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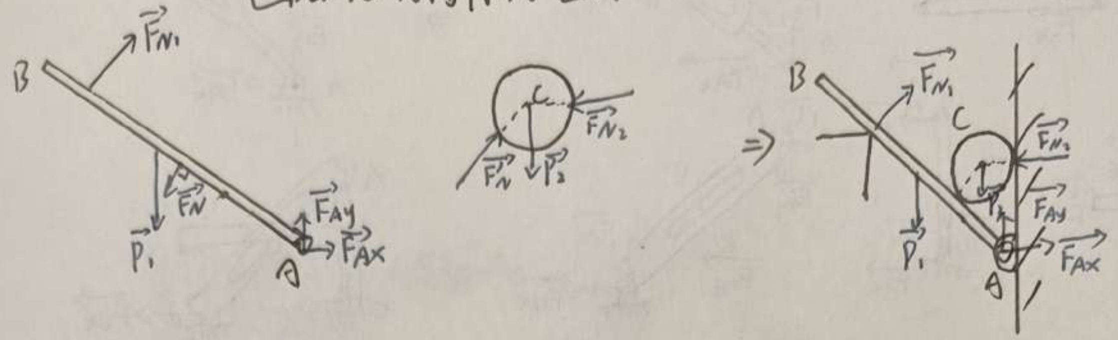
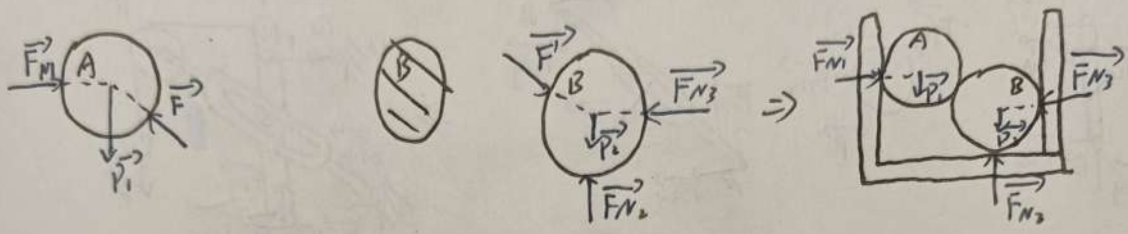
画法：先分离体再整体

1-2

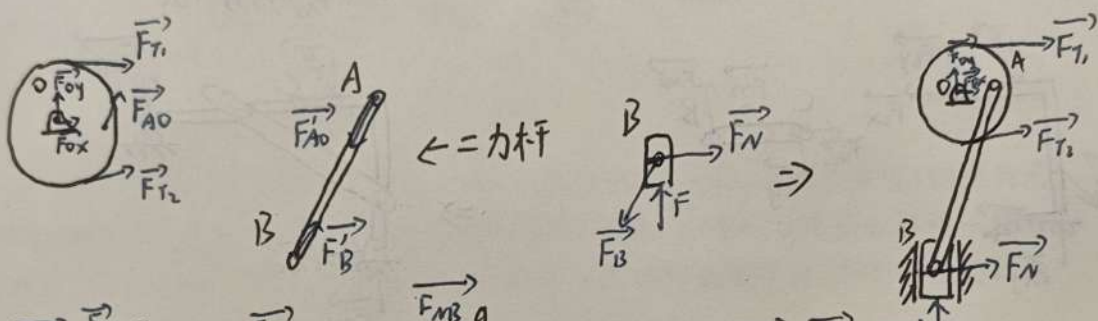
(a)



(b)



(c)



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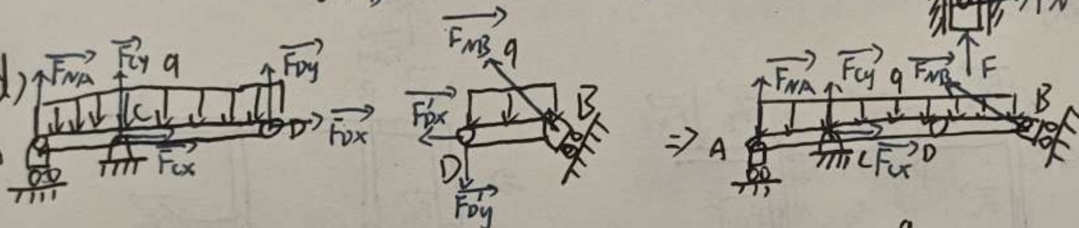
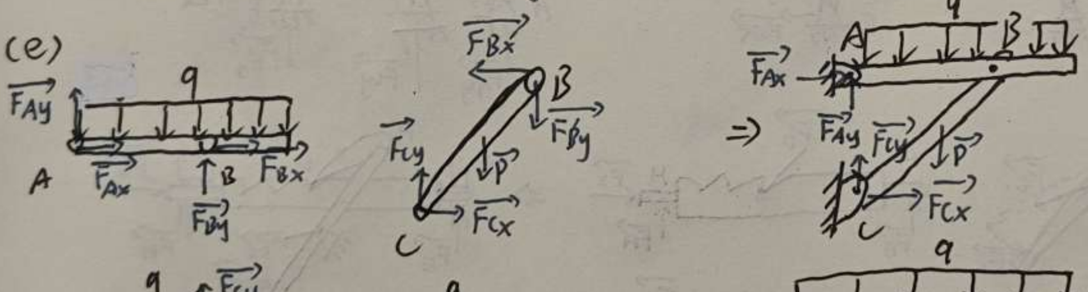
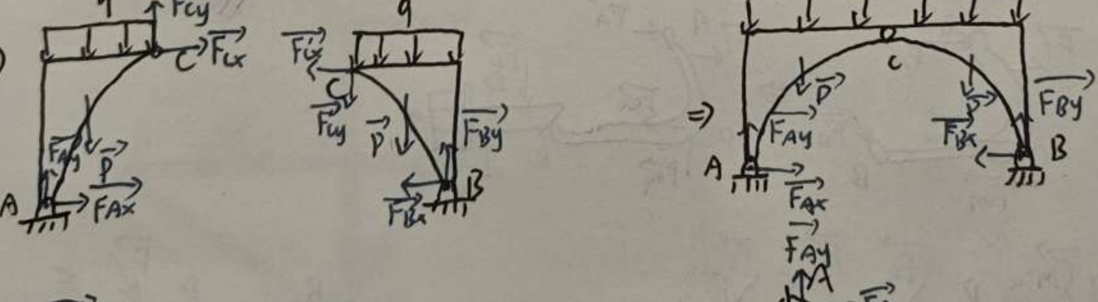


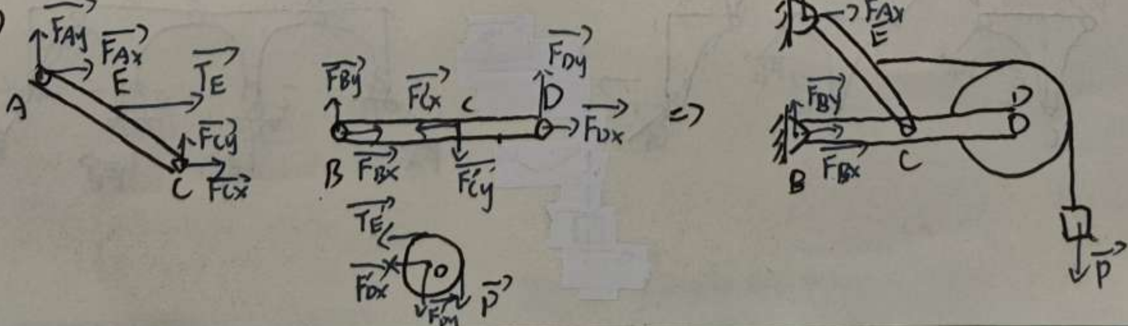
(e)



(f)



(g)



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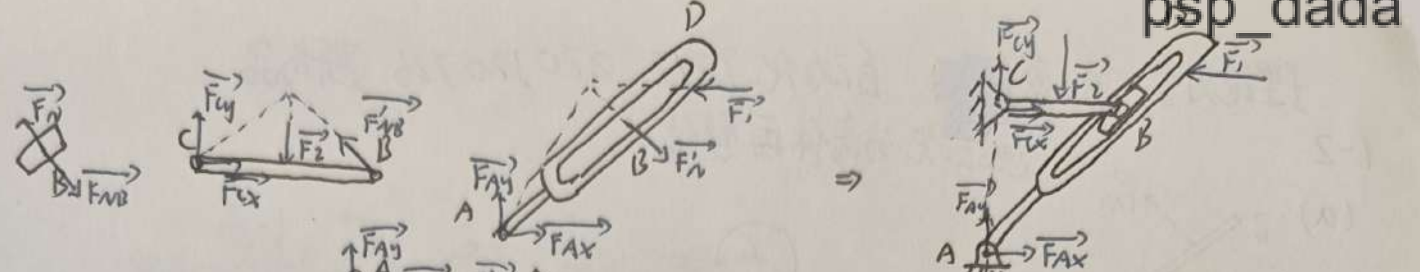
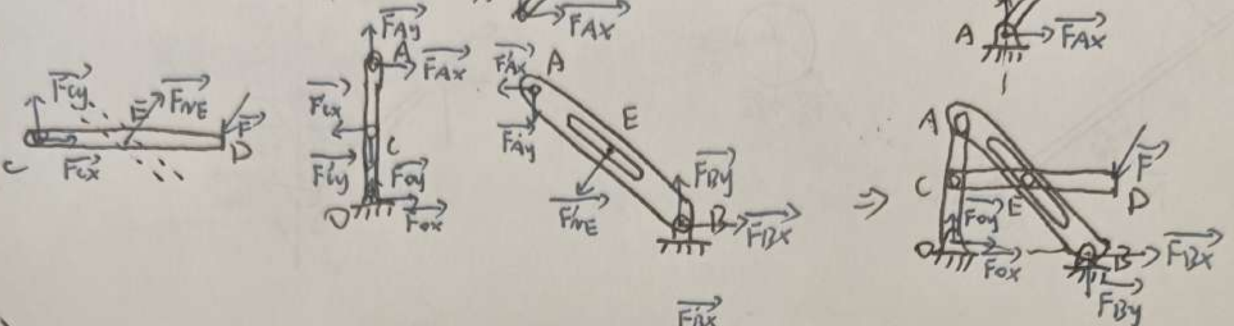
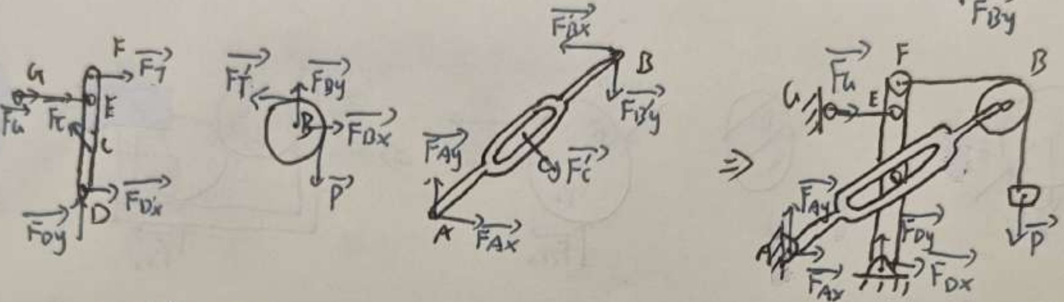
(h)



