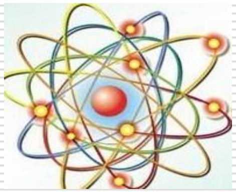
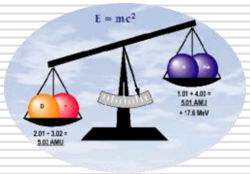




第4章　正弦电流电路

开课教师：　王灿

开课单位：机电学院--电气工程学科

4.6 正弦稳态电路的相量分析法

基本要求：熟练掌握正弦电流电路相量分析法原理及步骤、电路方程和电路定理的相量形式。

相量分析法的一般过程

（1） 将电阻推广为阻抗，将电导推广为导纳；

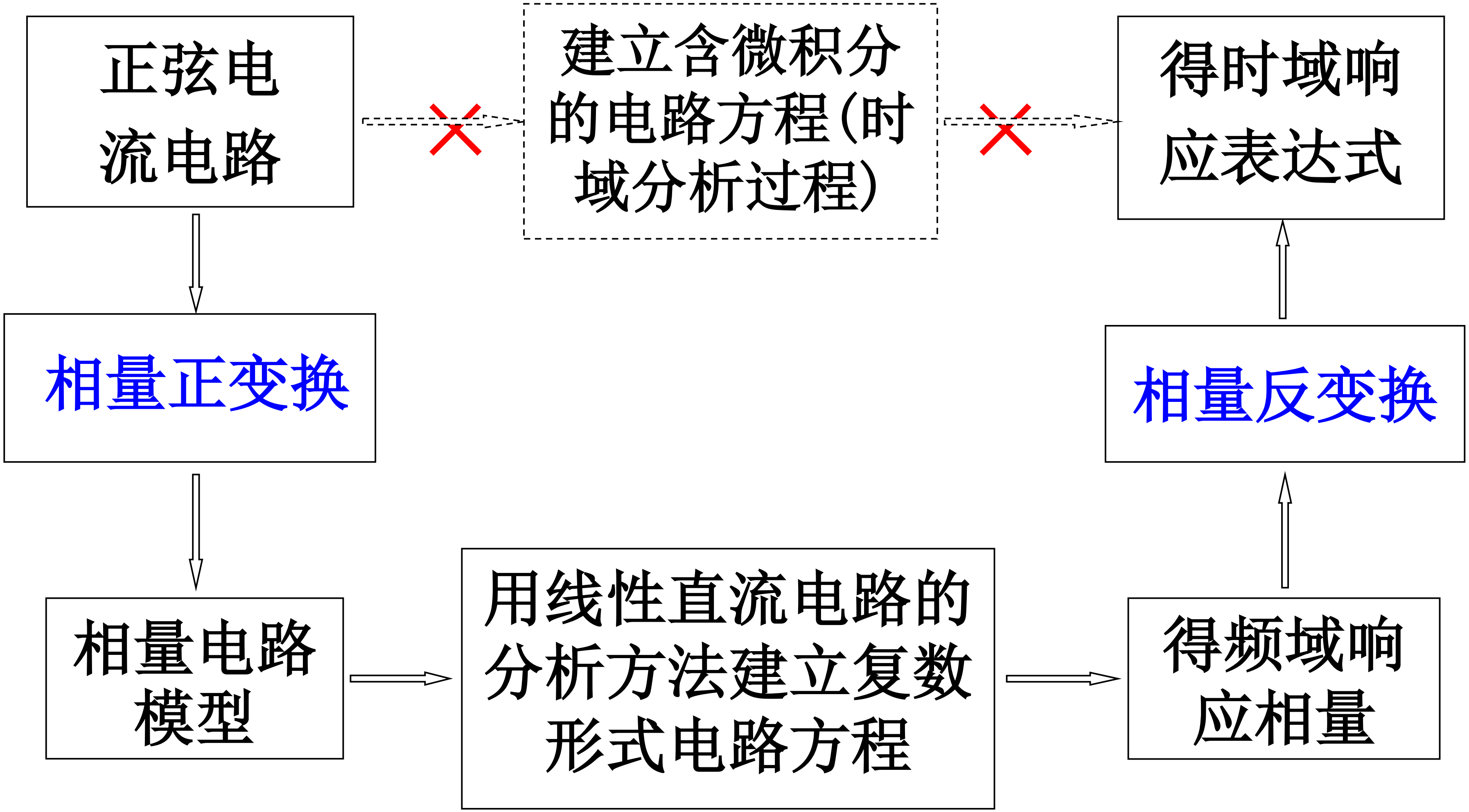
（2） 将激励用相量形式表示，恒定电压、电流推广为电压、电流的相量；

（3） 按线性直流电路分析方法计算相量模型电路；

（4） 将电压、电流相量计算结果变换成正弦表达式。

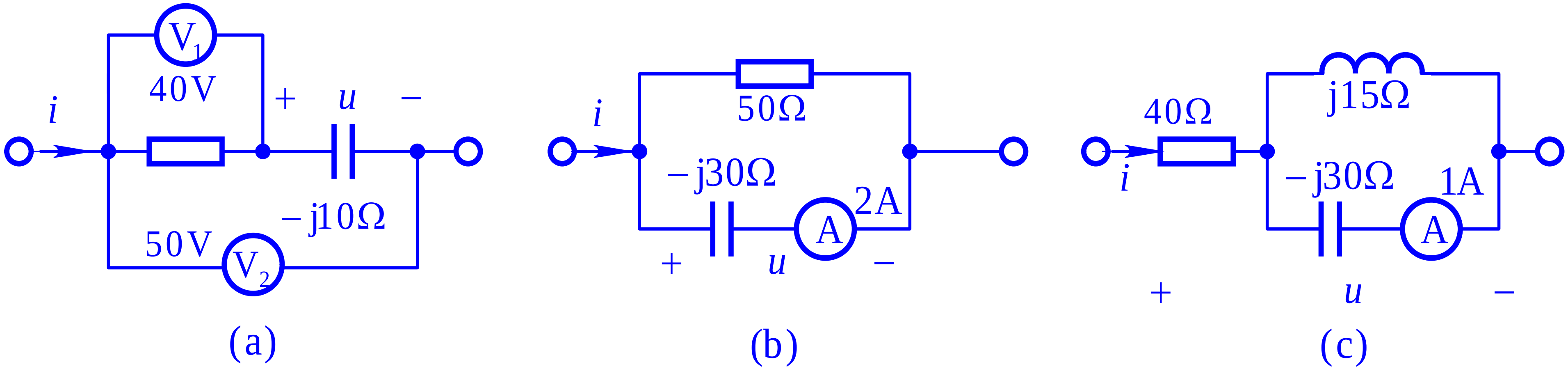
4.6 正弦稳态电路的相量分析法

正弦电流电路相量分析法过程示意



[补充4.4]

图示各电路中已标明电压表和电流表的读数，试求电压 ***u*** 和电流 ***i*** 的有效值。

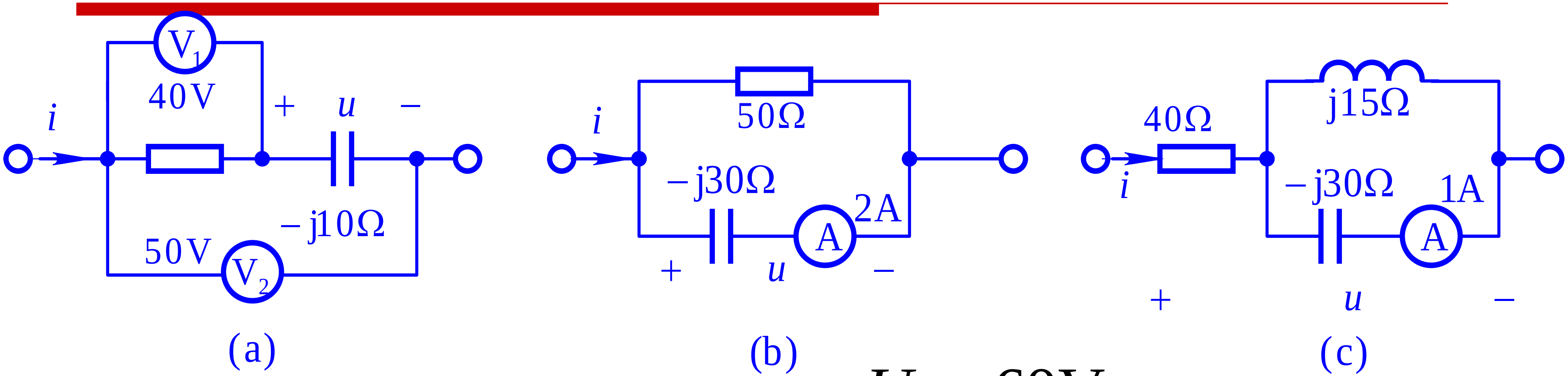


【解】 图(a): $\dot{U}_2 = \dot{U}_R + \dot{U}$

$$50\text{V} = \sqrt{(40\text{V})^2 + U^2} \qquad U = \sqrt{(50\text{V})^2 - (40\text{V})^2} = 30\text{V}$$

$$I = I_C = \frac{U}{|X_C|} = \frac{30\text{V}}{10\Omega} = 3\text{A}$$

[补充4.4]



