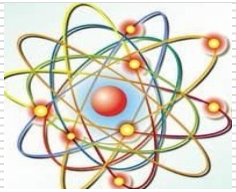

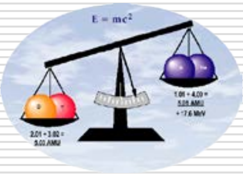

第8章　线性动态电路暂态过程的时域分析

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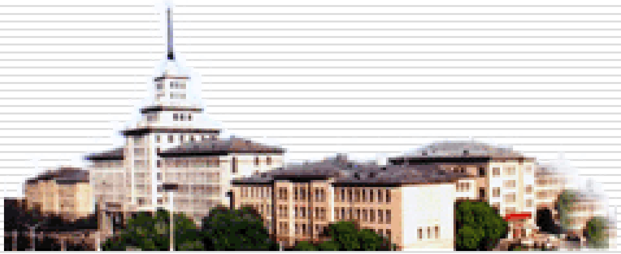
开课单位：机电学院--电气工程学科



8.5 一阶电路的阶跃响应

基本要求：理解零状态响应的定义、稳态分量与暂态分量的含义；掌握单位阶跃特性计算。

零状态响应:

电路中储能元件的原始储能为零［$u_C(0_-)=0$，$i_L(0_-)=0$］，仅由独立电源作用引起的响应。

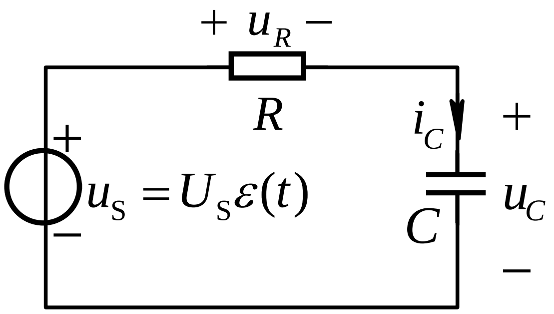
阶跃响应与单位阶跃特性

阶跃响应：电路在阶跃电源作用下的零状态响应

单位阶跃特性：$s(t)$ = 阶跃响应 / 阶跃电源幅值

8.5 一阶电路的阶跃响应

图示电路中$u_C(0_-)=0$，以$u_C(t)$为响应



阶跃响应：$u_C(t)$

单位阶跃特性：$s(t)=\dfrac{u_C(t)}{U_S}$　（无量纲）

$s(t)$在量值上为单位阶跃电源$\varepsilon(t)$引起的零状态响应

求解$u_C(t)$：

$$u_R+u_C=u_S$$

$u_R=Ri$

$i=C\mathrm{d}u_C/\mathrm{d}t$

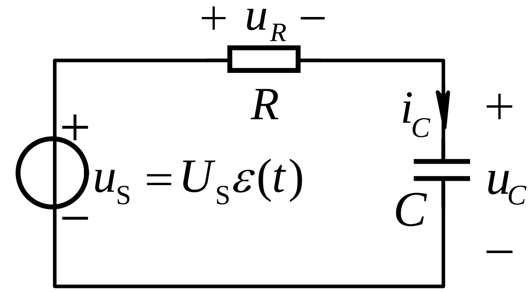
$u_S=U_S\,\varepsilon(t)$

$$\begin{cases} RC\dfrac{\mathrm{d}u_C}{\mathrm{d}t}+u_C=U_S\varepsilon(t) \\ u_C(0_+)=u_C(0_-)=0 \end{cases}$$

方程通解：$u_C(t)=u_{Cp}(t)+u_{Ch}(t)$

阶跃响应求解

$$\begin{cases} RC\dfrac{\mathrm{d}u_C}{\mathrm{d}t}+u_C=U_S\varepsilon(t) \\ u_C(0_+)=u_C(0_-)=0 \end{cases}$$