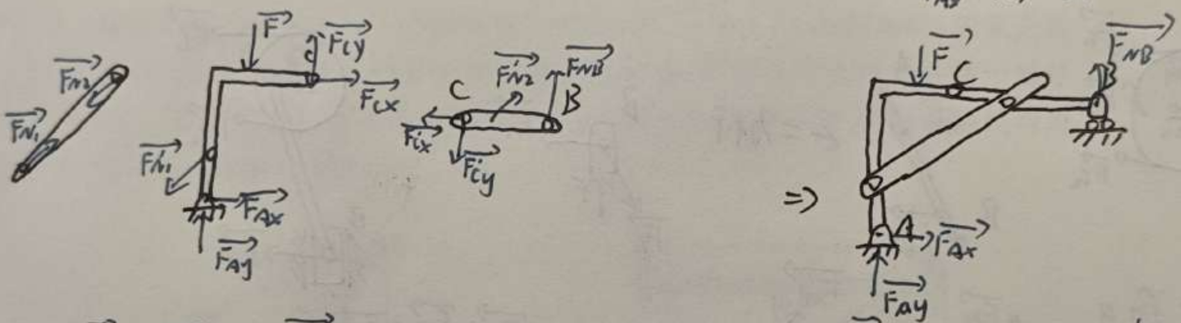
(i)



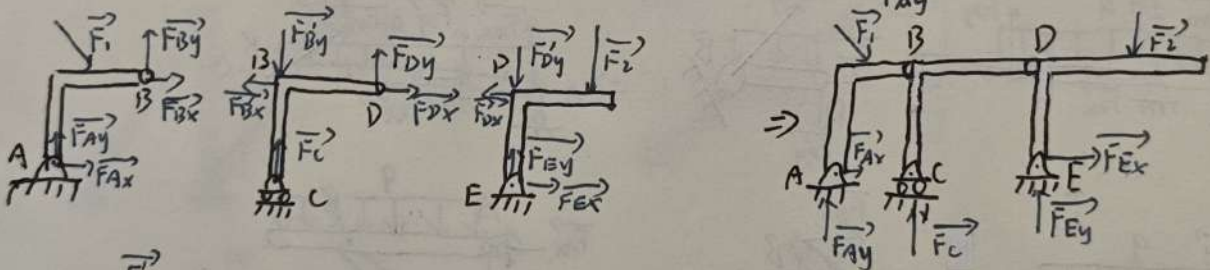
(j)



(k)



(l)



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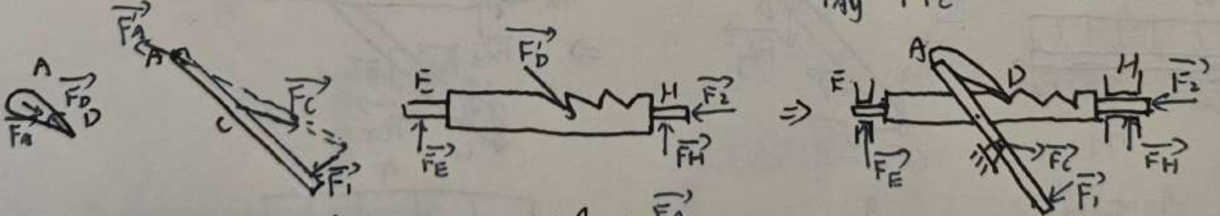
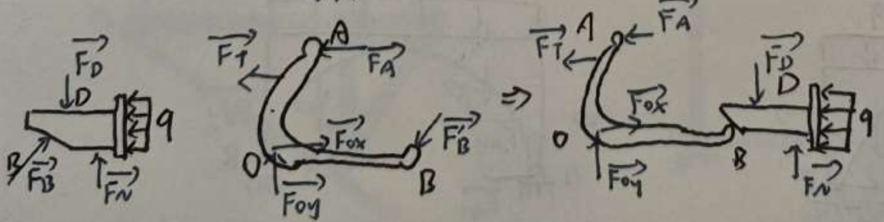
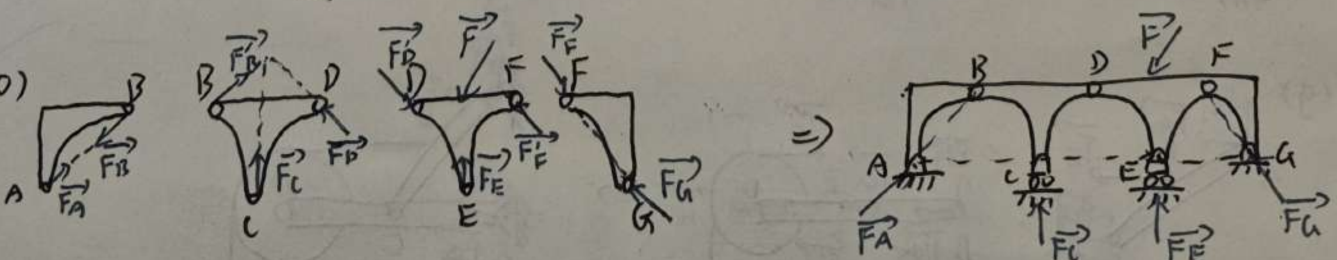


(n)



(o)



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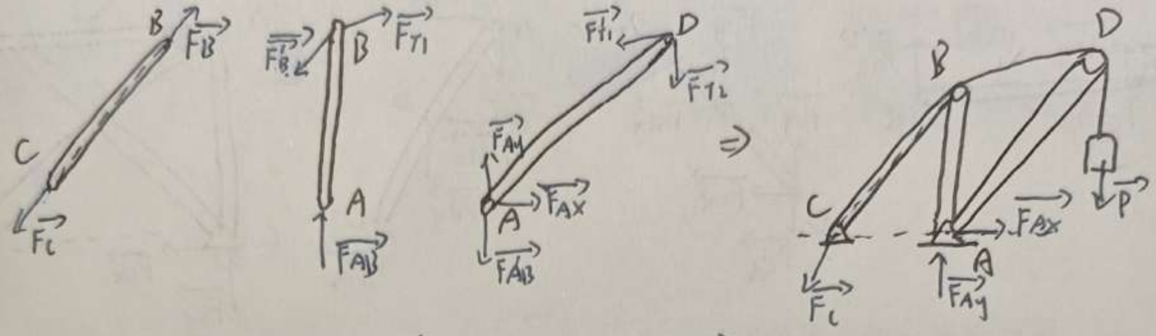
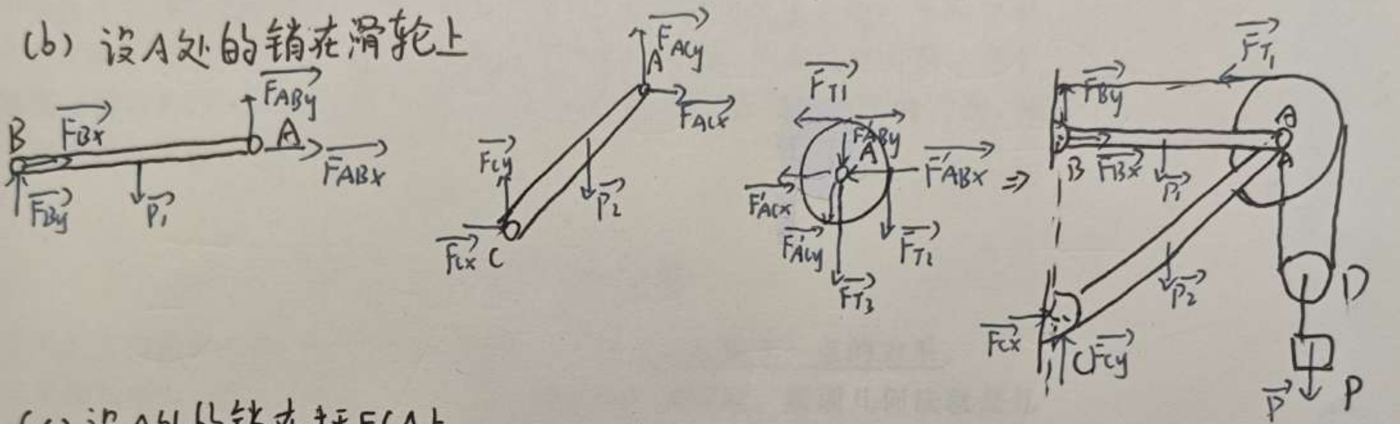
1-3

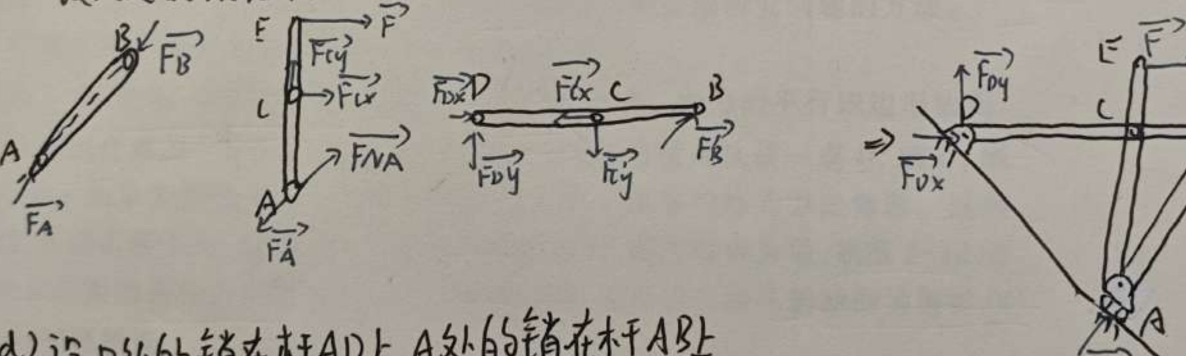
(a) 设B处的销在杆AB上，A处的销在杆AD上.