图(b):$U=30\Omega\times2A=60V$, $I_R=\dfrac{U}{R}=\dfrac{60V}{50\Omega}=1.2A$

$$I=\sqrt{I_C^{\ 2}+I_R^{\ 2}}=\sqrt{2^2+1.2^2}\,A=2.33A$$

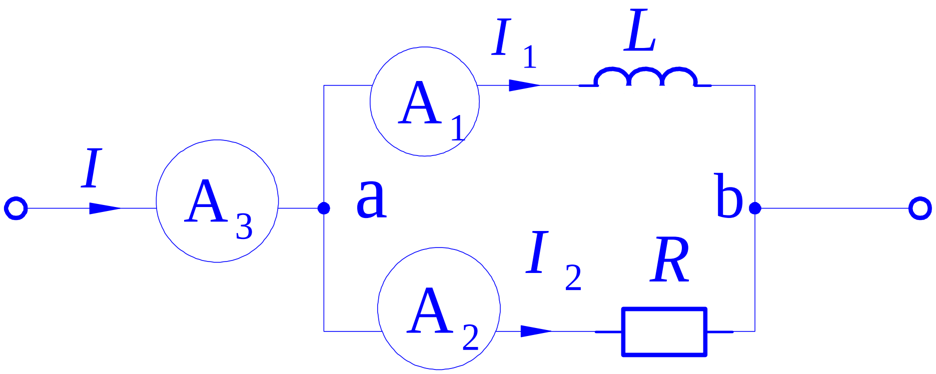
图(c): $U_C=|X_C|I_C=30\Omega\times1A=30V$

$$U_L=U_C=X_LI\Rightarrow I_L=\frac{U_C}{X_L}=\frac{30V}{15\Omega}=2A$$

$I=|I_L-I_C|=1A$ $U=\sqrt{U_C^{\ 2}+U_R^{\ 2}}=\sqrt{30^2+40^2}\,V=50V$

[补充4.5]

已知表1的读数是5A， ωL和R数值相等，求 表2和表3的读数。



L上电流滞后电压 90°

$$\dot{I}_1 = 5\angle -90^\circ \text{A}$$

$$\dot{I} = \dot{I}_1 + \dot{I}_2$$

$$= -\text{j}5 + 5 = 5\sqrt{2}\angle -45^\circ \text{A}$$

表3读数为 $5\sqrt{2}$ A

【解】

$$\dot{U}_{ab} = U_{ab}\angle 0^\circ$$

$$\frac{U_{ab}}{R} = \frac{U_{ab}}{\omega L} = 5\text{A}$$

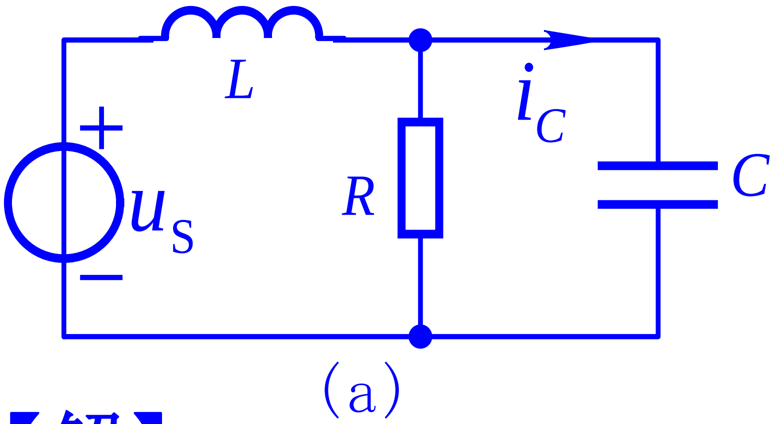
$$\Rightarrow I_1 = I_2 = 5\text{A}$$

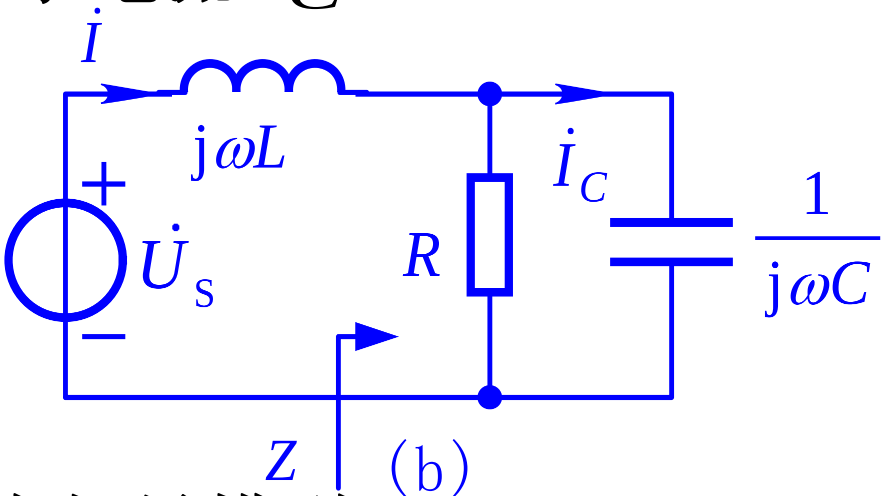
即 表2读数为 5A

注意：电流表读数均为有效值，有效值不满足KCL方程，而电流相量是满足KCL方程的。

[例4.9]

设图 (a)电路 $u_S = 60\sqrt{2}\cos(\omega t + 45^\circ)\text{V}$，$\omega = 100\ \text{rad/s}$，$C = 10^{-3}\text{F}$，$R = 10\Omega$，$L = 0.1\text{H}$ 求电流 i_C。



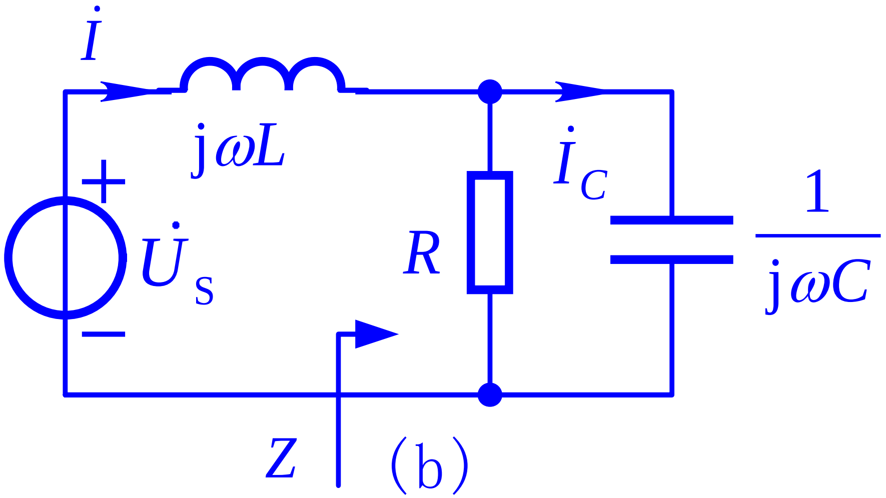


【解】 将时域电路模型变换为相量模型

$\dot{U}_S = 60\angle 45^\circ\text{V}$

$$Z = R // \frac{1}{j\omega C} = \frac{R \times \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{R}{1 + j\omega RC} = 5(1 - j)\Omega$$

