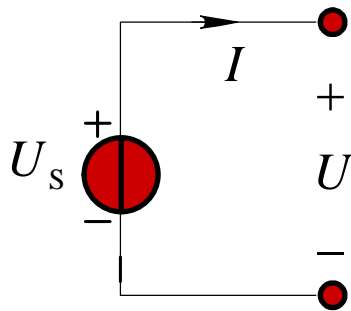


等效条件为

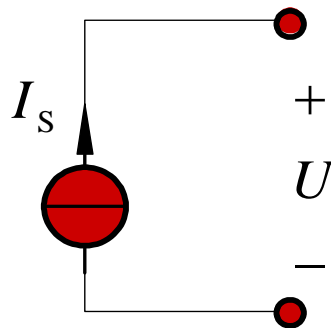
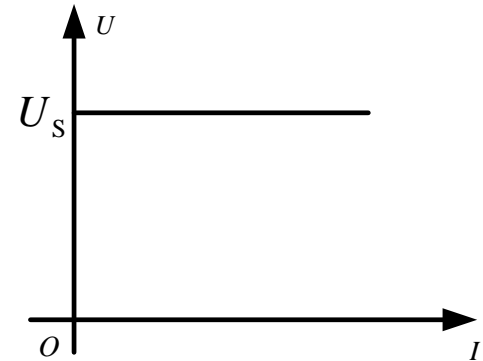
$$I_S = \frac{U_S}{R_i}, \quad G_i = \frac{1}{R_i}$$

2.2 含源支路的等效变换

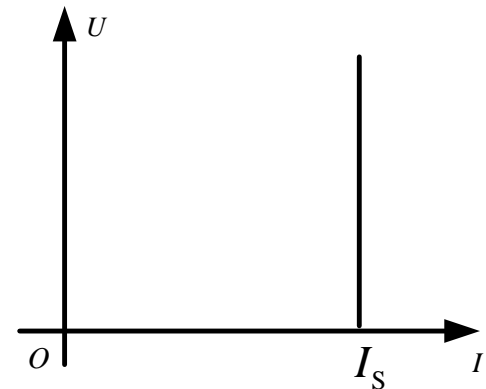
注：电压源内阻 $R_i=0$ ，而电流源内导 $G_i=0$ 时，即内阻等于无穷大，它们也称为理想电源。零不能取倒数，故电压源和电流源不能相互等效。