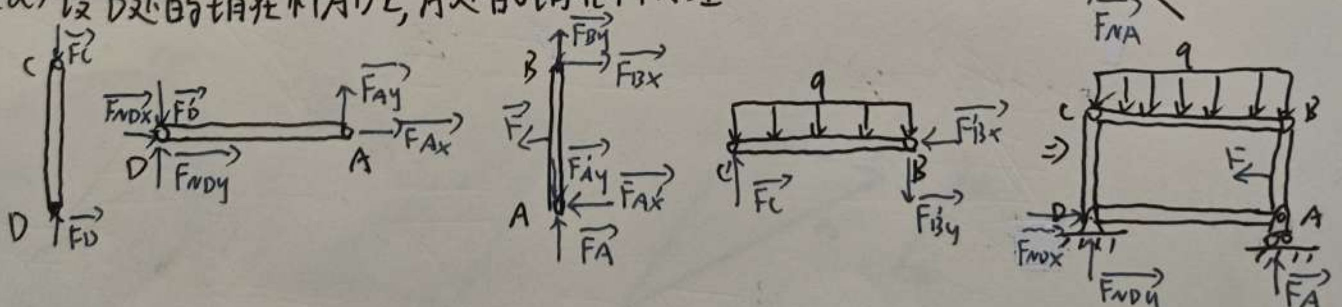
(b) 设A处的销在滑轮上



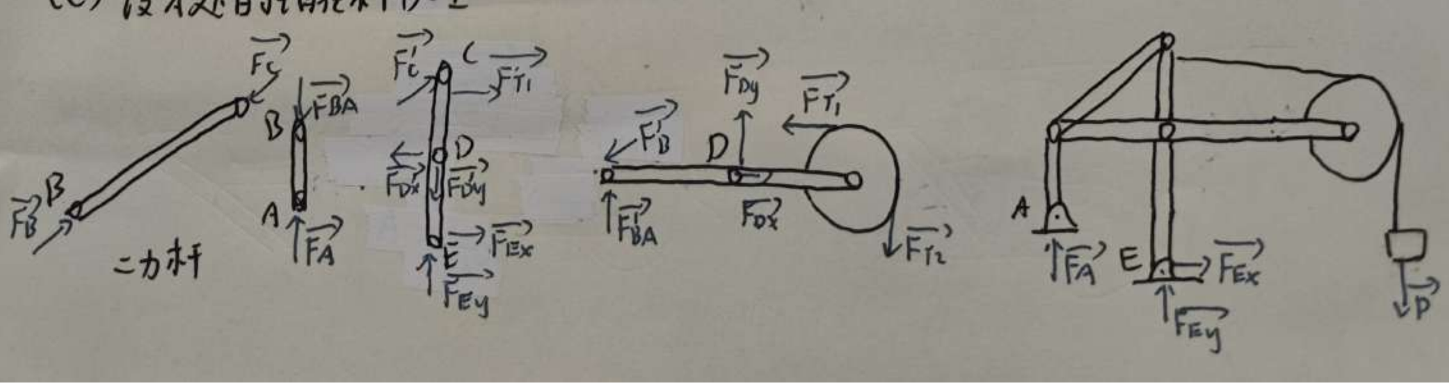
(c) 设A处的销在杆ECA上.



(d) 设D处的销在杆AD上，A处的销在杆AB上



(e) 设A处的销在杆BD上



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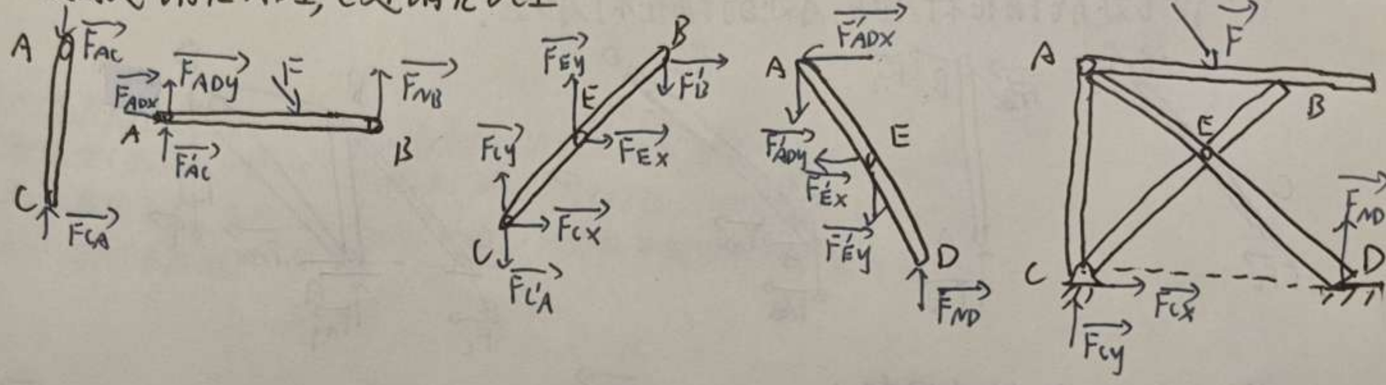
(f) 设A处销在AB上，C处销在BC上



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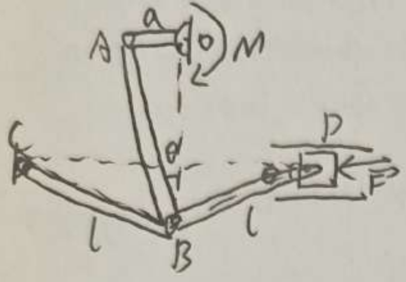
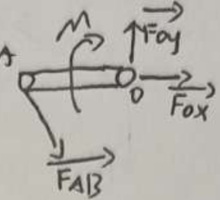
2-9 解：法一：解析法.



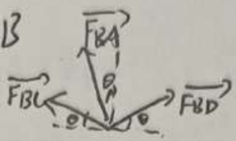
①分析杆AO



$\sum_i M_O(\vec{F_i}) = -M + F_{AB}a\cos\theta = 0$

$\Rightarrow F_{AB} = \frac{M}{a\cos\theta}$

②分析销钉B

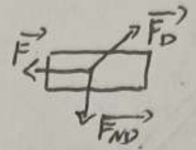


平衡方程：$\sum_i F_{xi} = F_{BA}\sin\theta + F_{BC}\cos\theta - F_{BD}\cos\theta = 0$

$\sum_i F_{yi} = F_{BA}\cos\theta + F_{BC}\sin\theta + F_{BD}\sin\theta = 0$

其中 $F_{BA} = F_{AB} \Rightarrow F_{BD} = -\frac{F_{AB}}{\tan2\theta}$

③分析物体D



其中 $F_D = -F_{BD} = \frac{M}{a\cos\theta\tan2\theta}$

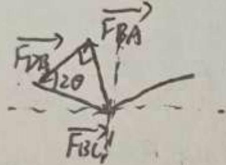
$\sum_i F_{xi} = F_D\cos\theta - F = 0$

$\Rightarrow M = Fa\tan2\theta$,

法二：解析+几何法

①分析物体D. $F_{DB}\cos\theta = F$.

②分析销钉B



由几何关系，$F_{BA} = F_{BD}\tan2\theta$

③分析杆AD, $\sum_i M_o(\vec{F_i}) = -M + F_{AB}a\cos\theta = 0 \Rightarrow M = Fa\tan2\theta$

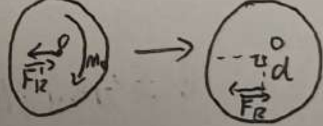
2-12 解：(1) ①先求主矢 $\Sigma F_x = F_1\cos45° - F_4 - F_2 = -150N$, $\Sigma F_y = F_1\sin45° - F_3 = 0$

主矢 $F_R' = 150N$，作用线经过点O，方向水平向左

②再求主矩. $M_O = F_2\cdot30 + F_3\cdot50 - F_4\cdot30 + F_1\cdot0 - M = -900\,Nmm$

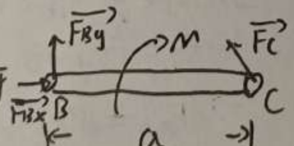
(2) 进一步合成，$d = \left|\frac{M_O}{F_R'}\right| = 6mm$

故力系合力 $F_R = 150N$，方向水平向左，作用线 $y=-6$



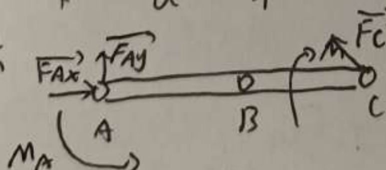
2-23

解：(a) ①先分析BC杆



$\sum_i M_B(\vec{F_i}) = -M + F_C a\cos\theta = 0 \Rightarrow F_C = \frac{M}{a\cos\theta}$

②再分析整体



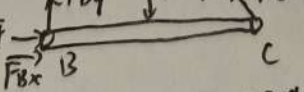
平衡方程：$\sum_i M_A(\vec{F_i}) = M_A - M + F_C 2a\cos\theta = 0 \Rightarrow M_A = -M$

$\Sigma F_x = F_{Ax} - F_C\sin\theta = 0 \Rightarrow F_{Ax} = \frac{M}{a}\tan\theta$

$\Sigma F_y = F_{Ay} + F_C\cos\theta = 0 \Rightarrow F_{Ay} = -\frac{M}{a}$

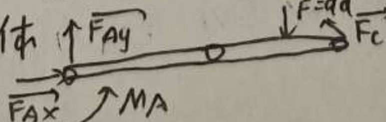
综上，$F_C = \frac{M}{a\cos\theta}$；$M_A = -M$；$\vec{F_A} = \frac{M}{a}\tan\theta\vec{i} - \frac{M}{a}\vec{j}$

(b) ①先分析BC杆



$\sum_i M_B(\vec{F_i}) = -qa\cdot\frac{a}{2} + F_C a\cos\theta = 0 \Rightarrow F_C = \frac{qa}{2\cos\theta}$

②再分析整体



平衡方程：$\sum_i M_A(\vec{F_i}) = M_A - qa\cdot\frac{3}{2}a + F_C\cdot2a\cos\theta = 0$

$\Rightarrow M_A = \frac{1}{2}qa^2$

$\Sigma F_x = F_{Ax} - F_C\sin\theta = 0 \Rightarrow F_{Ax} = \frac{1}{2}qa\tan\theta$

$\Sigma F_y = F_{Ay} + F_C\cos\theta - qa = 0 \Rightarrow F_{Ay} = \frac{1}{2}qa$

故 $\vec{F_A} = \frac{1}{2}qa\tan\theta\vec{i} + \frac{1}{2}qa\vec{j}$

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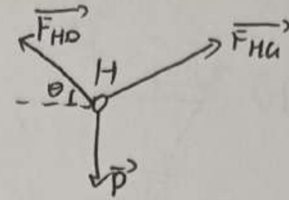
2-31 解：①先分析销钉H



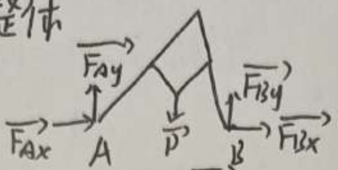
由几何关系，$\tan\theta=\frac{1}{2}$，$\sin\theta=\frac{\sqrt{5}}{5}$

平衡方程 $\Sigma F_x=F_{HD}\cos\theta-F_{HG}\cos\theta=0$

$\Sigma F_y=F_{HD}\sin\theta+F_{HG}\sin\theta-P=0$

$\Rightarrow F_{HG}=F_{HD}=\frac{\sqrt{5}}{2}P=500\sqrt{5}N$

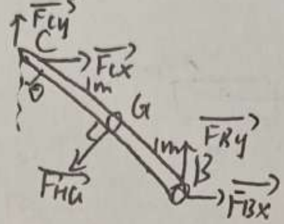
②再分析整体



平衡方程 $\Sigma M_A(\vec{F_i})=\Sigma M_B(\vec{F_i})=0 \Rightarrow F_{Ay}=F_{By}=\frac{P}{2}=500N$

$\Sigma F_x=F_{Ax}+F_{Bx}=0$ (1)

③再分析杆CGB

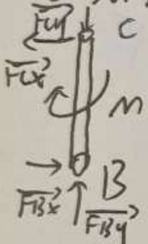


$\Sigma M_C(\vec{F_i})=-F_{By}\cdot 2\sin\theta-F_{Bx}2\cos\theta+F'_{HG}\cdot 1=0$