[例4.9]



$$\dot{I} = \frac{\dot{U}_\mathrm{s}}{Z + \mathrm{j}\omega L}$$

$$= \frac{60\angle 45^\circ \mathrm{V}}{[5(1-\mathrm{j}) + \mathrm{j}10]\Omega} = 6\sqrt{2}\ \mathrm{A}$$

$$\dot{I}_C = \frac{R}{R + \frac{1}{\mathrm{j}\omega C}} \times \dot{I} = \frac{\mathrm{j}\omega RC}{1 + \mathrm{j}\omega RC} \times \dot{I} = 6\angle 45^\circ\ \mathrm{A}$$

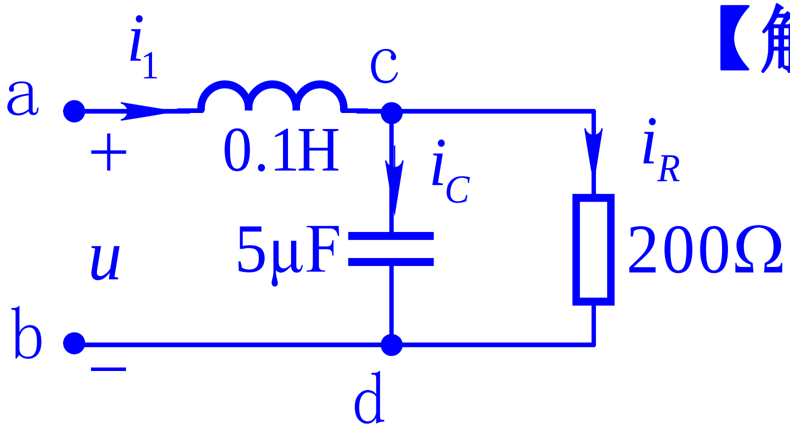
$$i_C = 6\sqrt{2}\cos(100t + 45^\circ)\mathrm{A}$$

[补充4.6]

在图示电路中已知 $i_R = \sqrt{2}\cos\omega t\text{A}$ ，ω=2×10^{3}rad/s 。

(1)求 ab 端的等效阻抗和等效导纳。

(2)求各元件的电压、电流及电源电压 u，并作各电压、电流的 相量图。



【解】

$$X_L = \omega L$$

$$= \left(2\times10^3\right)\text{rad/ s}\times0.1\text{H} = 200\Omega$$

$$X_C = -\frac{1}{\omega C}$$

$$= -\frac{1}{\left(2\times10^3\right)\text{rad/ s}\times\left(5\times10^{-6}\right)\text{F}} = -100\Omega$$

$$\text{(1)}\quad Z_{\text{cd}} = \frac{1}{\frac{1}{R}+\text{j}\omega C} = \frac{1}{\frac{1}{200}+\text{j}\frac{1}{100}} = 40(1-\text{j}2)\Omega$$

[补充4.6]

$$Z_{ab}=j\omega L+Z_{cd}=126.49\angle 71.56^\circ\Omega \qquad Y_{ab}=\frac{1}{Z_{ab}}=(2.5-j7.5)\text{mS}$$

(2) $\dot{U}_{cd}=\dot{I}_R\times R=200\angle 0^\circ\text{V}$

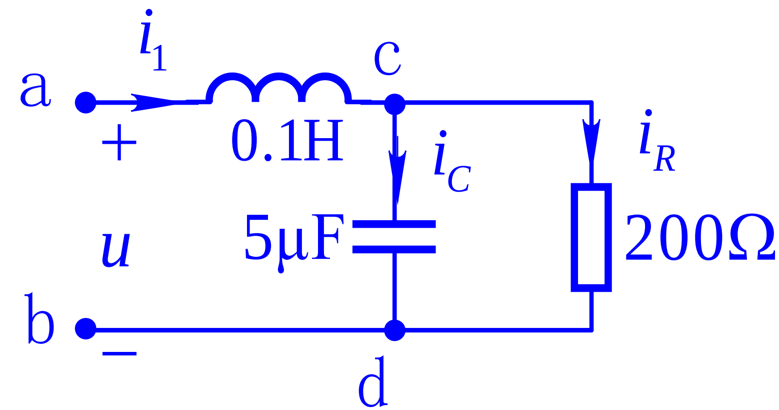
$$\dot{I}_C=j\omega C\dot{U}_{cd}=2\angle 90^\circ\text{A}$$

$$\dot{I}_1=\dot{I}_C+\dot{I}_R=2.236\angle 63.43^\circ\text{A}$$

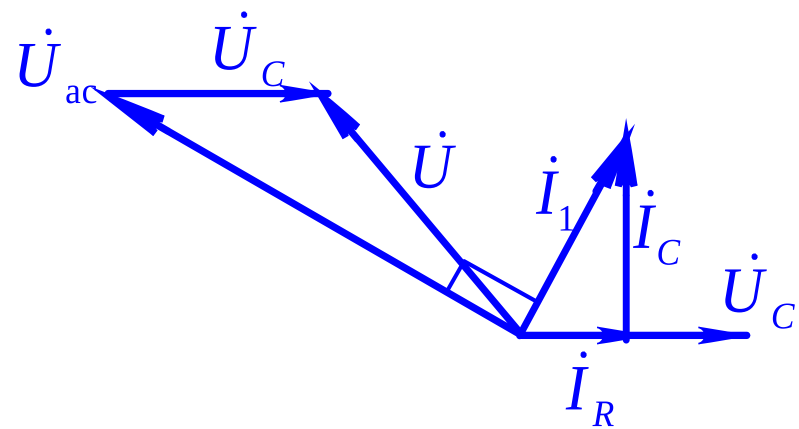
$$\dot{U}_{ac}=j\omega L\times\dot{I}_1=447.2\angle 153.43^\circ\text{V}$$

$$\dot{U}=Z_{ab}\cdot\dot{I}_1=282.83\angle 134.99^\circ\text{V}$$

$$u=282.83\sqrt{2}\cos(\omega t+134.99^\circ)\text{V}$$

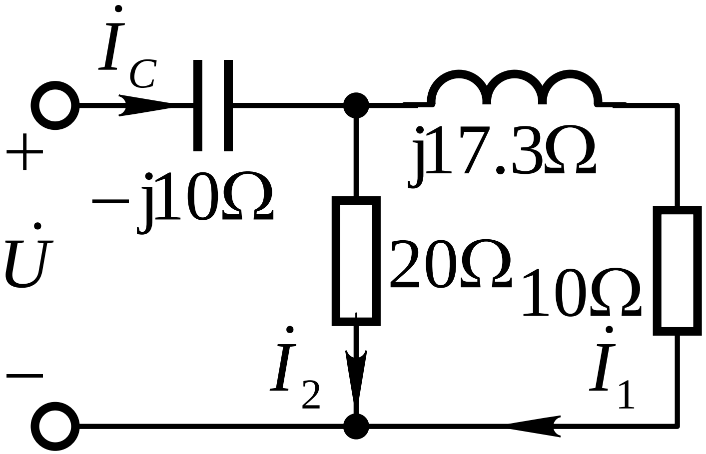


各电压、电流相量图



[补充4.7]

在图示电路中，各元件电压、电流取关联参考方向。设 $\dot{I}_1 = 1\angle 0^\circ$ A，写出各元件电压、电流相量。



【解】

L: $\dot{I}_L = \dot{I}_1 = 1\angle 0^\circ$ A,

$\dot{U}_L = 17.3\angle 90^\circ$ V

$\dot{U}_2 = (10 + j17.3)$ V

$\dot{I}_2 = \dot{U}_2 / 20\Omega = 1\angle 60^\circ$ A

R: $\dot{I}_R = \dot{I}_1 = 1\angle 0^\circ$ A,

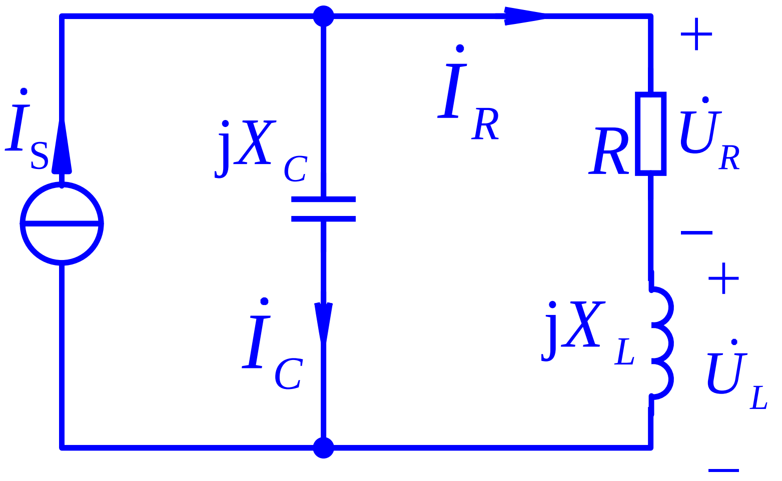
$\dot{U}_R = 10$V

C: $\dot{I}_C = \dot{I}_1 + \dot{I}_2 = 1.732\angle 30^\circ$ A

$\dot{U}_C = -j10\dot{I}_C = 17.32\angle -60^\circ$ V

[补充4.8]

已知图示电路中 $U_R = U_L = 10\text{V}$，$R = 10\Omega$，$X_C = -10\Omega$，求I_S .