***RC*电路的阶跃响应**

方程通解: $u_C(t)=u_{Cp}(t)+u_{Ch}(t)$

（1）求特解 $u_{Cp}(t)$：$t\to\infty$时，电路达到稳态 $u_C(\infty)=u_S(\infty)=U_S$

$$u_{Cp}(t)=U_s$$

（2）求齐次方程通解 $u_{Ch}(t)$： $u_{Ch}(t)=Ae^{-t/RC}=Ae^{-t/\tau}$

（3）求非齐次方程通解 $u_C(t)$： $u_C(t)=U_s+Ae^{-t/\tau}$

（4）确定积分常数 A： $u_C(0_+)=U_s+A=0 \quad\to\quad A=-U_s$

通解: $u_C(t)=U_s-U_s e^{-t/\tau}$

*RC*电路单位阶跃特性

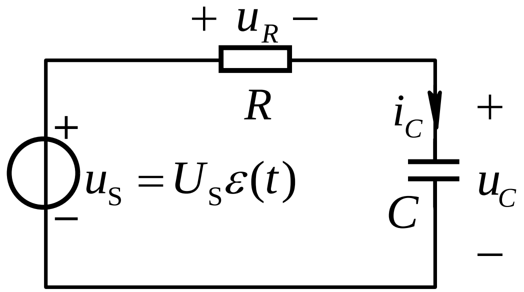
阶跃响应：$u_C(t)=U_s-U_s\mathrm{e}^{-t/\tau}$ $t>0$

$$u_C(t)=U_s(1-\mathrm{e}^{-t/\tau})\varepsilon(t)$$ $-\infty<t<\infty$

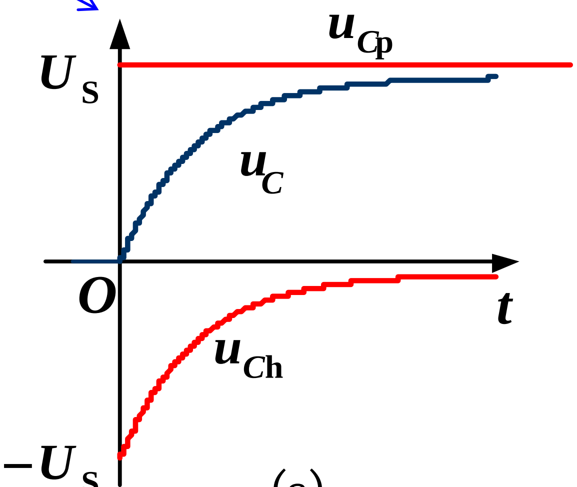
单位阶跃特性：$s(t)=\dfrac{u_C(t)}{U_s}=(1-\mathrm{e}^{-t/\tau})\varepsilon(t)$

$$i_C(t)=C\frac{\mathrm{d}u_C}{\mathrm{d}t}=\frac{U_S}{R}\mathrm{e}^{-t/\tau}\varepsilon(t)$$

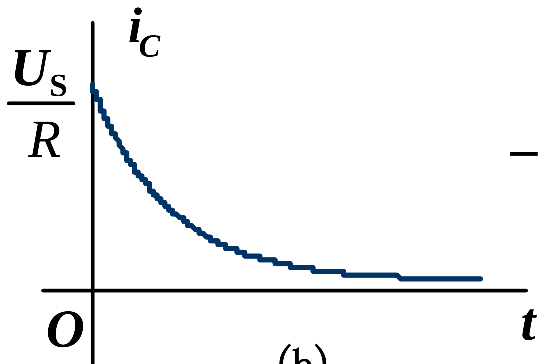
$$=\frac{U_S-u_C(t)}{R}=i_R$$



*RC*电路的阶跃响应

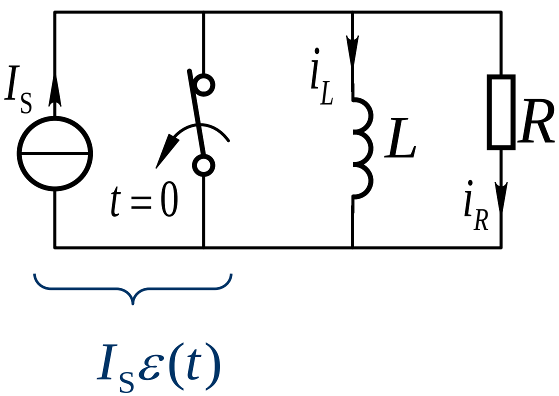


(a)



(b)