$\Rightarrow F_{Bx}=\frac{F'_{HG}-2\sin\theta F_{By}}{2\cos\theta}=375N$

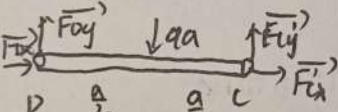
$\Rightarrow F_{Ax}=-375N$

2-36. 解：设B处的销钉位于杆DAB



①先分析杆BC $\Sigma M_B(\vec{F_i})=-M+F_{Cx}\cdot a=0 \Rightarrow F_{Cx}=\frac{M}{a}=qa$

②再分析杆CD

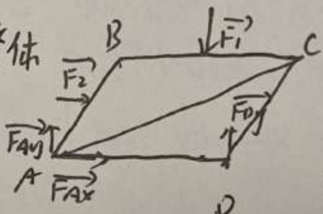


$\Sigma M_C(\vec{F_i})=qa\cdot\frac{1}{2}a-F_{Dy}\cdot a=0 \Rightarrow F_{Dy}=\frac{1}{2}qa$

$\Sigma F_x=F_{Dx}+F'_{Cx}=0 \Rightarrow F_{Dx}=-qa$

即铰链D所受为力 $\vec{F}=-qa\vec{i}+\frac{1}{2}qa\vec{j}$

2-45. 解：①先分析整体

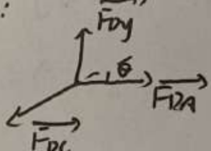


$\Sigma F_x=F_{Ax}-F_2=0 \Rightarrow F_{Ax}=-75kN$

$\Sigma M_A(\vec{F_i})=F_{Dy}\cdot 9-F_1\cdot 7.5-F_2\cdot 2=0 \Rightarrow F_{Dy}=\frac{350}{3}kN$

$\Sigma F_y=F_{Dy}+F_{Ay}-F_1=0 \Rightarrow F_{Ay}=\frac{10}{3}kN$

②再分析销钉D：



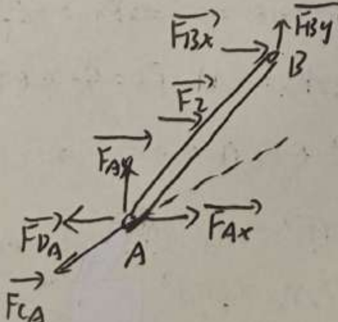
由几何关系，$\sin\theta=\frac{4}{5}$，$\cos\theta=\frac{3}{5}$

$\Sigma F_y=F_{Dy}-F_{DC}\sin\theta=0$；$\Sigma F_x=F_{DA}-F_{DC}\cos\theta=0$

$\Rightarrow F_{DA}=87.5kN$；$F_{DC}=\frac{875}{6}kN\approx 145.8kN$

设A处的销钉在杆FAB上

③分析杆FAB，未知量 F_{Bx}、F_{By}、F_{CA}



平衡方程：$\Sigma M_B(\vec{F_i})=F_2\cdot 2+F_{Ax}\cdot 4-F_{Ay}\cdot 3-F_{DA}\cdot 4-F_{CA}\cdot\frac{9\sqrt{10}}{10}=0$

$\Rightarrow F_{CA}=-179.2kN$

答：F_{AD}为87.5kN压力 F_{CA}为179.2kN拉力

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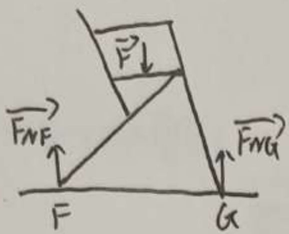
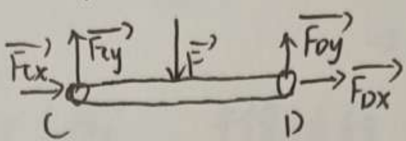
2-54 解：①先分析整体 $\sum F_y = F_{NF}+F_{NG}-F=0$；$\sum M_G(\vec{F_i}) = F\cdot 4a - F_{NF}\cdot 8a = 0 \Rightarrow F_{NF}=F_{NG}=\frac{1}{2}F$

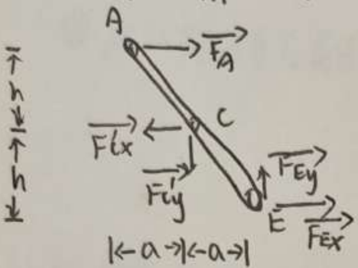


②再分析杆CD，设D处的销钉在杆BG上



由①理，可知 $F_{Dy}=F_{Cy}=\frac{1}{2}F$

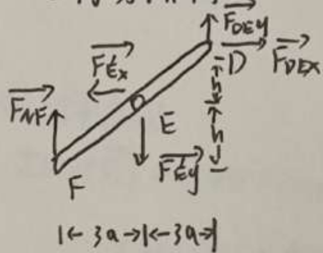
③再分析杆ACE



$\sum F_x = F_A + F_{Ex} - F'_{Cx} = 0$；$\sum F_y = F_{Ey} - F'_{Cy} = 0 \Rightarrow F_{Ey} = \frac{1}{2}F$

$\sum M_C(\vec{F_i}) = -F_A h + F_{Ex} h + F_{Ey} a = 0$

④再分析杆FED

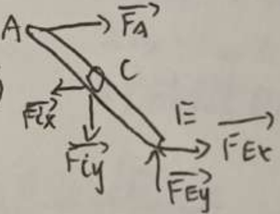


$\sum M_D(\vec{F_i}) = -F'_{Ex}\cdot h + F'_{Ey}\cdot 3a - F_{NF}\cdot 6a = 0$

$\Rightarrow F_{Ex} = F'_{Ex} = -\frac{3Fa}{2h}$，代入上式，有

$F_A = -\frac{Fa}{h}$，$F_{Cx} = -\frac{5Fa}{2h}$

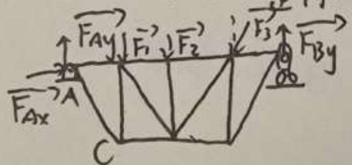
综上，杆ACE上受力为



$F_A = -\frac{a}{h}F$　$F'_{Cx} = -\frac{5a}{2h}F$，$F'_{Cy}=\frac{1}{2}F$

$F_{Ex} = -\frac{3a}{2h}F$，$F_{Ey}=\frac{1}{2}F$

2-58 解：①先分析整体



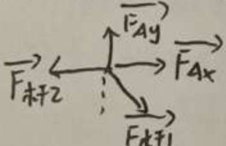
平衡方程：$\sum F_x = F_{Ax} - F_3\sin 30^\circ = 0$

$\sum F_y = F_{Ay}+F_{By}-F_1-F_2-F_3\cos 30^\circ=0$

$\sum M_A(\vec{F_i}) = -F_1 a - 2F_2 a - 3F_3 a\cos 30^\circ + 4F_{By} a = 0$

$\Rightarrow F_{Ax}=10kN$，$F_{Ay}=21.82kN$，$F_{By}=25.50kN$

②再分析A处销钉

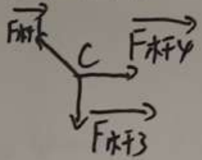


$\sum F_x = F_{Ax} + \frac{\sqrt{2}}{2}F_{杆1} - F_{杆2}=0$

$\sum F_y = F_{Ay} - \frac{\sqrt{2}}{2}F_{杆1}=0$

$\Rightarrow F_{杆1}=30.83kN$；$F_{杆2}=31.82kN$

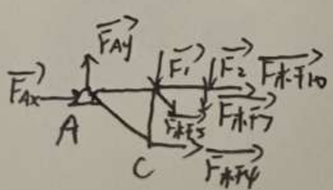
③再分析C点，如图



同理，$F_{杆4}=21.82kN$

$F_{杆3}=21.82kN$

④取截面4-5-7-10取左半部分分析。



平衡方程：$\sum F_y = F_{Ay}-F_1-F_2-F_{杆7}-\frac{\sqrt{2}}{2}F_{杆5}=0$

$\sum F_x = F_{Ax}+F_{杆10}+F_{杆4}+\frac{\sqrt{2}}{2}F_{杆5}=0$

$\sum M_A(\vec{F_i}) = -F_1 a - F_2 2a - F_{杆7}2a + F_{杆4}a - \frac{\sqrt{2}}{2}F_{杆5}a=0$

$\Rightarrow F_{杆5}=16.71kN$ 拉力；$F_{杆7}=-20.0kN$ 压力；$F_{杆10}=-43.7kN$ 压力

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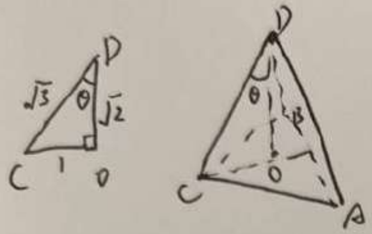
3-2 解：四个球的质心组成一正四面体

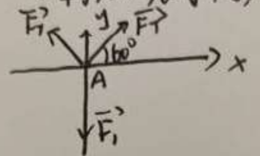


过D作DO⊥面ABC于点O，连接CO，由几何关系知，$\sin\theta=\frac{\sqrt{3}}{3}$，$\cos\theta=\frac{\sqrt{6}}{3}$

①先研究球D，设两球之间压力大小为F_N，有

z方向：$F_{Nz}=F_N\cos\theta$，$3F_{Nz}=P \Rightarrow F_N=\frac{\sqrt{6}}{6}P$

②再研究球A，受力如图



其中F_1是F_N在水平面上的投影，$F_1=F_N\sin\theta=\frac{\sqrt{2}}{6}P$

$\sum F_y=F_1-2F_T\sin60^\circ=0 \Rightarrow F_T=\frac{\sqrt{6}}{18}P\approx0.136P$

3-9 解：①先求主矢：$F'_{Rx}=\sum F_x=F_4-F_1=0$　$F'_{Ry}=\sum F_y=F_2-F_5=0$，$F'_{Rz}=F_3-F_6=0 \Rightarrow \vec{F'_R}=\vec{0}$

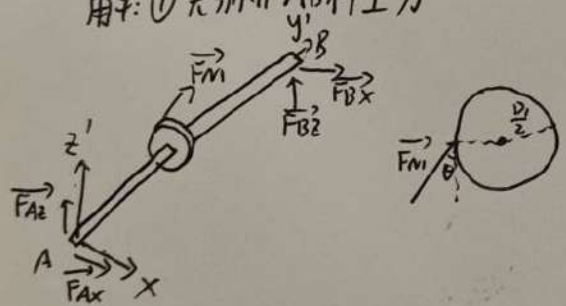
②再求主矩：$M_{Ox}=\sum M_x=-F_6a-F_2a=-40N\cdot m$；$M_{Oy}=\sum M_y=-F_1a-F_3a=-40N\cdot m$

$M_{Oz}=\sum M_z=F_5a-F_4a=0$

$\Rightarrow \vec{M_O}=-40\vec{i}-40\vec{j}\ (N\cdot m)$

原力系向O点简化为一力偶，其矢矩为$\vec{M_O}$

3-16 解：①先分析AB杆上力



平衡方程：$\sum F_x=F_{Ax}+F_{Bx}+F_{N1}\sin20^\circ=0$

$\sum F_y=0=0$

$\sum F_z=F_{Az}+F_{Bz}+F_{N1}\cos20^\circ=0$

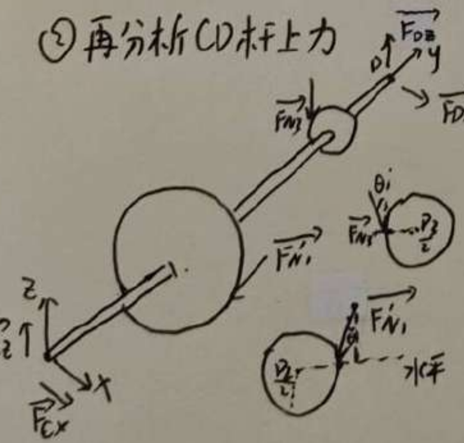
$\sum M_{y'}=F_{N1}\cos20^\circ\cdot\frac{D_1}{2}-M_1=0$