电压源



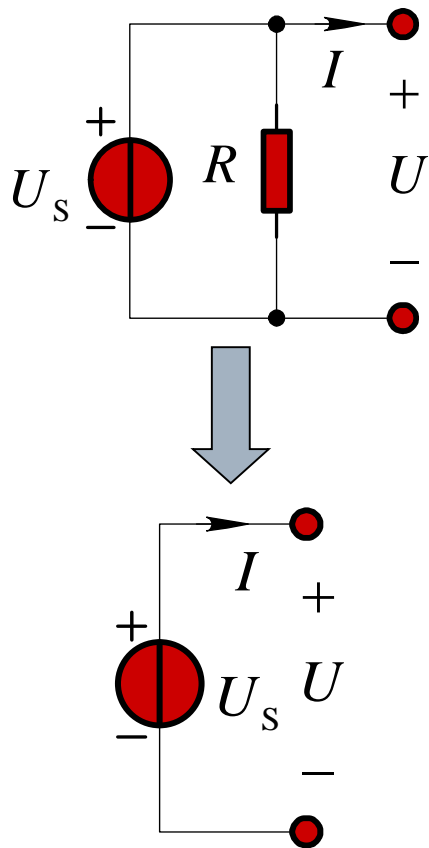
电流源



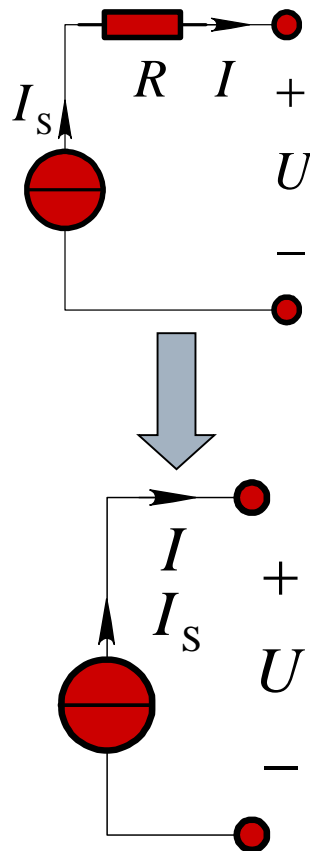
2.2 含源支路的等效变换

2 其它含源支路的等效

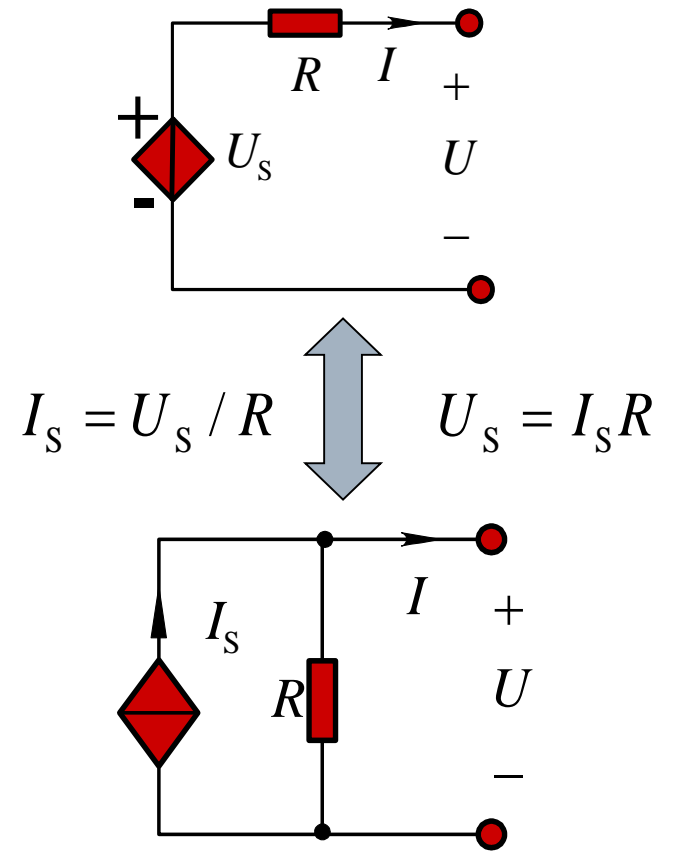
电压源并电阻



电流源串电阻

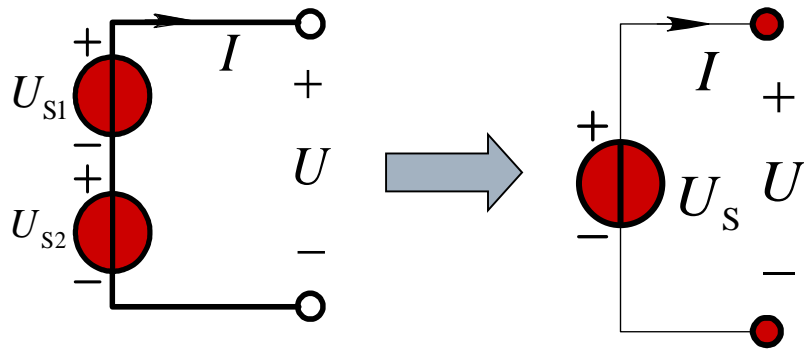