RL一阶电路零状态响应、单位阶跃特性



$$i_L(0_+)=0$$

$$\frac{L}{R}\frac{di_L}{dt}+i_L=I_S$$

$$\frac{di_L}{dt}+\frac{1}{L/R}i_L=\frac{1}{L}RI_S$$

$$i_L(t)=i_{LP}+i_{Lh}=i_{LP}+Ae^{-\frac{t}{L/R}}=(I_S-I_Se^{-\frac{t}{\tau}})=I_S(1-e^{-\frac{t}{\tau}})\varepsilon(t)$$

$$(t\geq 0) \qquad (-\infty<t<+\infty)$$

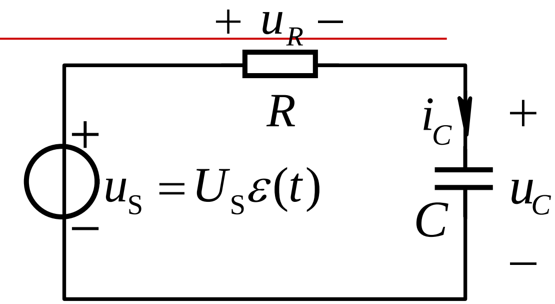
$$s_L(t)=\frac{i_L(t)}{I_S}=(1-e^{-\frac{t}{L/R}})\varepsilon(t)$$

若$t=t_0$时换路，即$i_S=I_S\varepsilon(t-t_0)$

$$i_L(t)=I_S(1-e^{-\frac{t-t_0}{L/R}})\varepsilon(t-t_0) \qquad s_L(t)=(1-e^{-\frac{t-t_0}{L/R}})\varepsilon(t-t_0)$$

脉冲响应

延迟阶跃响应：(即$t=t_0$时换路)



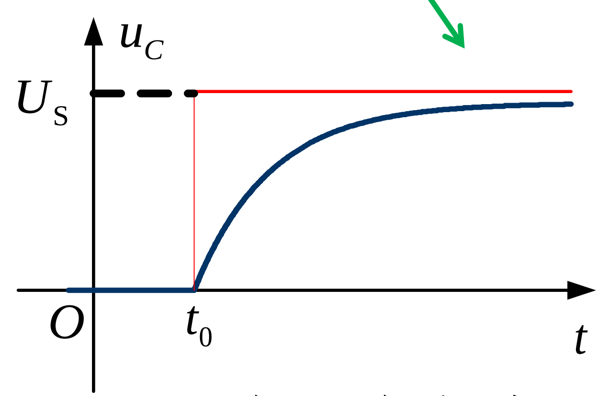
$$u_s=U_S\varepsilon(t) \longrightarrow u_C(t)=U_S(1-e^{-t/\tau})\varepsilon(t)$$

$$u_s=U_S\varepsilon(t-t_0) \longrightarrow u_C(t)=U_S(1-e^{-(t-t_0)/\tau})\varepsilon(t-t_0)$$

脉冲响应：

$$u_S=U_S[\varepsilon(t)-\varepsilon(t-t_0)]$$

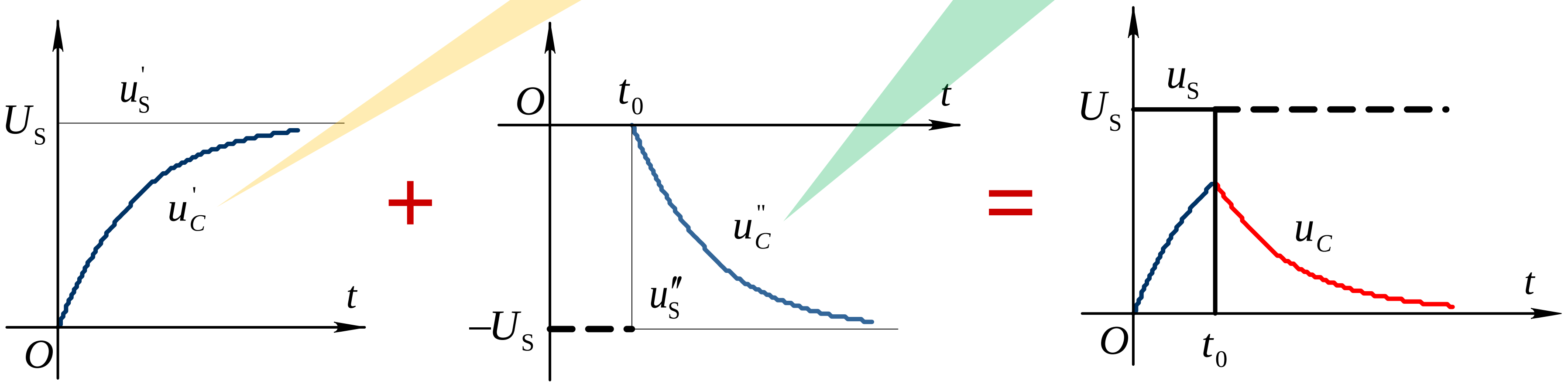
$$\longrightarrow u_C(t)=U_S(1-e^{-t/\tau})\varepsilon(t) - U_S(1-e^{-(t-t_0)/\tau})\varepsilon(t-t_0)$$