【解】

设 $\dot{U}_R = 10\angle 0^\circ \text{V}$,

则 $\dot{U}_L = 10\angle 90^\circ \text{V}$

$$\dot{I}_R = \dot{U}_R / R = 1\angle 0^\circ \text{A}$$

$$\dot{I}_C = \frac{\dot{U}_R + \dot{U}_L}{jX_C} = \frac{10 + j10}{-j10} = (-1 + j)\ \text{A}$$

$$\dot{I}_S = \dot{I}_R + \dot{I}_C = 1\angle 0^\circ - 1 + j = j = 1\angle 90^\circ \text{A}$$

$$I_S = 1\ \text{A}$$

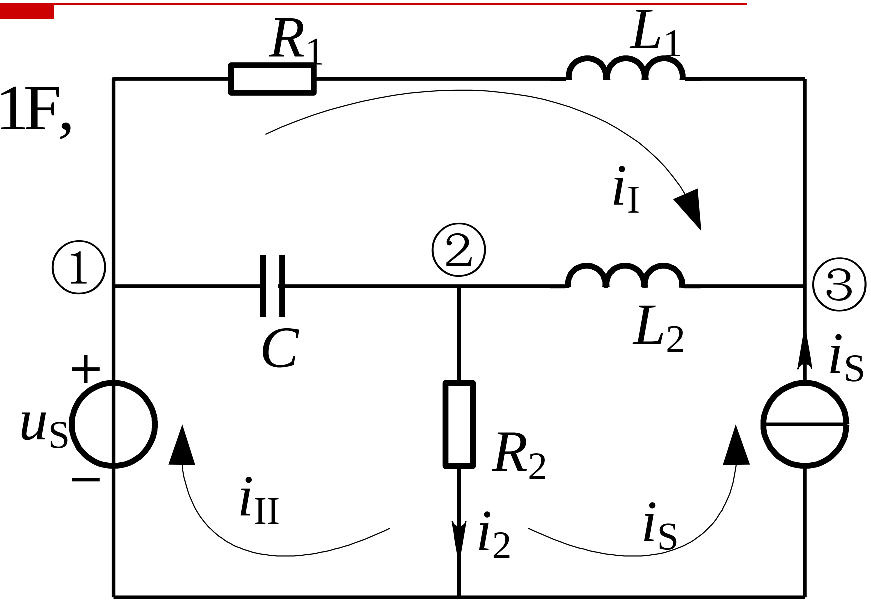
[补充4.9]

$R_1 = R_2 = 1\Omega,\ L_1 = L_2 = 0.01\text{H},\ C = 0.01\text{F},$

$u = 4\cos(100t - 45°)\text{V},$

$i = 2.236\sqrt{2}\cos(100t + 153.43°)\text{A}$

求电流$i_2(t)$。



【解】

回路电流法

$$\dot{U}_S = 2\sqrt{2}\angle 45°\text{V} = (2 - \text{j}2)\text{V}$$

$$\dot{I}_S = 2.236\angle 153.43°\text{A} = (-2 + \text{j})\text{A}$$

$$(R_1 + \text{j}\omega L_1 + \text{j}\omega L_2 + \frac{1}{\text{j}\omega C})\dot{I}_\text{I} - \frac{1}{\text{j}\omega C}\dot{I}_\text{II} + \text{j}\omega L_2 \dot{I}_S = 0$$

$$-\frac{1}{\text{j}\omega C}\dot{I}_\text{I} + (R_2 + \frac{1}{\text{j}\omega C})\dot{I}_\text{II} + R_2 \dot{I}_S = \dot{U}_S$$

[补充4.9]

$$(R_1 + j\omega L_1 + j\omega L_2 + \frac{1}{j\omega C})\dot{I}_{\mathrm{I}} - \frac{1}{j\omega C}\dot{I}_{\mathrm{II}} + j\omega L_2 \dot{I}_S = 0$$

$$-\frac{1}{j\omega C}\dot{I}_{\mathrm{I}} + (R_2 + \frac{1}{j\omega C})\dot{I}_{\mathrm{II}} + R_2 \dot{I}_S = \dot{U}_S$$

$$\begin{cases} (1+j)\Omega \dot{I}_{\mathrm{I}} + j\Omega \dot{I}_{\mathrm{II}} = (1+2j)\mathrm{V} \\ j\Omega \dot{I}_{\mathrm{I}} + (1-j)\Omega \dot{I}_{\mathrm{II}} = (4-3j)\mathrm{V} \end{cases}$$

$$\begin{cases} \dot{I}_{\mathrm{I}} = -j\mathrm{A} = 1\angle 90^\circ \mathrm{A} \\ \dot{I}_{\mathrm{II}} = 3\mathrm{A} \end{cases} \Rightarrow \dot{I}_2 = \dot{I}_{\mathrm{II}} + \dot{I}_S = \sqrt{2}\angle 45^\circ \mathrm{A}$$

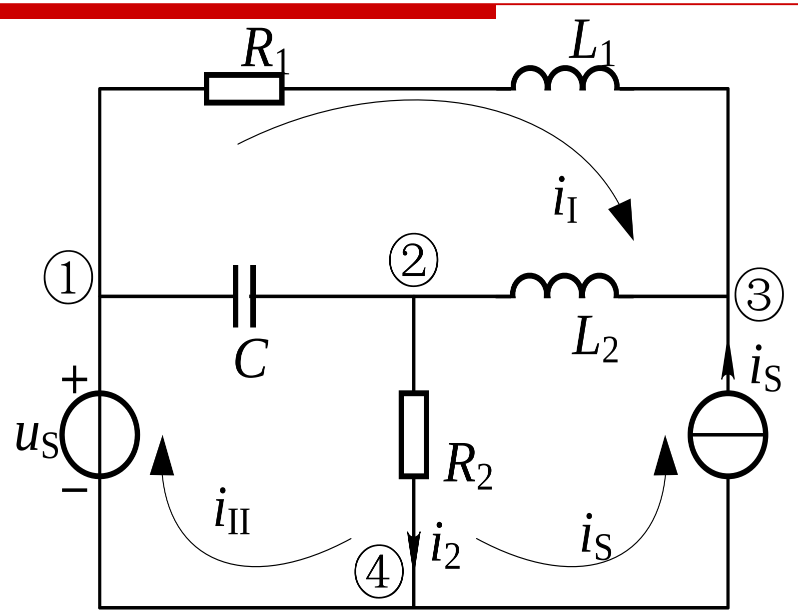
$$i_2(t) = 2\cos(100t + 45^\circ)\mathrm{A}$$

[补充4.9]

节点电压法

以节点④为参考节点

$\dot{U}_{n1} = \dot{U}_S$



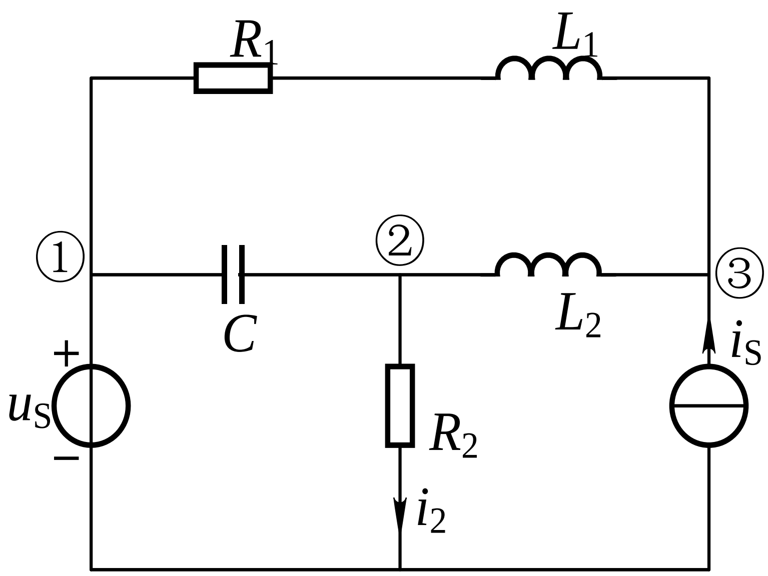
$$-\mathrm{j}\omega C\dot{U}_{n1} + (\mathrm{j}\omega C + \frac{1}{R_2} + \frac{1}{\mathrm{j}\omega L_2})\dot{U}_{n2} - \frac{1}{\mathrm{j}\omega L_2}\dot{U}_{n3} = 0$$