含受控源支路的等效

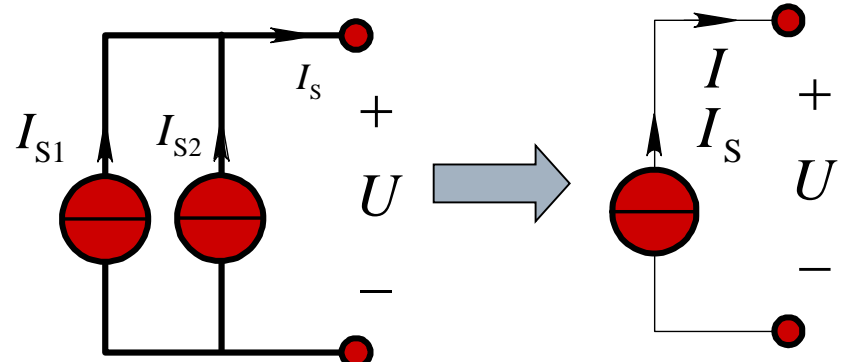


2.2 含源支路的等效变换

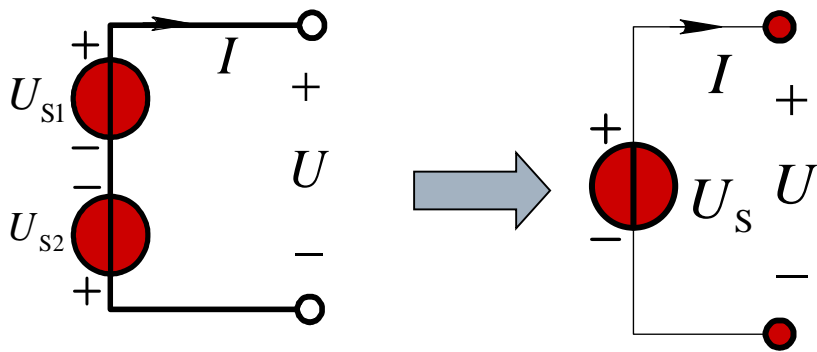
电压源的正向，反向串联 电流源的正向，反向并联



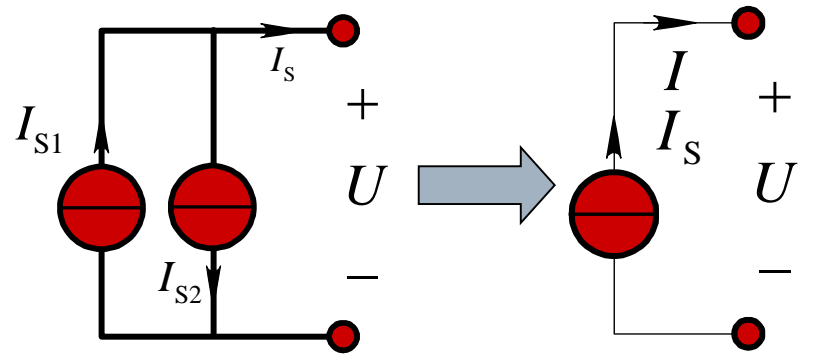