延迟阶跃响应波形

脉冲响应

$$u_C(t) = U_S(1-e^{-t/\tau})\varepsilon(t) - U_S(1-e^{-(t-t_0)/\tau})\varepsilon(t-t_0)$$



脉 冲 响 应 的 电 压 波 形

u_C表示为分段函数

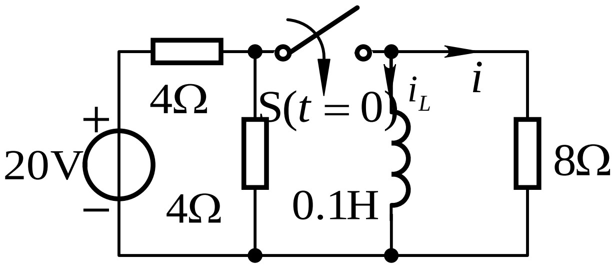
$$u_C(t)=\begin{cases}0, & (t<0)\\ U_S(1-e^{-t/\tau}) & (0\le t<t_0)\\ U_S(1-e^{-t/\tau})-U_S(1-e^{-(t-t_0)/\tau})=U_S(1-e^{-t_0/\tau})e^{-(t-t_0)/\tau} & (t\ge t_0)\end{cases}$$

$(0\le t<t_0)$ → 零状态响应

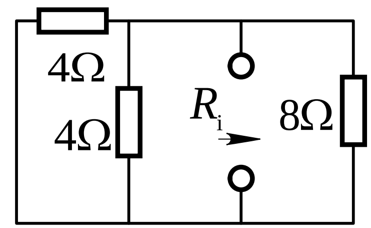
$U_S(1-e^{-t_0/\tau})e^{-(t-t_0)/\tau}$：零 输 入 响 应

【补充例题3】

图(a) 电路原处于稳态，$t=0$ 时接通。求$t>0$ 时的电流i 。



(a)



(b)

【解】 开关原断开，由换路定律 $i_L(0_+)=i_L(0_-)=0$

达到稳态时电感短路，故 $i_L(\infty)=20/4=5\text{A}$

求等效电阻的电路如图(b)所示。 $R_\text{i}=(4//4)//8=1.6\Omega$

时间常数 $\tau=L/R_\text{i}=(1/16)\text{s}$

$t>0$后电路为零状态响应

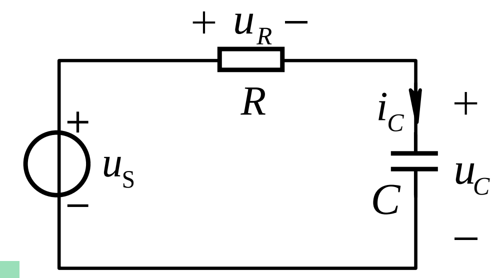
$$i_L(t)=i_L(\infty)(1-\text{e}^{-t/\tau})=5(1-\text{e}^{-16t})\text{A} \quad (t\geq 0)$$

$$i(t)=\frac{u_L}{8\Omega}=(L\frac{\text{d}i_L}{\text{d}t})/8\Omega=\frac{0.1\times 5\times 16\times \text{e}^{-16t}}{8}=\text{e}^{-16t}\text{A} \quad (t>0)$$