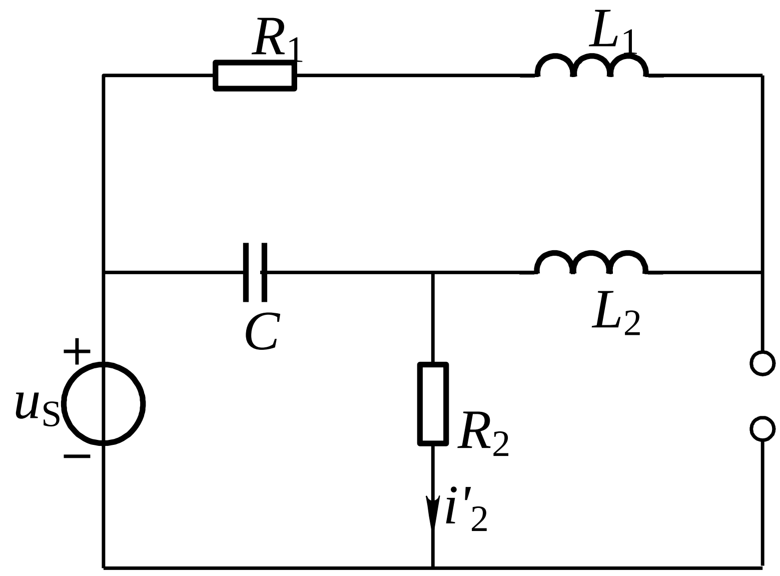
$$-\frac{1}{R_1 + \mathrm{j}\omega L_1}\dot{U}_{n1} - \frac{1}{\mathrm{j}\omega L_2}\dot{U}_{n2} + (\frac{1}{R_1 + \mathrm{j}\omega L_1} + \frac{1}{\mathrm{j}\omega L_2})\dot{U}_{n3} = \dot{I}_S$$

$$\begin{cases} \dot{U}_{n2} = (1+\mathrm{j})\mathrm{V} = \sqrt{2}\angle 45^\circ \mathrm{V} \\ \dot{U}_{n3} = (1-\mathrm{j})\mathrm{V} = \sqrt{2}\angle -45^\circ \mathrm{V} \end{cases} \Rightarrow \dot{I}_2 = \dot{U}_{n2} / R_2 = \sqrt{2}\angle 45^\circ \mathrm{A}$$

[补充4.9]

叠加定理

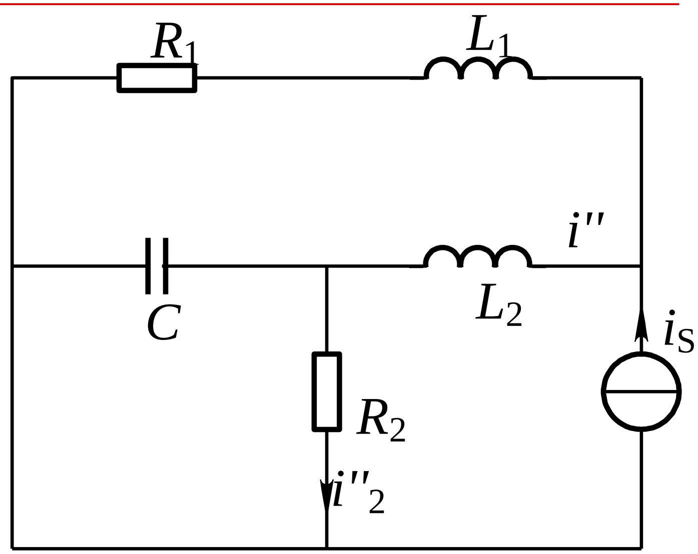




$$\dot{I}_2' = \frac{\dot{U}_S}{R_2 + \dfrac{(R_1 + j\omega L_1 + j\omega L_2)\cdot \dfrac{1}{j\omega C}}{R_1 + j\omega L_1 + j\omega L_2 + \dfrac{1}{j\omega C}}} = \frac{(2 - j2)\text{V}}{\dfrac{3}{1+j}\Omega} = \frac{4}{3}\text{A}$$

[补充4.9]

叠加定理



$$Z = \mathrm{j}\omega L_2 + \frac{R_2 \cdot \frac{1}{\mathrm{j}\omega C}}{R_2 + \frac{1}{\mathrm{j}\omega C}} = \frac{1}{1-\mathrm{j}}\Omega$$

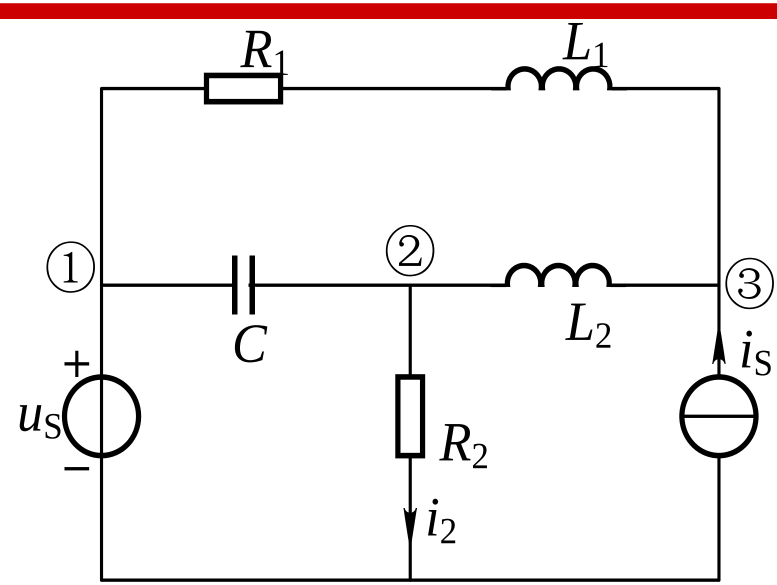
$$\dot{I}'' = \frac{R_1 + \mathrm{j}\omega L_1}{Z + R_1 + \mathrm{j}\omega L_1}\dot{I}_S = \frac{2}{3}(-2+\mathrm{j})\mathrm{A}$$

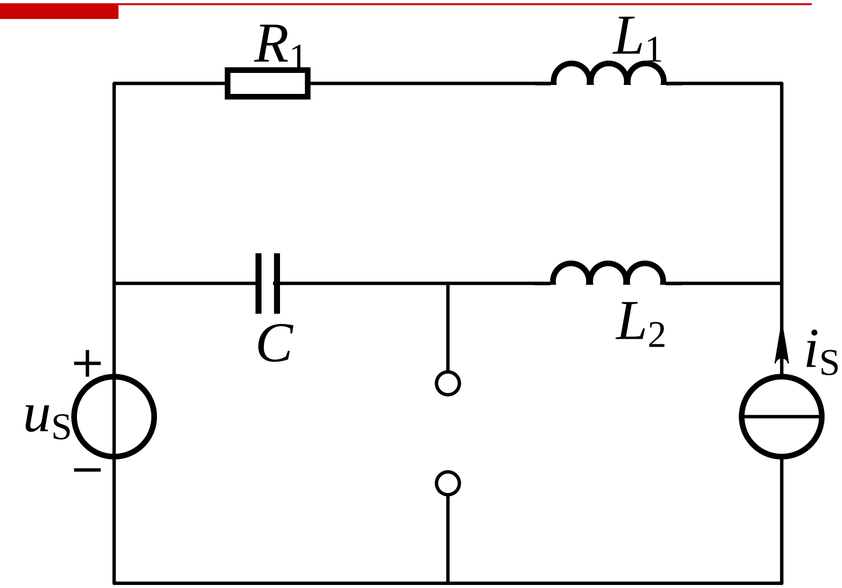
$$\dot{I}_2'' = \frac{\frac{1}{\mathrm{j}\omega C}}{R_2 + \frac{1}{\mathrm{j}\omega C}}\dot{I}'' = (-\frac{1}{3}+\mathrm{j})\mathrm{A}$$

$$\dot{I}_2 = \dot{I}_2' + \dot{I}_2'' = (1+\mathrm{j})\mathrm{A} = \sqrt{2}\angle 45^\circ\mathrm{A}$$