$$U_S = U_{S1} + U_{S2}$$



$$I_S = I_{S1} + I_{S2}$$



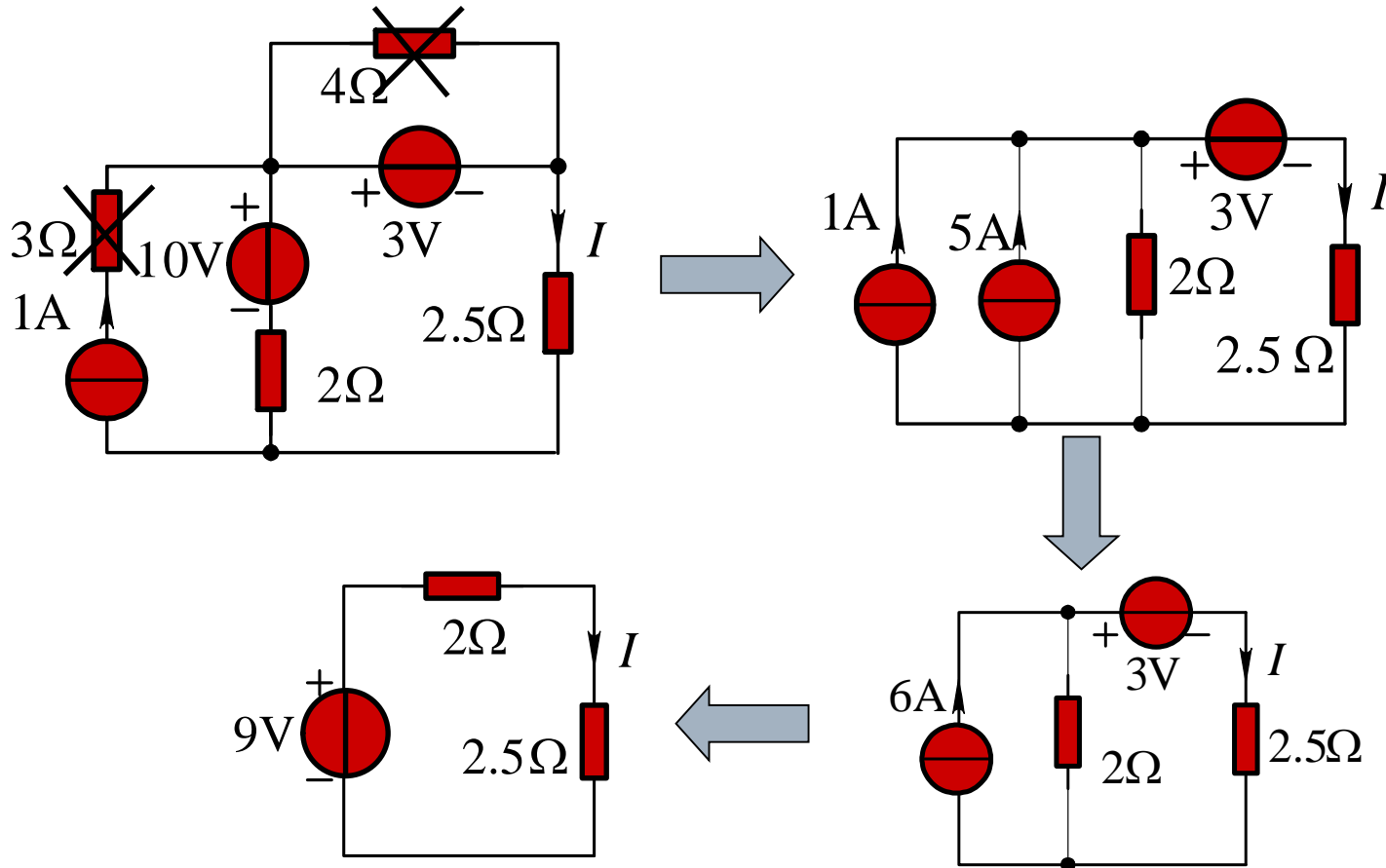
$$U_S = U_{S1} - U_{S2}$$



$$I_S = I_{S1} - I_{S2}$$

2.2 含源支路的等效变换

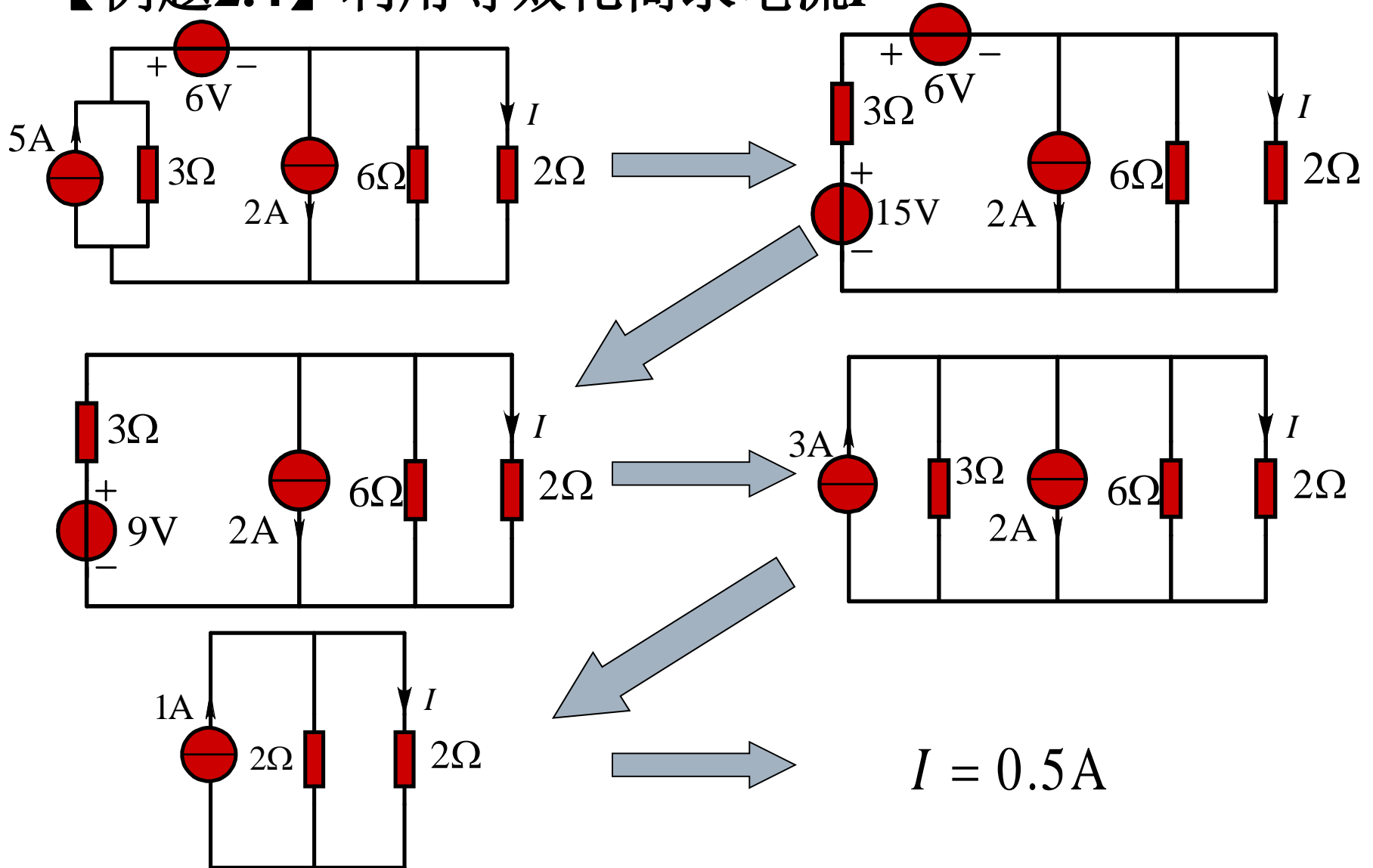