8.6 一阶电路的冲激响应

冲激响应： 电路在冲激电源作用下的零状态响应

单位冲激特性： $h(t)$ =冲激响应 /冲激强度



$$u_S = U_s\varepsilon(t) \longrightarrow RC\frac{\mathrm{d}u_C}{\mathrm{d}t} + u_C = U_s\varepsilon(t)$$

$\times\frac{1}{U_s}$

$$\times\frac{1}{U_s} \longrightarrow RC\frac{\mathrm{d}(u_C/U_s)}{\mathrm{d}t} + (u_C/U_s) = \varepsilon(t)$$

$$u_S = \varepsilon(t) \longrightarrow RC\frac{\mathrm{d}s(t)}{\mathrm{d}t} + s(t) = \varepsilon(t)$$

u_C $\times\frac{1}{U_s}$

$$s(t) = \frac{u_C}{U_s}$$

$\frac{\mathrm{d}}{\mathrm{d}t}$

$$\frac{\mathrm{d}}{\mathrm{d}t} \longrightarrow RC\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}s(t)}{\mathrm{d}t}\right) + \frac{\mathrm{d}s(t)}{\mathrm{d}t} = \frac{\mathrm{d}\varepsilon(t)}{\mathrm{d}t} = \delta(t)$$

$\frac{\mathrm{d}}{\mathrm{d}t}$

$$h(t) = \frac{\mathrm{d}s(t)}{\mathrm{d}t}$$

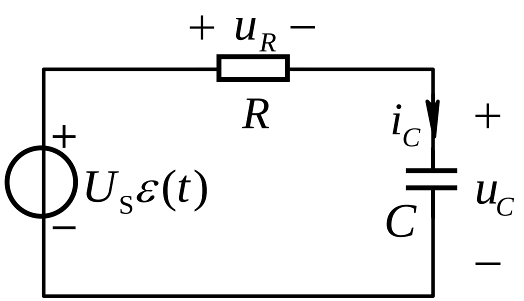
$$u_S = \delta(t) \longrightarrow RC\frac{\mathrm{d}h(t)}{\mathrm{d}t} + h(t) = \delta(t)$$

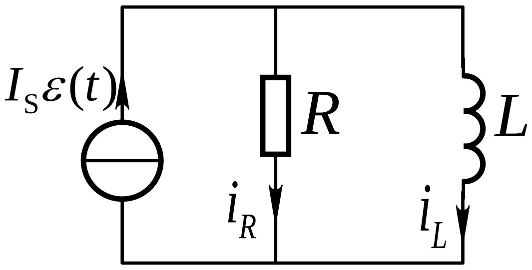
$$s(t) = \int_{0_-}^{t} h(\xi)\mathrm{d}\xi$$

一般激励导数（积分）的响应等于激励响应的导数（积分）

一阶电路单位冲激特性$h(t)$等于单位阶跃特性$s(t)$的导数

单位冲激特性





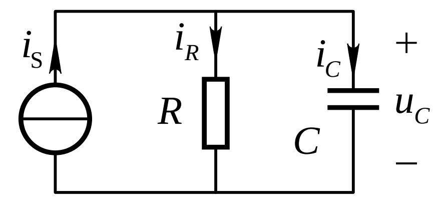
$$s(t)=(1-\mathrm{e}^{-t/\tau})\varepsilon(t)$$

$$h(t)=\frac{\mathrm{d}s(t)}{\mathrm{d}t}=\frac{1}{\tau}\mathrm{e}^{-t/\tau}\varepsilon(t)+(1-\mathrm{e}^{-t/\tau})\delta(t)=\frac{1}{\tau}\mathrm{e}^{-t/\tau}\varepsilon(t)$$

利用单位阶跃特性的导数获得单位冲激特性，再乘以任意冲激强度，便得到对该冲激激励的零状态响应。
这是计算冲激响应的重要方法。

RC、RL电路冲激响应

$$C\frac{\mathrm{d}u_C}{\mathrm{d}t}+\frac{1}{R}u_C=Q\delta(t) \longleftarrow \quad i_C+i_R=i_S$$