[补充4.9]

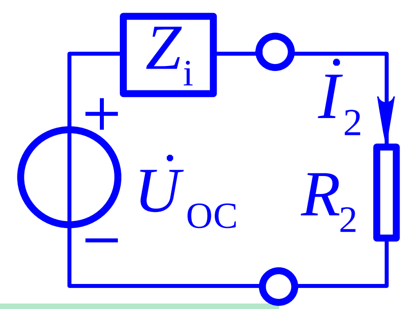
等效电源定理





$$\dot{U}_{OC}=\frac{1}{j\omega C}\dot{I}_C+\dot{U}_S=\frac{1}{j\omega C}\cdot\frac{R_1+j\omega L_1}{R_1+j\omega L_1+j\omega L_2+\frac{1}{j\omega C}}\dot{I}_S+\dot{U}_S=3\text{V}$$

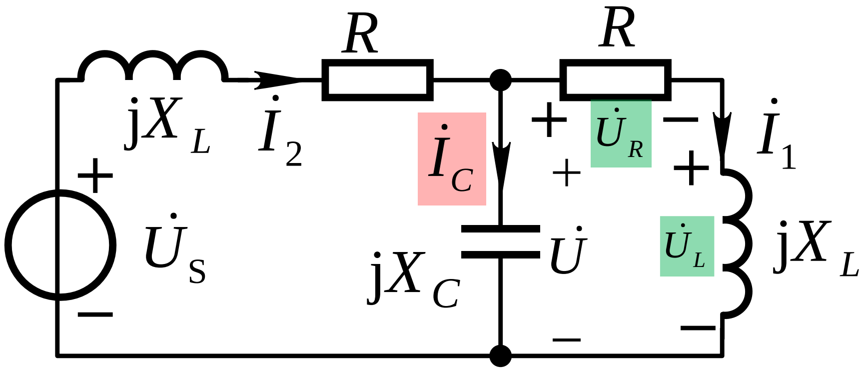
$$Z_i=\frac{(R_1+j\omega L_1+j\omega L_2)\cdot\frac{1}{j\omega C}}{R_1+j\omega L_1+j\omega L_2+\frac{1}{j\omega C}}=(0.5-j1.5)\Omega$$

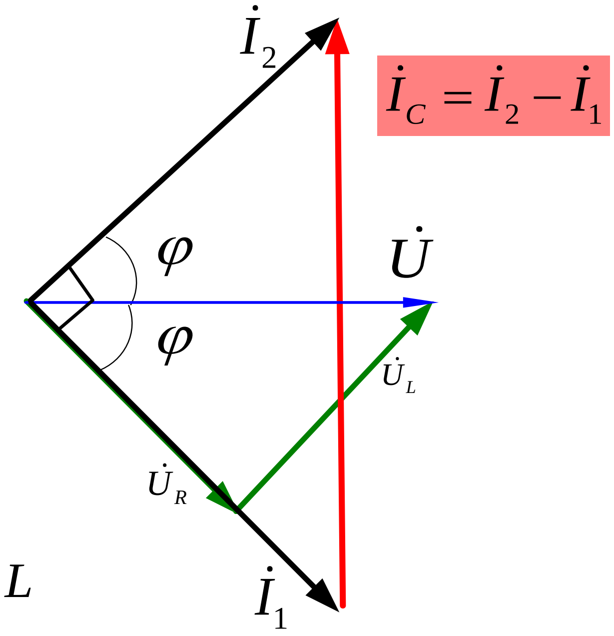


$$\dot{I}_2=\frac{\dot{U}_{OC}}{R_2+Z_i}=\sqrt{2}\angle 45^\circ\text{A}$$

[补充4.10]

已知图示电路中的感抗 X_L，要求 $\dot{I}_2 = j\dot{I}_1$。以电压$\dot{U}$为参考相量画出相量图，求电阻**R**和容抗 X_C 。





【解】

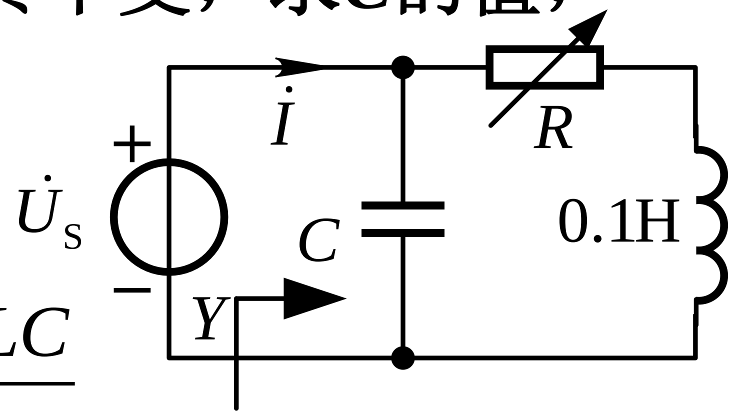
$\varphi = 45^\circ$ → $U_R = U_L$ → $R = X_L$

$$I_C = \sqrt{2} I_1 \rightarrow \frac{U}{|X_C|} = \frac{\sqrt{2}U}{\sqrt{R^2 + X_L^2}} = \frac{\sqrt{2}U}{\sqrt{2}|X_L|} \rightarrow |X_C| = |-X_L|$$

$$X_C = -X_L$$

[补充4.11]

图示电路，$\dot{U}_S = 10\text{V}$，角频率ω=10³rad/s。要求无论R怎样改变，电流有效值I始终不变，求C的值，并分析电流I的辐角变化情况。



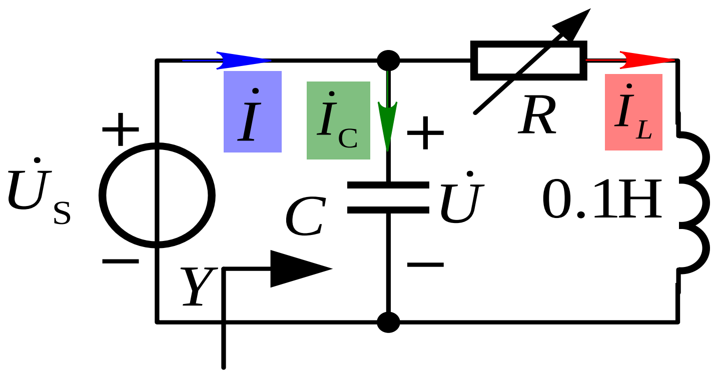
【解】

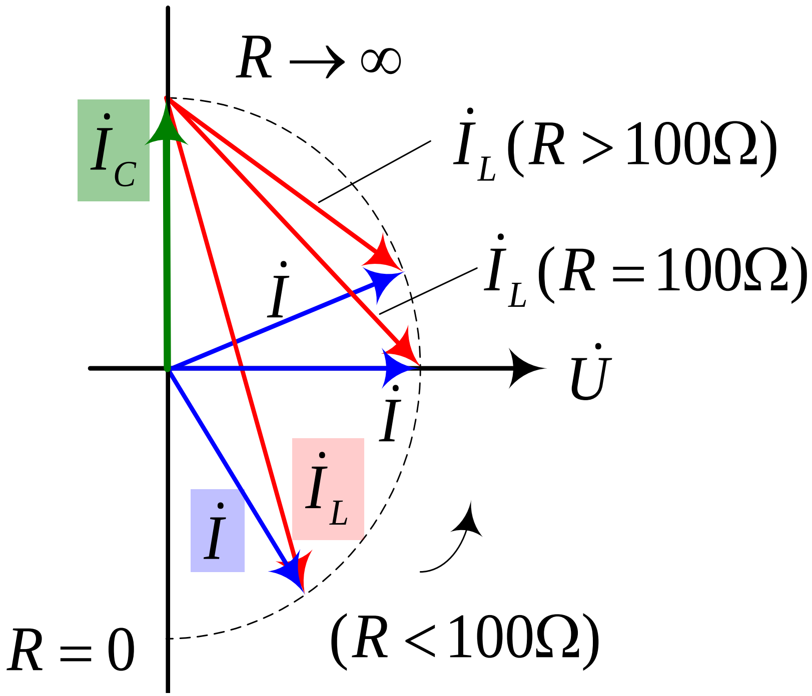
$$Y = \mathrm{j}\omega C + \frac{1}{R+\mathrm{j}\omega L} = \frac{\mathrm{j}\omega RC + 1 - \omega^2 LC}{R+\mathrm{j}\omega L}$$

$$= \omega C \frac{\mathrm{j}R - \omega L + \dfrac{1}{\omega C}}{R+\mathrm{j}\omega L} = \omega C \frac{\sqrt{R^2 + \left(-\omega L + \dfrac{1}{\omega C}\right)^2}}{\sqrt{R^2 + (\omega L)^2}} \angle\varphi$$