$\sum M_x=F_{N1}\cos20^\circ\cdot0.2+F_{Bz}\cdot0.58=0$

$\sum M_{z'}=-F_{N1}\sin20^\circ\cdot0.2-F_{Bx}\cdot0.58=0$

$\Rightarrow F_{N1}=9.272kN$，$F_{Ax}=-2.078kN$，$F_{Az}=-5.709kN$，$F_{Bx}=-1.093kN$，$F_{Bz}=-3.004kN$

②再分析CD杆上力



平衡方程：$\sum F_x=F_{Cx}+F_{Dx}+F_{N3}\sin20^\circ-F'_{N1}\sin20^\circ=0$

$\sum F_y=0=0$

$\sum F_z=F_{Cz}+F_{Dz}-F_{N3}\cos20^\circ-F'_{N1}\cos20^\circ=0$

$\sum M_x=-F'_{N1}\cos20^\circ\cdot0.2-F_3\cos20^\circ\cdot0.435+F_{Dz}\cdot0.58=0$

$\sum M_y=F'_{N1}\cdot\cos20^\circ\cdot\frac{D_2}{2}-F_{N3}\cos20^\circ\cdot\frac{D_3}{2}=0$

$\sum M_z=F'_{N1}\sin20^\circ\cdot0.2-F_{N3}\sin20^\circ\cdot0.435-F_{Dx}\cdot0.58=0$

$\Rightarrow F_{N3}=28.724kN$，$F_{Cx}=-0.379kN$，$F_{Cz}=12.456kN$，$F_{Dx}=-6.275kN$，$F_{Dz}=23.248kN$

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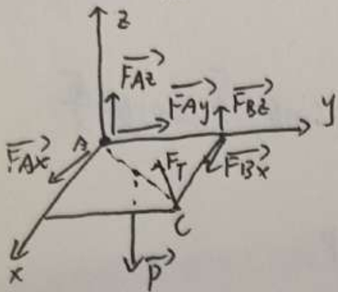
3-18 解：CE可视为二力杆，设AD=1，则CD=$\sqrt{3}$，分析板，如图



$F_{Tz}=F_T\sin30°=\frac{1}{2}F_T$,

$F_{Ty}=F_T\cos30°\sin30°=\frac{\sqrt{3}}{4}F_T$

$F_{Tx}=F_T\cos30°\cos30°=\frac{3}{4}F_T$

平衡方程 $\Sigma F_x=F_{Ax}+F_{Bx}-F_{Tx}=0$

$\Sigma F_y=F_{Ay}-F_{Ty}=F_{Ay}-\frac{3}{4}F_T=0$

$\Sigma F_z=F_{Az}+F_{Bz}+F_{Tz}-P=0$

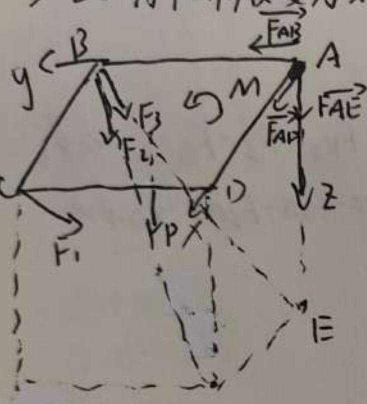
$\Sigma M_x=F_{Bz}\cdot\sqrt{3}+F_{Tz}\cdot\sqrt{3}-\frac{\sqrt{3}}{2}\cdot P=0$

$\Sigma M_y=-F_{Tz}\cdot1+\frac{1}{2}P=0$

$\Sigma M_z=-F_{Bx}\cdot\sqrt{3}-F_{Ty}\cdot1+F_{Tx}\cdot\sqrt{3}=0$

解得：$F_T=200N$，$F_{Ax}=86.6N$，$F_{Ay}=150N$，$F_{Az}=100N$，$F_{Bx}=F_{Bz}=0$

3-22 解：板受力如图



平衡方程

$M_{AB}=-\frac{\sqrt{2}}{2}F_1a-\frac{1}{2}aP=0$ 得 $F_1=-\frac{\sqrt{2}}{2}P$

$M_{AE}=-M-\frac{\sqrt{3}}{3}F_2\cdot a-\frac{\sqrt{2}}{2}F_1\cdot a=0$ 得 $F_2=\frac{\sqrt{2}}{2}P-\frac{\sqrt{3}}{a}M$

$M_{AD}=\frac{\sqrt{2}}{2}F_1a+\frac{a}{2}P+\frac{\sqrt{3}}{3}F_2a+\frac{\sqrt{2}}{2}F_3a=0$ 得 $F_3=\frac{\sqrt{2}}{a}M-\frac{\sqrt{2}}{2}P$

$\Sigma F_x=F_{AD}+\frac{\sqrt{3}}{3}F_2=0$ 得 $F_{AD}=\frac{M}{a}-\frac{P}{2}$

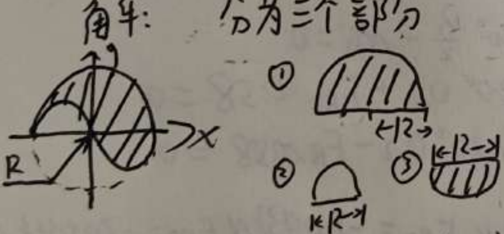
$\Sigma F_y=F_{AB}-\frac{\sqrt{2}}{2}F_1-\frac{\sqrt{2}}{2}F_3-\frac{\sqrt{3}}{3}F_2=0$ 得 $F_{AB}=-\frac{P}{2}$

$\Sigma F_z=F_{AE}+P+\frac{\sqrt{2}}{2}F_1+\frac{\sqrt{2}}{2}F_3+\frac{\sqrt{3}}{3}F_2=0$

即 $F_{AE}+P+F_{AB}=0$ 得 $F_{AE}=-\frac{P}{2}$

3-26. 已知半圆 $x_c=\frac{4R}{3\pi}$

解：分为三个部分



① $S=\frac{1}{2}\pi R^2$，$x_c=0$，$y_c=\frac{4R}{3\pi}$

② $S=-\frac{1}{8}\pi R^2$，$x_c=-\frac{R}{2}$，$y_c=\frac{2R}{3\pi}$

③ $S=\frac{1}{8}\pi R^2$，$x_c=\frac{R}{2}$，$y_c=-\frac{2R}{3\pi}$

整体：$x_c=\frac{\Sigma x_iS_i}{S_{总}}=\frac{\frac{1}{2}\pi R^2\cdot0+\frac{1}{8}\pi R^2\cdot\frac{R}{2}+\frac{1}{8}\pi R^2\cdot\frac{R}{2}}{\frac{1}{2}\pi R^2}=\frac{R}{4}$

$y_c=\frac{\Sigma y_iS_i}{S_{总}}=\frac{4\cdot\frac{4R}{3\pi}-1\cdot\frac{2R}{3\pi}-1\cdot\frac{2R}{3\pi}}{4}=\frac{R}{\pi}$

综上，重心坐标为 $(\frac{R}{4},\frac{R}{\pi})$

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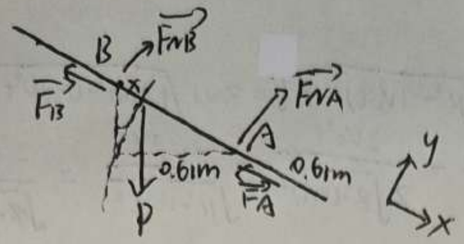
T4-5 解:



受力如图所示, P作用在杆的中点.

由几何关系, $x=\overline{AB}-0.61=\frac{0.61}{\cos 30^\circ}-0.61=0.094m$

假设能平衡, 平衡方程: $\Sigma F_y = F_{NA}+F_{NB}-\frac{\sqrt{3}}{2}P=0$

$\Sigma M_B = -\frac{\sqrt{3}}{2}P\cdot x+\frac{0.61}{\cos 30^\circ}F_{NA}=0$

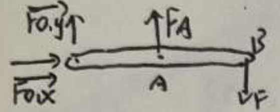
$\Rightarrow F_{NA}=30.86N,\ F_{NB}=200.37N$

$\Sigma F_x = P_x-(F_A+F_B)=0$

其中 $P_x=133.5N$, $(F_A+F_B)_{max}=f_s(F_{NA}+F_{NB})=115.61N$

故 $F_A+F_B<P_x$, 假设不成立, 杆不能平衡

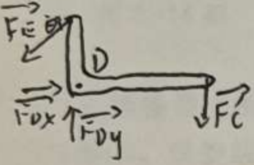
T4-12. 解: ① 先研究杆 O_1AB, AC为二力杆



$\Sigma M_O = 0.5F_A-0.75F_B=0$

$\Rightarrow F_A=\frac{3}{2}F_B=300N$

② 再研究杆EDC



$\Sigma M_D=F_E\cos\theta\cdot 0.25-F_C'\cdot 0.5=0$

$\Rightarrow F_E\cos\theta=2F_C'=600N$

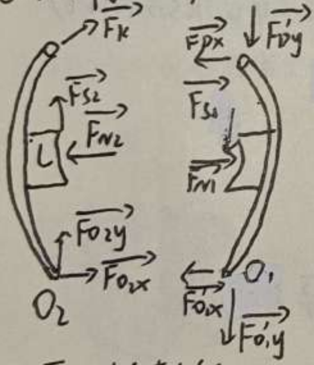
$\Sigma F_x=F_{Dx}-F_E\cos\theta=0$

$\Rightarrow F_{Dx}=600N$

由几何关系知 $\tan\theta=\frac{1}{2},\ \cos\theta=\frac{2\sqrt{5}}{5},\ \sin\theta=\frac{\sqrt{5}}{5}$