$$i_S=Q\delta(t)$$

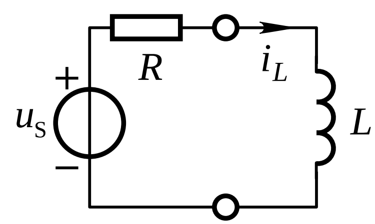
$$C\int_{0_-}^{0_+}\frac{\mathrm{d}u_C}{\mathrm{d}t}\mathrm{d}t+\frac{1}{R}\int_{0_-}^{0_+}u_C\mathrm{d}t=Q\int_{0_-}^{0_+}\delta(t)\mathrm{d}t=Q$$

$$\frac{1}{R}\int_{0_-}^{0_+}u_C\mathrm{d}t=0$$

$$C\int_{0_-}^{0_+}\frac{\mathrm{d}u_C}{\mathrm{d}t}\mathrm{d}t=C\int_{u_C(0_-)}^{u_C(0_+)}\mathrm{d}u_C=C[u_C(0_+)-u_C(0_-)]=Q$$

对偶原理

$$u_C(0_+)=\frac{Q}{C}+u_C(0_-) \qquad i_L(0_+)=\frac{\Psi}{L}+i_L(0_-)$$

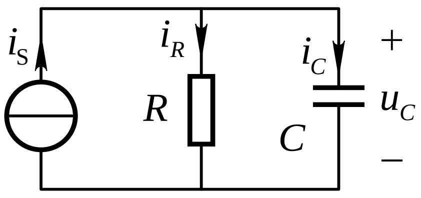


初值： $u_C(0_-)=0$ $\qquad i_L(0_-)=0$

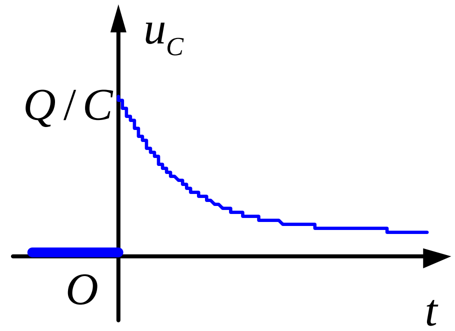
零输入： $u_C(t)=u_C(0_+)\mathrm{e}^{-\frac{t}{\tau}}=\frac{Q}{C}\mathrm{e}^{-\frac{t}{\tau}}$ $\qquad i_L(t)=\frac{\Psi}{L}\mathrm{e}^{-\frac{t}{\tau}}$ $\qquad u_S=\psi\delta(t)$

$$h(t)=\frac{1}{C}\mathrm{e}^{-t/\tau}\varepsilon(t) \qquad h(t)=\frac{1}{L}\mathrm{e}^{-t/\tau}\varepsilon(t)$$

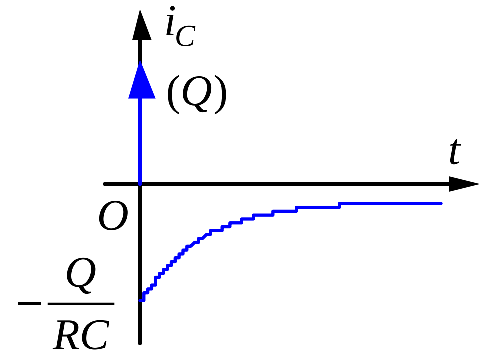
冲激激励下电容电压/电流波形



$$i_S = Q\delta(t)$$



图(b)



图(c)

$$u_C(t) = u_C(0_+)\mathrm{e}^{-\frac{t}{\tau}} = \frac{Q}{C}\mathrm{e}^{-\frac{t}{\tau}} = \frac{Q}{C}\mathrm{e}^{-\frac{t}{\tau}}\varepsilon(t)$$

电压波形如图(b)

$$(t > 0)$$

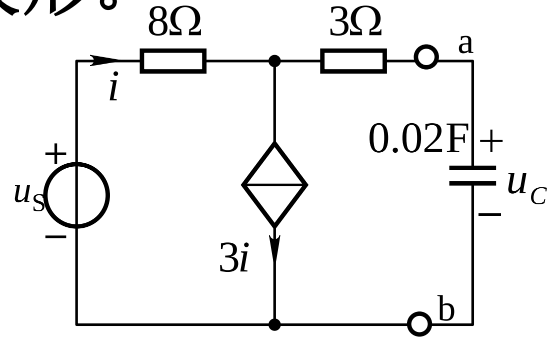
$$i_C(t) = C\frac{\mathrm{d}u_C}{\mathrm{d}t} = Q\mathrm{e}^{-t/\tau}\delta(t) - \frac{Q}{\tau}\mathrm{e}^{-t/\tau}\varepsilon(t) = Q\delta(t) - \frac{Q}{RC}\mathrm{e}^{-t/\tau}\varepsilon(t)$$