【例题2.3】用等效变换求图示电路中电流 I 。



$$I = \frac{9\text{V}}{2\Omega + 2.5\Omega} = 2\text{A}$$

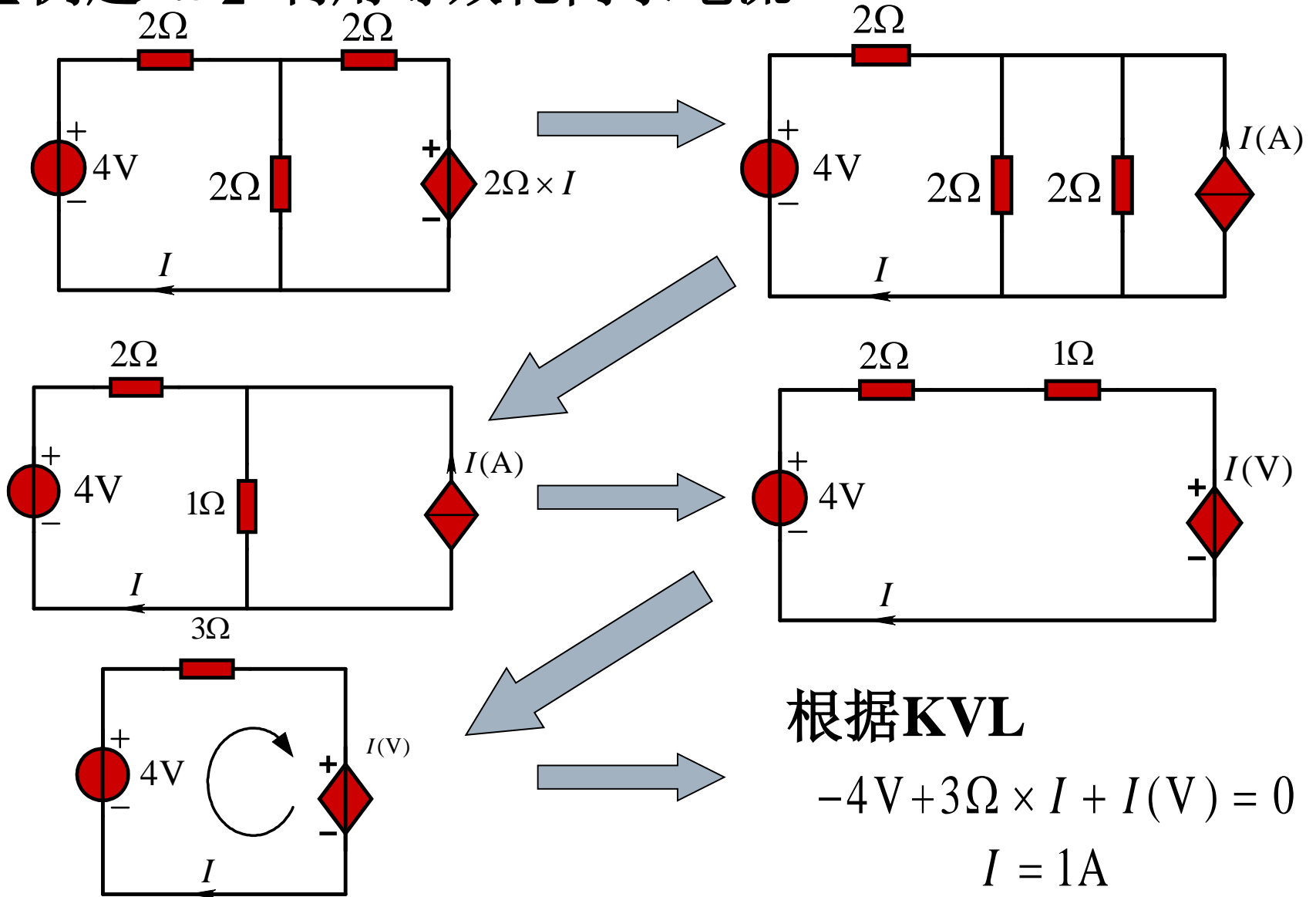
2.2 含源支路的等效变换

【例题2.4】利用等效化简求电流 I



2.2 含源支路的等效变换

【例题2.5】利用等效化简求电流 I





谢

谢！