令$Y = \omega C \angle\varphi$ → $-\omega L + \dfrac{1}{\omega C} = \pm\omega L$ → $C = \dfrac{1}{2\omega^2 L} = 5\mu\text{F}$

[补充4.11]





当 $R = 0$， $\dot{I}$ 滞后 $\dot{U}_S$ 为 $-90°$；

当 $0 < R < 100\Omega$， $\dot{I}$ 滞后 $\dot{U}_S$ 为从 $-90°$ 向 0 变化；

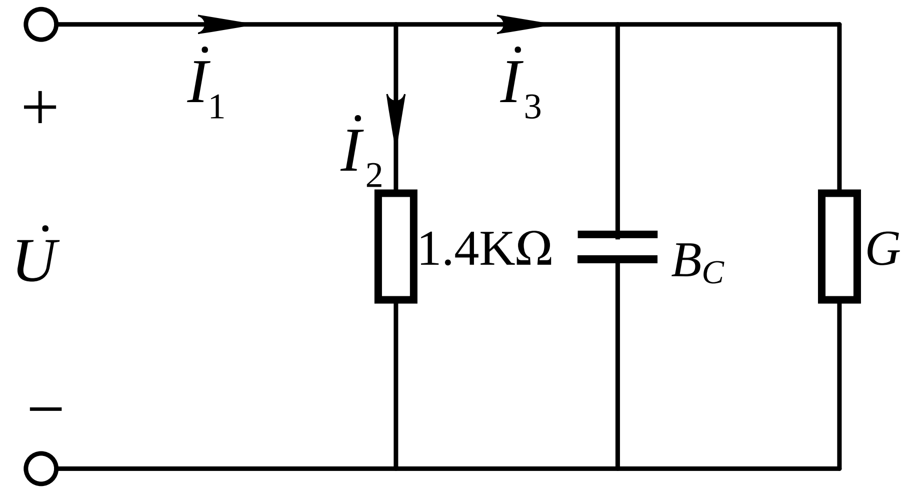
当 $R = 100\Omega$， $\dot{I}$ 与 $\dot{U}_S$ 同相位；

当 $R > 100\Omega$， $\dot{I}$ 越前 $\dot{U}_S$ 为从 0 向 $90°$ 变化；

当 $R \to \infty$， $\dot{I}$ 越前 $\dot{U}_S$ 为 $90°$。

[补充4.12]

已知I_1=0.4A，I_2=0.1A，I_3=0.38A，求B_C和G。



【解】

$$U = 1.4\text{K}\Omega I_2$$

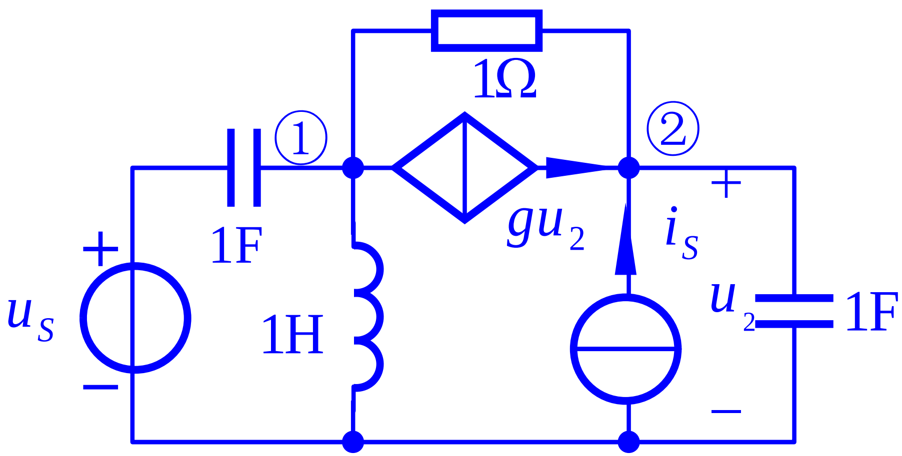
$$I_3 = \sqrt{(B_C U)^2 + (GU)^2} = 0.38\text{A}$$

$$I_1 = \sqrt{(B_C U)^2 + (I_2 + GU)^2} = 0.4\text{A}$$

[补充4.13]

已知图示电路中 g =1S, $u_S = 10\sqrt{2}\sin\omega t\text{V}$, $i_S = 10\sqrt{2}\cos\omega t\text{A}$

ω =1rad/s。求受控电流源的电压 u_{12}。



【解】

$$\dot{U}_S = 10\angle -90^\circ \text{ V},$$

$$\dot{I}_S = 10\angle 0^\circ \text{ A}$$

列写节点电压方程：

$$\begin{cases} n_1: \left(j\omega C_1 + \dfrac{1}{j\omega L} + \dfrac{1}{R} \right)\dot{U}_{n_1} - \dfrac{1}{R}\dot{U}_{n_2} = j\omega C_1 \dot{U}_S - g\dot{U}_2 \\ n_2: -\dfrac{1}{R}\dot{U}_{n_1} + \left(j\omega C_2 + \dfrac{1}{R} \right)\dot{U}_{n_2} = \dot{I}_S + g\dot{U}_2 \end{cases} \qquad \dot{U}_2 = \dot{U}_{n2}$$

[补充4.13]

$$\begin{cases} n_1: \left(\mathrm{j}\omega C_1 + \dfrac{1}{\mathrm{j}\omega L} + \dfrac{1}{R} \right) \dot{U}_{n_1} - \dfrac{1}{R} \dot{U}_{n_2} = \mathrm{j}\omega C_1 \dot{U}_S - g\dot{U}_2 \\ n_2: -\dfrac{1}{R} \dot{U}_{n_1} + \left(\mathrm{j}\omega C_2 + \dfrac{1}{R} \right) \dot{U}_{n_2} = \dot{I}_S + g\dot{U}_2 \end{cases}$$

$$\dot{U}_2 = \dot{U}_{n2}$$

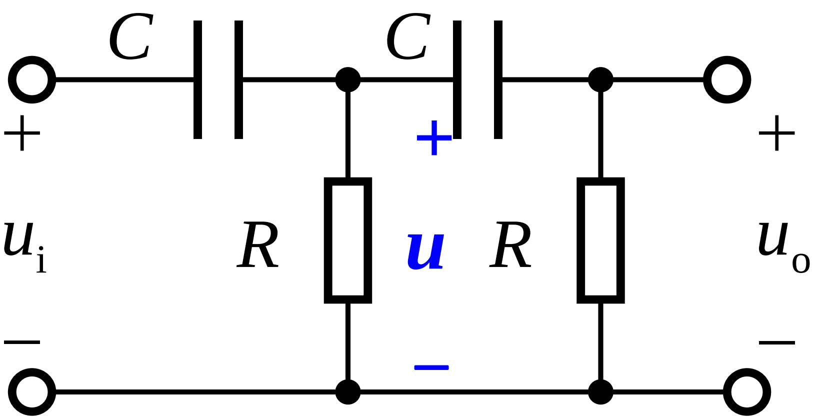
解得 $\dot{U}_{n1} = 10\,\mathrm{V} \quad \dot{U}_{n2} = -\mathrm{j}20\,\mathrm{V}$

$$\dot{U}_{12} = \dot{U}_{n_1} - \dot{U}_{n_2} = (10 + \mathrm{j}20)\,\mathrm{V} = 22.36\angle 63.43^\circ\,\mathrm{V}$$

$$u_{12} = 22.36\sqrt{2}\cos\left(\omega t + 63.43^\circ\right)\,\mathrm{V}$$

[补充4.14]