$$= Q\delta(t) - \frac{U}{R}\mathrm{e}^{-t/\tau}\varepsilon(t)$$

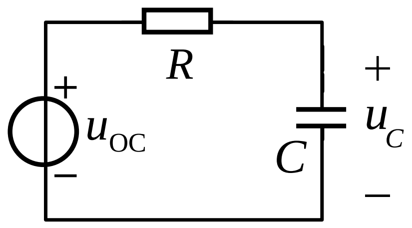
电流波形如图(c)

【补充例题4】

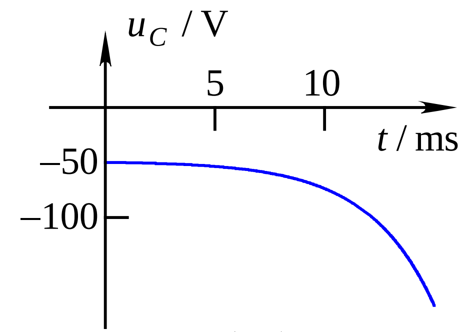
求图(a)所示电路单位阶跃特性$s(t)$和单位冲激特性$h(t)$， 并画出$h(t)$波形。



图(a)



图(b)



图(c)

【解】方法一

$$u_S = \varepsilon(t)\ \text{V}$$

戴维南电路如图(b)

$$i = 3i \Rightarrow i = 0 \rightarrow u_{OC} = u_S$$

$$\left.\begin{aligned} i_{SC} &= i - 3i = -2i \\ 8i + 3i_{SC} &= u_S \end{aligned}\right\} \quad i_{SC} = -u_S$$

$$R = \frac{u_{OC}}{i_{SC}} = -1\ \Omega$$

$$\tau = -1 \times 0.02 = -0.02\ \text{s}$$

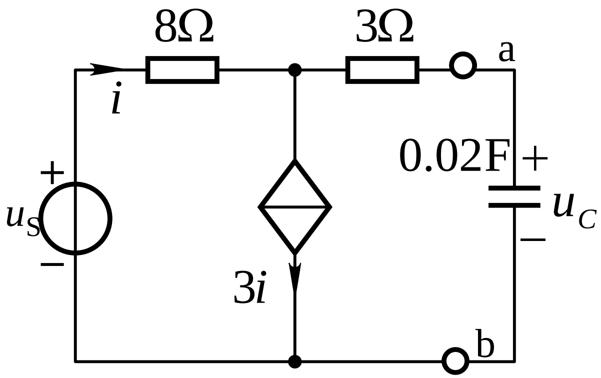
$$s(t) = (1 - e^{-t/\tau})\varepsilon(t) = (1 - e^{50t})\varepsilon(t)$$

$$h(t) = \frac{ds(t)}{dt} = -50e^{50t}\varepsilon(t)$$

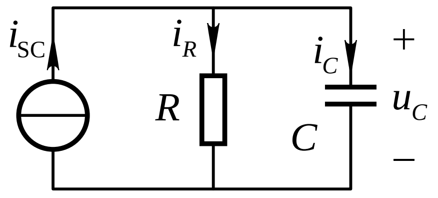
$h(t)$波形如图(c)所示

【补充例题4】

求图(a)所示电路单位冲激特性$h(t)$ 。



图(a)



$$i_{SC}=Q\delta(t)=-\delta(t)$$

【解】方法二 $u_S=1\text{Wb}\times\delta(t)$

$$u_C(t)=\frac{Q}{C}e^{-t/\tau}\varepsilon(t)$$

$$R=-1\ \Omega$$

$$\tau=-0.02\ \text{s}$$

$$h(t)=\frac{-1}{0.02}e^{-\frac{t}{-0.02}}\varepsilon(t)=-50e^{50t}\varepsilon(t)$$

$$i_{SC}=-u_S=-\delta(t)$$