$\Rightarrow F_E=670.82N$,

③ ~~很~~再研究曲杆 O_2LK 与 O_1D, 如图,



假设鼓轮顺时针转

$\Sigma M_{O_2}=F_{N2}-2F_K\cos\theta=0$

$\Rightarrow F_{N2}=2F_K\cos\theta=1200N$

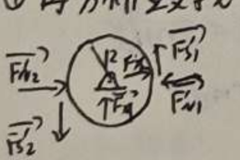
$\Sigma M_{O_1}=-F_{N1}+2F_{Dx}=0$

$F_{N1}=2F_{Dx}=1200N$

则 $F_{S2}=f_sF_{N2}=600N$

$F_{S1}=f_sF_{N1}=600N$

④ 再分析鼓轮



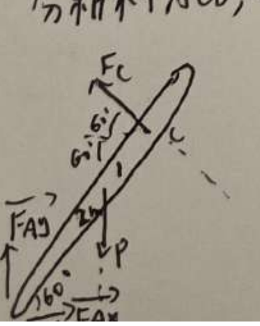
制动力矩

$M=F_{S1}'\cdot R+F_{S2}'R=300N\cdot m$

T4-25. 解: 由于BC为二力杆, 受力须沿杆

故B处C处摩擦角 φ_B、φ_C 不小于 30°

分析杆ACD, 平衡方程



$\Sigma M_A=-P\cdot 1m+F_C\cdot\frac{3}{2}\sqrt{3}m=0$

$\Rightarrow F_C=\frac{2\sqrt{3}}{9}P$

$\Sigma F_x=F_{Ax}-F_C\cos 60^\circ=0$

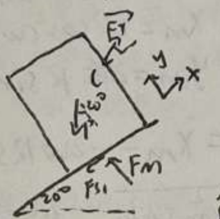
$F_{Ax}=\frac{\sqrt{3}}{9}P$

$\Sigma F_y=F_{Ay}+F_C\sin 60^\circ-P=0\Rightarrow F_{Ay}=\frac{2}{3}P$

摩擦角 $\varphi_A=\arctan\frac{F_{Ax}}{F_{Ay}}=16.1^\circ$

T4-15 解:

(1) 若 F_T 较大, 箱子上滑, 受力如下



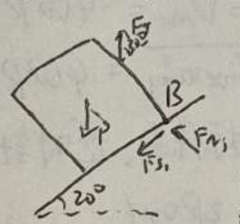
$\Sigma F_x=F_T\cos 30^\circ-F_{S1}-P\sin 20^\circ=0$

$\Sigma F_y=F_T\sin 30^\circ+F_{N1}-P\cos 20^\circ=0$

且有 $F_{S1}=f_sF_{N1}$

解得 $P=F_T=109.72kN$

(2) 若 F_T 较大, 箱子向上翻倒, 受力如下



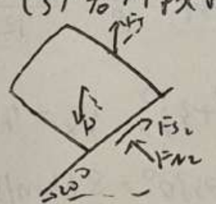
$\Sigma M_B=-F_T\cos 30^\circ a+P\sin 20^\circ\frac{h}{2}+P\cos 20^\circ\frac{b}{2}=0$

解得 $P=F_T=104.2kN$

由于 $109.72kN>104.2kN$

重力最大为 $104.2kN$

(3) 若 F_T 较小, 箱子下滑, 如下



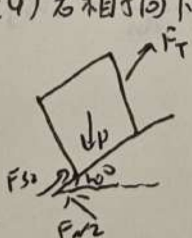
$\Sigma F_x=F_T\cos 30^\circ+F_{S2}-P\sin 20^\circ=0$

$\Sigma F_y=F_T\sin 30^\circ+F_{N2}-P\cos 20^\circ=0$

且有 $F_{S2}=f_sF_{N2}$

解得 $P=F_T=40.21kN$

(4) 若箱子向下翻倒, 如下



$\Sigma M_A=+P\sin 20^\circ\frac{h}{2}-P\cos 20^\circ\frac{b}{2}+F_T\sin 30^\circ b-F_T\cos 30^\circ a=0$

$\Rightarrow F_T=-24.12kN$ 不成立舍去

综上, 重物的重量为 $40.21kN\le P\le 104.2kN$

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5-3解：设杆AB长为l

(1) B相对于地面

X：$x_B=0$，$v_{Bx}=\dot{x}_B=0$；Y：$y_B=\sqrt{R^2-(v_0t)^2}+l=0.01\sqrt{(8m)^2-(1m/s)^2t^2}+l$ $(0\le t\le 8s)$

$$v_{By}=\dot{y}_B=-\frac{2v_0^2t}{2\sqrt{R^2-(v_0t)^2}}=\frac{-v_0^2t}{\sqrt{R^2-v_0^2t^2}}=\frac{-0.01(m/s)^2t}{\sqrt{(8m)^2-(1m/s)^2t^2}}\quad(0\le t\le 8s)$$

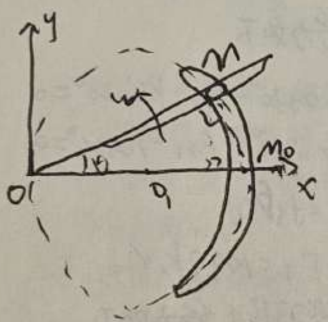
(2) B相对于凸轮

X：$x_B=v_0t=(0.01m/s)t$　$v_B=v_0=0.01m/s$

Y：与(1)中情况一致，由于两个参考系在Y方向上无相对运动

$$y_B=0.01\sqrt{(8m)^2-(1m/s)^2t^2}+l,\quad v_{By}=\frac{-0.01(m/s^2)t}{\sqrt{(8m)^2-(1m/s)^2t^2}}\quad(0\le t\le 8s)$$

5-7解：(1)直角坐标法：取O为坐标原点，建立如图坐标系



$x_m=2R\cos^2\theta$，$y_m=2R\cos\theta\sin\theta$. 其中$\theta=\omega t$

$$\Rightarrow\begin{cases}x_m=2R\cos^2(\omega t)\\ y_m=\cancel{2R}\,R\sin(2\omega t)\end{cases}$$

$v_{mx}=\dot{x}_m=-2\omega R\sin(2\omega t)$　$v_{my}=\dot{y}_m=2\omega R\cos(2\omega t)$

$v=\sqrt{v_{mx}^2+v_{my}^2}=2\omega R$

$a_{mx}=\dot{v}_{mx}=-4\omega^2R\cos(2\omega t)$，$a_{my}=\dot{v}_{my}=-4\omega^2R\sin(2\omega t)$

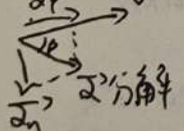
$a=\sqrt{a_{mx}^2+a_{my}^2}=4\omega^2R$

(2)自然法 取M_0为弧坐标原点，逆时针为正向

$s_m=2R\cdot\theta=2R\omega t$　　$v=\dot{s}_m=2R\omega$

$a_\tau=\frac{dv}{dt}=0$，$a_n=\frac{v^2}{\rho}=\frac{4R^2\omega^2}{R}=4R\omega^2\Rightarrow a=\sqrt{a_n^2+a_\tau^2}=4R\omega^2$

T5-10 解：由题，$v=4\vec{i}+3\vec{j}$，$v_x=4m/s$，$v_y=3m/s$，$v=\sqrt{v_x^2+v_y^2+v_z^2}=5m/s$



$a_\tau=a\cos\beta=10\cos30^\circ=8.66m/s^2$，$a_n=a\sin\beta=10\sin30^\circ=5m/s^2$

由$a_n=\frac{v^2}{\rho}$知$\rho=\frac{v^2}{a_n}=5m$

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题6-9. 解: 设B点距轴AA'为l_1, C点距l_2, 设I轮动角速度ω_1,

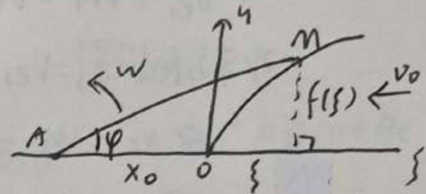
B点速度 $v_B=\omega_1\frac{D}{2}$, $l_1=\frac{d}{2}\sin(45^\circ-\theta)$

则 $\omega_{AA'}=\frac{v_B}{l}=\frac{\omega_1 D}{d\sin(45^\circ-\theta)}$

又 $l_2=\frac{d}{2}\sin(45^\circ+\theta)$ 有 $v_C=\omega_{AA'}l_2=\frac{\omega_1 D\sin(45^\circ+\theta)}{2\sin(45^\circ-\theta)}$

故 $\omega_2=\frac{v_C}{\frac{D}{2}}=\frac{\omega_1\sin(45^\circ+\theta)}{\sin(45^\circ-\theta)}=\omega_1\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\Rightarrow n_2=n_1\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$

题6-13. 解: 如图, 设O在定系Axy中坐标$(x_0,0)$



$\dot x_0=-v_0$, $\sin\varphi=\frac{f(\xi)}{l}$

$(x_0+\xi)^2+f^2(\xi)=l^2$

故 $x_0=\sqrt{l^2-f^2(\xi)}-\xi$

左右两边同时求导,得 $\dot x_0=\frac{-2f(\xi)f'(\xi)\dot\xi}{2\sqrt{l^2-f^2(\xi)}}-\dot\xi\Rightarrow\dot\xi=\frac{v_0\sqrt{l^2-f^2(\xi)}}{\sqrt{l^2-f^2(\xi)}+f(\xi)f'(\xi)}$

$\dot\varphi\cos\varphi=\frac{f'(\xi)\dot\xi}{l}$

从而 $\omega=\dot\varphi=\frac{f'(\xi)\dot\xi}{\cos\varphi\, l}=\frac{f'(\xi)}{x_0+\xi}\frac{v_0\sqrt{l^2-f^2(\xi)}}{\sqrt{l^2-f^2(\xi)}+f(\xi)f'(\xi)}=\frac{v_0f'(\xi)}{\sqrt{l^2-f^2(\xi)}+f(\xi)f'(\xi)}$

(2) M在定系Axy中坐标 $(l\cos\varphi, l\sin\varphi)$

动系Oξη中坐标 $(x-x_0, l\sin\varphi)$

设AM角速度为ω, 取水平时为时间零点, $x_0=l-v_0t$

从而动系中坐标为 $\xi=l\cos\omega t-l+v_0t$ $\eta=l\sin\omega t$ 一直在曲线$f(\xi)$上,

消去时间t,得 $\xi=\sqrt{l^2-\eta^2}-l+\frac{v_0}{\omega}\arcsin\frac{\eta}{l}$, 此即凸轮轮廓方程

题6-17 解: 由 $\vec{v_A}=200\vec{j}$ mm/s, $\vec{a_B^t}=150\vec{i}$ mm/s² 知圆盘逆时针减速转动

$\omega=\frac{v_A}{R}=2$ rad/s $\vec{\omega}$方向朝纸指向外侧

$\alpha=\frac{a_B^t}{R}=1.5$ rad/s² 方向朝纸指向内侧

C点: $v_A=200$ mm/s, $\theta=45^\circ\Rightarrow\vec{v_A}=(100\sqrt{2}\vec{i}+100\sqrt{2}\vec{j})$ mm/s

$a_C^t=150$ mm/s² $\Rightarrow\vec{a_C^t}=(-\frac{150}{\sqrt{2}}\vec{i}-\frac{150}{\sqrt{2}}\vec{j})$ mm/s²

$a_C^n=\omega^2R=400$ mm/s² $\Rightarrow\vec{a_C^n}=(-\frac{400}{\sqrt{2}}\vec{i}+\frac{400}{\sqrt{2}}\vec{j})$ mm/s²

$\vec{a_C}=\vec{a_C^n}+\vec{a_C^t}=(-\frac{550}{\sqrt{2}}\vec{i}+\frac{250}{\sqrt{2}}\vec{j})$ mm/s²

$=(-388.9\vec{i}+176.78\vec{j})$ mm/s²

psp_dada

题7-12. 解:

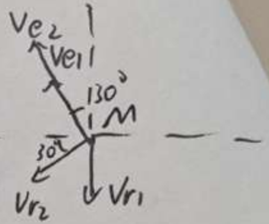
① 以M为动点，圆盘O为动系，相对运动沿圆盘导槽直线运动

牵连运动：绕轴O定轴转动

$$\vec{v_a} = \vec{v_{e1}} + \vec{v_{r1}}$$

② 以M为动点，杆OA为动系，相对运动：沿杆OA的直线运动

牵连运动：绕轴O定轴转动



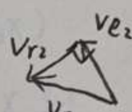
$$\vec{v_a} = \vec{v_{e2}} + \vec{v_{r2}}$$

$$\Rightarrow \vec{v_{e1}} + \vec{v_{r1}} = \vec{v_{e2}} + \vec{v_{r2}}$$

水平方向投影：$v_{e1}\sin30° = v_{e2}\sin30° + v_{r2}\cos30°$

$$\Rightarrow v_{r2} = (v_{e1} - v_{e2})\tan30° = \frac{b\tan30°}{\cos30°}(\omega_1 - \omega_2) = 0.4\,m/s$$

由 $\vec{v_a} = \vec{v_{r2}} + \vec{v_{e2}}$，其中 $v_{e2} = \frac{b}{\cos30°}\omega_2 = 0.346\,m/s$



$$v_a = \sqrt{v_{e2}^2 + v_{r2}^2} = \sqrt{0.3464^2 + 0.4^2} = 0.529\,m/s,$$

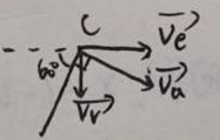
角度 $\theta = \arctan\frac{v_{e2}}{v_{r2}} = 40.9°$ 为与OA的夹角

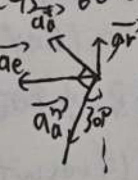
题7-19 解：以工作台为动系，滑块C为动点，相对运动：铅直方向的直线运动

牵连运动：水平方向的平移

由 $s = 0.2\sin\frac{\pi t}{6}$ 知 $v_e = \dot{s} = (\frac{\pi}{30}\cos\frac{\pi}{6}t)\,m/s$，$a_e = \ddot{s} = (-\frac{\pi^2}{180}\sin\frac{\pi}{6}t)\,m/s^2$

当 $t = 1s$，有相对速度 $v_e|_{t=1s} = \frac{\sqrt{3}}{60}\pi\,m/s$，$a_e|_{t=1s} = -\frac{\pi^2}{360}\,m/s^2$，方向如下图





$$v_r = \frac{v_e}{\sqrt{3}} = \frac{\pi}{60}\,m/s = 0.052\,m/s, \quad v_a = \frac{2}{\sqrt{3}}v_e = 0.105\,m/s, \quad \omega = \frac{v_a}{OC} = 0.175\,rad/s \text{ 顺时针}$$

	a_e	a_r	a_a^t	a_a^n
大小	✓			✓
方向	✓	✓	✓	✓

由 $\vec{a_e} + \vec{a_r} = \vec{a_a^t} + \vec{a_a^n}$ 其中 $a_a^n = \frac{v_a^2}{OC} = 0.030625\,m/s^2$

向 $\vec{a_a^n}$ 方向投影得 $a_a^n = a_e\sin30° - a_r\cos30° \Rightarrow a_r = -\frac{2}{\sqrt{3}}a_a^n + \frac{a_e}{\sqrt{3}} = -0.0195\,m/s^2$

向 $\vec{a_e}$ 方向投影，$a_e = a_a^t\cos30° + a_a^n\cos60°$

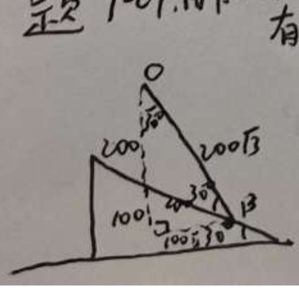
$$\Rightarrow a_a^t = \frac{1}{\cos30°}(a_e - a_a^n\cos60°) = 0.01398\,m/s^2$$

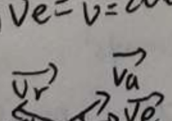
$$\alpha = \frac{a_a^t}{OC} = 0.0233\,rad/s^2，\text{逆时针方向}$$

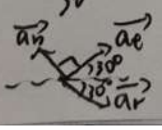
题7-24 解：几何关系如图：设杆OB上B点为动点，尖劈为动系，相对运动：沿斜面直线运动

牵连运动：平移

有 $v_e = v = 200\,mm/s$，$a_e = 0$







由 $\vec{v_a} = \vec{v_r} + \vec{v_e}$ 知 $v_a = \frac{v_e}{\sqrt{3}} = \frac{200}{\sqrt{3}}\,mm/s$，$\omega = \frac{v_a}{r} = \frac{1}{3}\,rad/s$

由 $\vec{a_r} = \vec{a_n} + \vec{a_t}$ 知 $a_t = \frac{1}{\sqrt{3}}a_n$，其中 $a_n = \omega^2 r = \frac{200\sqrt{3}}{9}\,mm/s^2$

$$\Rightarrow a_t = \frac{200}{9}\,mm/s^2 = 0.022\,m/s^2 \quad \alpha = \frac{a_t}{r} = \frac{\sqrt{3}}{27}\,rad/s^2$$

psp_dada

第7题 解：如图，取圆环M为动点，杆OA为动系

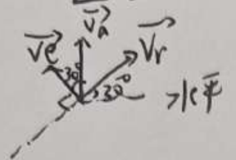
绝对运动：沿圆环圆周运动；相对运动：沿杆直线运动

牵连运动：定轴转动

速度分析 $\vec{v_a} = \vec{v_e} + \vec{v_r}$ ⇒ $v_a = \frac{4\sqrt{3}}{3}\omega R$

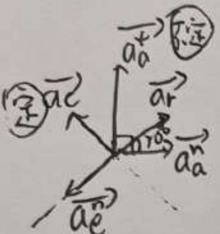
大小 ? $\omega 2R$? $v_r = \frac{2\sqrt{3}}{3}\omega R$

✓ ✓ ✓



加速度分析：匀角速 ⇒ $\vec{a_e^t}=0$，$a_C = 2\omega v_r = \frac{4\sqrt{3}}{3}\omega^2 R$；$a_e^n = \omega^2 2R$

$a_a^n = \frac{v_a^2}{R} = \frac{16}{3}\omega^2 R$



$\vec{a_a^t} + \vec{a_a^n} = \vec{a_e^n} + \vec{a_r} + \vec{a_C}$

? $\frac{16}{3}\omega^2 R$ $\omega^2 2R$? $\frac{4\sqrt{3}}{3}\omega^2 R$

✓ ✓ ✓ ✓ ✓

要求 a_a^t，沿 $\vec{a_C}$ 方向分解，有：$a_a^t\cos30° - a_a^n\sin30° = 0+0+a_C$

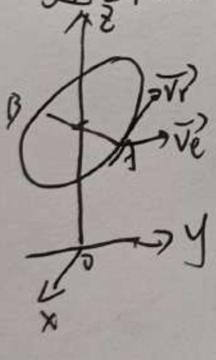
⇒ $a_a^t = \frac{2}{\sqrt{3}}\left(\frac{1}{2}a_a^n + a_C\right) = \frac{16\sqrt{3}+24}{9}\omega^2 R \approx 5.746\,\omega^2 R$

⇒ $a_a = \sqrt{a_a^{t2} + a_a^{n2}} \approx 7.840\,\omega^2 R$

题7-36 解：设动点为A，动系为铅垂轴 绝对：复杂曲线运动；相对：v_r的圆周运动

牵连：ω的定轴转动

建坐标系有 $\vec{v_r} = (0, \frac{v_r}{2}, \frac{\sqrt{3}v_r}{2})$，$\vec{v_e} = (0, \omega R, 0)$



$\vec{v_a} = \vec{v_r} + \vec{v_e} = (0, \frac{v_r}{2} + \omega R, \frac{\sqrt{3}}{2}v_r)$

⇒ $v_a = \sqrt{(\frac{v_r}{2} + \omega R)^2 + (\frac{\sqrt{3}}{2}v_r)^2} = \sqrt{v_r^2 + \omega^2 R^2 + v_r \omega R}$

加速度：$a_r^n = \frac{v_r^2}{R}$，$a_e^n = \omega^2 R$，~~ae~~ $a_C = 2\omega v_r \sin30° = \omega v_r$

$\vec{a_a} = \vec{a_e^n} + \vec{a_e^t} + \vec{a_r^n} + \vec{a_r^t} + \vec{a_C}$

? $\omega^2 R$ 0 $\frac{v_r^2}{R}$ 0 ωv_r

? ✓ ✓ ✓

$a_a = \omega^2 R + \frac{v_r^2}{R} + \omega v_r$

方向从A指向B.

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T8-6. 解：$v_A = \omega_{OA} OA = 0.2 m/s$，取A为基点，$\vec{v_B} = \vec{v_A} + \vec{v_{BA}}$，

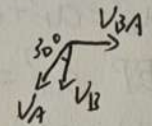
由几何关系可知，$v_{BA} = v_A \tan\varphi = 0.2\tan 30° = 0.115 m/s$，$AB = 0.05 + \frac{0.1}{\sqrt{3}} \approx 0.1077 m$

故 $\omega_{ABD} = \frac{v_{BA}}{AB} = 1.07 rad/s$

又有 $\vec{v_D} = \vec{v_A} + \vec{v_{DA}}$ 故 $v_D = v_A + \omega_{ABD} \cdot AD = 0.253 m/s$

T8-10 解：杆AO_1、BO_1 轮I 定轴转动，AB平面运动

$v_A = AO_1 \cdot \omega = 4.5 m/s$，以A为基点，B处



$\vec{v_B} = \vec{v_A} + \vec{v_{BA}}$

$\Rightarrow v_B = 2.25\sqrt{3} m/s$，$v_{BA} = 2.25 m/s$

$\Rightarrow \omega_{AB} = \frac{v_{BA}}{BA} = 1.5 rad/s$，$\omega_{O_1B} = \frac{v_B}{O_1B} = 3.75 rad/s$，设E为两齿轮的接触点

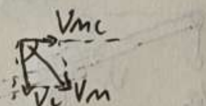
$v_E = v_B - \omega_{AB} r_2 = 1.8\sqrt{3}$ $\Rightarrow \omega_2 = \frac{v_E}{r_2} = 6 rad/s$

T8-17 解：

杆ACB作平移，则 $v_C = v_A = OA\,\omega = 0.08 m/s$　设齿轮A.C接触点为D.D'

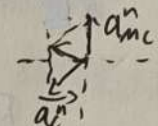
$\vec{v_D} = \vec{v_A} + \vec{v_{DA}}$，$\vec{v_{D'}} = \vec{v_C} + \vec{v_{D'C}}$ 且 $\vec{v_D} = \vec{v_{D'}}$，$\vec{v_A} = \vec{v_C}$ $\Rightarrow v_{DA} = v_{D'C}$

即 $r \cdot \omega_A = r\omega_C \Rightarrow \omega_C = \omega_A = 0.2 rad/s$，取C为基点，