在图示 ***RC*** 移相电路中设 $R=1/(\omega C)$，试求输出电压 u_o 和输入电压 u_i 的相位差。



$$\frac{\dot{U}}{\dot{U}_i}=\frac{\dfrac{R(R+1/\mathrm{j}\omega C)}{R+R+1/\mathrm{j}\omega C}}{1/\mathrm{j}\omega C+\dfrac{R(R+1/\mathrm{j}\omega C)}{R+R+1/\mathrm{j}\omega C}}$$

$$=\frac{1}{3}(1+\mathrm{j})$$

【解】

$$\frac{\dot{U}_o}{\dot{U}}=\frac{R}{R+1/\mathrm{j}\omega C}$$

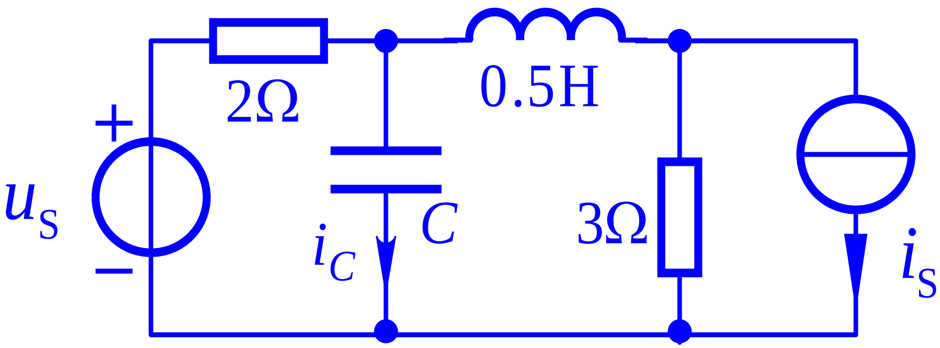
$$=\frac{R}{R-\mathrm{j}R}=\frac{1}{1-\mathrm{j}}$$

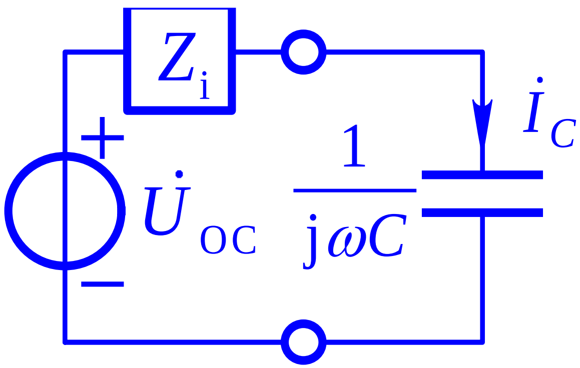
$$\frac{\dot{U}_o}{\dot{U}_i}=\frac{\dot{U}_o}{\dot{U}}\times\frac{\dot{U}}{\dot{U}_i}=\frac{1}{1-\mathrm{j}}\times\frac{1+\mathrm{j}}{3}=\frac{1}{3}\mathrm{j}$$

u_o 越前于 u_i 的相位差为 90°

[例4.12]

图示电路中,C=0.05F 时,$i_C = 5\sqrt{2}\cos(10t-60^\circ)\text{A}$ ，求当 C=0.25F 时， i_C = ?





$$Z_i = \frac{2\times(3+j5)}{2+3+j5}\Omega = (1.6+j0.4)\Omega$$

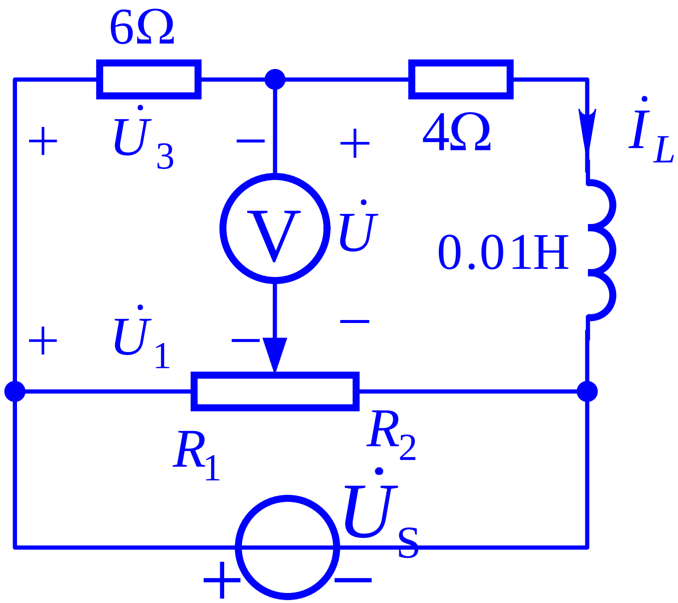
当 $C=0.05\text{F}$时， $\dot{U}_{oc} = (Z_i+\frac{1}{j\omega C})\dot{I}_C = (Z_i - j2)\times 5\angle-60^\circ = 8\sqrt{2}\angle-105^\circ\text{V}$

当 $C=0.25\text{F}$时， $\dot{I}_C = \frac{\dot{U}_{oc}}{Z_i+1/j\omega C} = 5\sqrt{2}\angle-105^\circ\text{A}$

$$i_C = \sqrt{2}\times 5\sqrt{2}\cos(10t-105^\circ)\text{A} = 10\cos(10t-105^\circ)\text{A}$$

[例4.13]

图示电路，正弦电压源角频率为 ω=1000rad/s，电压表为理想的。求R_1/R_2为何值时，电压表的读数为最小？



(a)

【解】

设$R_1/R_2=r$，R_1分得分压为

$$\dot{U}_1=\frac{R_1\dot{U}_S}{R_1+R_2}=\frac{r}{r+1}\dot{U}_S \qquad (1)$$

6Ω电阻电压为

$$\dot{U}_3=\frac{6\dot{U}_S}{(6+4)+j\omega L}=\frac{6\dot{U}_S}{10+j10} \qquad (2)$$

电压表两端电压为

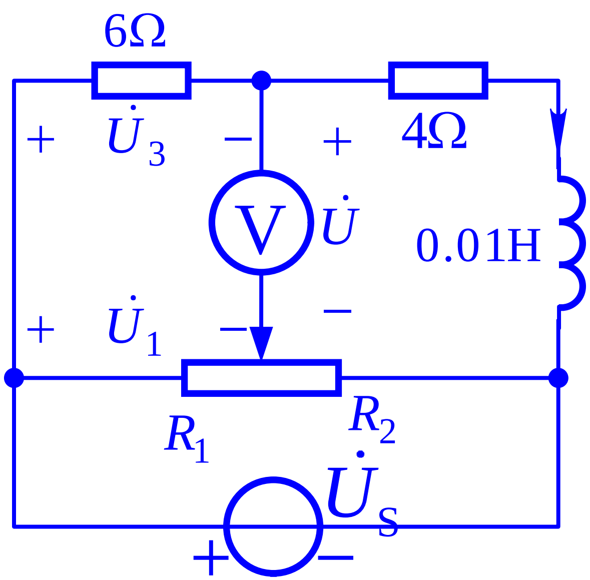
$$\dot{U}=-\dot{U}_3+\dot{U}_1=(\frac{r}{r+1}-0.3+j0.3)\dot{U}_S \qquad (3)$$

[例4.13]

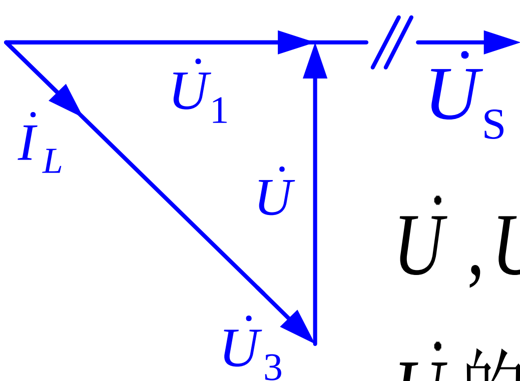
$$\dot{U} = -\dot{U}_3 + \dot{U}_1 = (\frac{r}{r+1} - 0.3 + \text{j}0.3)\dot{U}_S \quad (3)$$

实部为零时电压表的读数便是最小

$$\frac{r}{r+1} - 0.3 = 0 \quad 即 \quad r = \frac{R_1}{R_2} = \frac{3}{7}$$



(a)


(b)

$\dot{U}, \dot{U}_1$为垂直关系时，$\dot{U}$的长度最短。