$\vec{v_m} = \vec{v_C} + \vec{v_{mc}}$，$v_{mc} = \omega_C \cdot CM = 0.02 m/s$

$v_m = \sqrt{v_{mc}^2 + v_c^2 - 2v_{mc}v_c\cos 150°} = 0.0978 m/s$

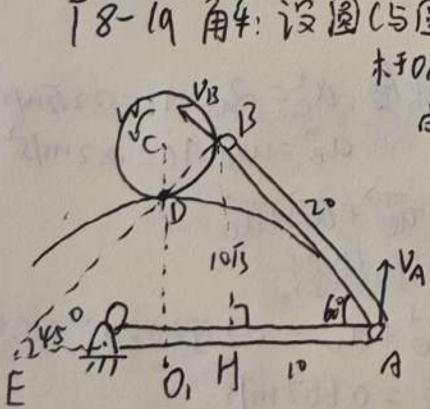


$\vec{a_m} = \vec{a_c} + \vec{a_{mc}} = \vec{a^n_c} + \vec{a^n_{mc}}$

其中 $a^n_c = \omega^2 O_1A = 0.016 m/s^2$，$a^n_{mc} = \omega_c^2 CM = 0.004 m/s^2$

$a_m = \sqrt{a_c^{n2} + a_{mc}^{n2} - 2a^n_c a^n_{mc}\cos(30°)} = 0.0127 m/s^2$

T8-19 解：设圆C与圆弧接触点D，圆C做纯滚动，速度瞬心为点D



杆OA作定轴转动，转轴为O，杆AB作平面运动，速度瞬心为点E，

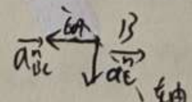
由几何关系得 $BE = \sqrt{2}BH = 10\sqrt{6} cm$，$EA = (10 + 10\sqrt{3}) cm$，从而 $v_C = \omega \cdot DC = r\omega$

$v_B = \omega \cdot DB = 10\sqrt{2} m/s$，$\omega_{AB} = \frac{v_B}{BE} = \frac{1}{\sqrt{3}} rad/s$，$v_A = \omega_{AB} \cdot EA = (10 + \frac{10}{\sqrt{3}}) cm/s$

从而 $\omega_{OA} = \frac{v_A}{OA} = (\frac{1}{2} + \frac{1}{2\sqrt{3}}) rad/s \approx 0.789 rad/s$

加速度：先求C点加速度，$a^n_C = \frac{v_C^2}{4r} = \frac{r\omega^2}{4} = 5 cm/s^2$　$a^t_C = 0$

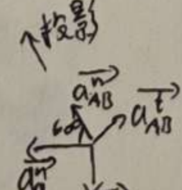
再求B点加速度 $\vec{a_B} = \vec{a_C} + \vec{a_{BC}} = \vec{a^n_C} + \vec{a^n_{BC}}$



$\Rightarrow \vec{a_A} = \vec{a_B} + \vec{a_{AB}}$ 即

$\vec{a^n_A} + \vec{a^t_A} = \vec{a^n_C} + \vec{a^n_{BC}} + \vec{a^n_{AB}} + \vec{a^t_{AB}}$

	$\vec{a^n_A}$	$\vec{a^t_A}$	$\vec{a^n_C}$	$\vec{a^n_{BC}}$	$\vec{a^n_{AB}}$	$\vec{a^t_{AB}}$
大小	$OA\omega_{OA}^2$	?	✓	$r\omega^2$	$AB\omega_{AB}^2$	?
方向	✓	✓	✓	✓	✓	✓



沿$\vec{a^n_{AB}}$方向投影，$OA\omega_{OA}^2\cos\varphi - OA\alpha_{OA}\sin\varphi = -5\sin\varphi + r\omega^2\cos\varphi + AB\omega_{AB}^2$

$\Rightarrow \alpha_{OA} = (\frac{4\sqrt{3}}{9} - \frac{5}{12}) rad/s^2$

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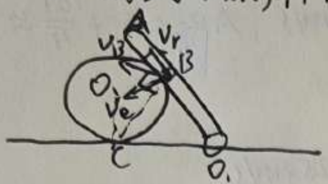
T8-26 解：由于轮O只滚不滑 ⇒ C点为其速度瞬心，$\omega_0=\frac{v_0}{OC}=\frac{v_0}{R}=0.4rad/s$

$v_B=BC\cdot\omega_0=\sqrt{3}R\cdot\omega_0=0.2\sqrt{3}m/s$，$\alpha_0=0$

以销B为动点，杆AO_1为动系 ⇒ 绝对运动：绕C旋转 相对：直线运动

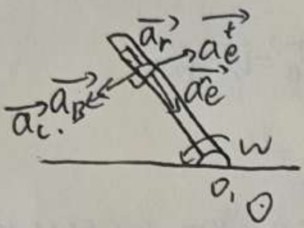
牵连：定轴转动



$\vec{v_B}=\vec{v_r}+\vec{v_e}$ $v_e=v_B\cos60^\circ=0.1\sqrt{3}m/s$，$v_r=v_B\sin60^\circ=0.3m/s$

$\omega_{AO_1}=\frac{v_e}{BO_1}=\frac{0.1\sqrt{3}}{0.5\sqrt{3}}=0.2rad/s$

以O为基点 先求a_B，$\vec{a_B}=\vec{a_O}+\vec{a_{BO}^t}+\vec{a_{BO}^n}=\vec{a_{BO}^n}$，$a_B=\omega_0^2R=0.08m/s^2$



$\vec{a_B}=\vec{a_r}+\vec{a_e}+\vec{a_c}$

即 $\vec{a_B}=\vec{a_r}+\vec{a_e^t}+\vec{a_e^n}+\vec{a_c}$

大小 0.1 ? ? ω^2BO_1 $2\omega v_r$

方向 ✓ ✓ ✓ ✓ ✓

向轴$\vec{a_B}$方向投影有：$a_B=0+(-a_e^t)+0+a_c$ ⇒ $a_e^t=a_c-a_B$

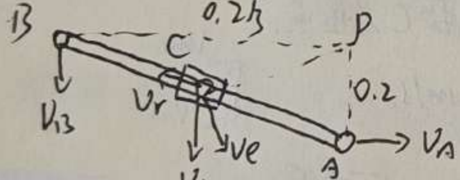
$=2\omega v_r-0.08$

$=0.04m/s^2$

从而$\alpha_{O_1A}=\frac{a_e^t}{O_1B}=\frac{0.04}{0.5\sqrt{3}}=0.0462\ rad/s^2$

T8-29. 解：以套筒上C点为动点，杆AB为动系，绝对运动：竖直方向直线运动

相对运动：沿AB直线 牵连运动：绕P定轴转

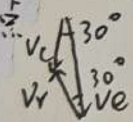


P为杆AB的速度瞬心

$\omega_{AB}=\frac{v_A}{AP}=1rad/s$，$v_B=PB\omega_{AB}=0.2\sqrt{3}m/s$，$v_e=0.2m/s$

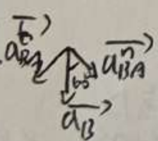
C点有 $\vec{v_c}=\vec{v_e}+\vec{v_r}$，几何关系如图，有$v_c=\frac{v_e}{\sqrt{3}}=0.115m/s$

C点



下求加速度，以点A为基点，$\vec{a_B}=\vec{a_A}+\vec{a_{BA}}=\vec{a_{BA}^t}+\vec{a_{BA}^n}$

B点



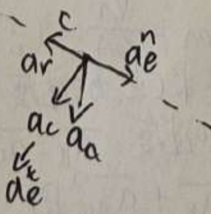
$a_{BA}^n=\omega_{AB}^2\cdot AB=0.4m/s^2$ ⇒ $a_{BA}^t=\sqrt{3}a_{BA}^n=0.4\sqrt{3}m/s^2$

$\alpha_{AB}=\frac{a_{BA}^t}{AB}=\sqrt{3}rad/s^2$

以点A为基点 求点C的牵连加速度，$a_e^t=\alpha_{AB}\cdot AC=0.2\sqrt{3}m/s^2$

$a_e^n=\omega_{AB}^2\cdot AC=0.2m/s^2$

C点



$\vec{a_a}=\vec{a_e^t}+\vec{a_e^n}+\vec{a_r}+\vec{a_c}$

向轴$\vec{a_e^t}$方向投影，

$\frac{\sqrt{3}}{2}a_a=a_e^t+a_c=0.2\sqrt{3}+2\cdot1\cdot v_r=0.2\sqrt{3}+\frac{0.4}{\sqrt{3}}$

$\Rightarrow a_a=\frac{2}{3}=0.667m/s^2$

综上，杆CD的速度$v_c=0.115m/s$，加速度$a_a=0.667m/s^2$

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T8-32

解：(1) 角速度：取套筒D为动点，杆AB为动系

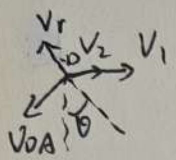
绝对运动：圆周运动　相对：沿杆AB直线　牵连：平面运动

$\vec{v}_D = \vec{v}_1 = \vec{v}_r + \vec{v}_e$

以A为基点，$\vec{v}_e = \vec{v}_A + \vec{v}_{DA} = \vec{v}_2 + \vec{v}_{DA}$

$\Rightarrow \vec{v}_1 = \vec{v}_r + \vec{v}_2 + \vec{v}_{DA}$

	$\vec{v}_1$	$\vec{v}_r$	$\vec{v}_2$	$\vec{v}_{DA}$
大小	✓	?	✓	?
方向	✓	✓	✓	✓

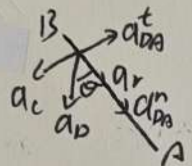


沿 $\vec{v}_{DA}$ 投影得 $-v_1\cos\theta = 0 + (-v_2\cos\theta) + v_{DA} \Rightarrow v_{DA} = (v_2 - v_1)\cos\theta,\ v_r = (v_2 - v_1)\sin$

由 $v_{DA} = AD\omega_{AB} = \frac{2R\omega_{AB}}{\cos\theta} \Rightarrow \omega_{AB} = \frac{(v_2 - v_1)\cos^2\theta}{2R}$　方向：$v_2 > v_1$ 时 ω_{AB} 为逆时针

(2) 角加速度：

绝对：$a_a = a_D = v_1^2/R = \frac{v_1^2}{R}$，$a_c = 2\omega_{AB}\cdot v_r = \frac{(v_2 - v_1)^2\sin\theta\cos^2\theta}{R}$



$\vec{a}_D = \vec{a}_e + \vec{a}_r + \vec{a}_c = \vec{a}_A + \vec{a}^n_{DA} + \vec{a}^t_{DA} + \vec{a}_r + \vec{a}_c$

即 $\vec{a}_D = \vec{a}^n_{DA} + \vec{a}^t_{DA} + \vec{a}_r + \vec{a}_c$

	$\vec{a}_D$	$\vec{a}^n_{DA}$	$\vec{a}^t_{DA}$	$\vec{a}_r$	$\vec{a}_c$
大小	$\frac{v_1^2}{R}$	?	?	?	✓
方向	✓	✓	✓	✓	✓

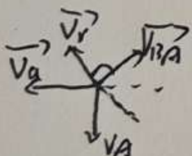
向 a^t_{DA} 方向投影，$-a_D\sin\theta = 0 + a^t_{DA} + 0 - a_c$

$\Rightarrow a^t_{DA} = a_c - a_D\sin\theta = \frac{1}{R}\left[(v_2 - v_1)^2\cos^2\theta - v_1^2\right]\sin\theta$

$\alpha_{AB} = \frac{a^t_{DA}}{DA} = \frac{\left[(v_2 - v_1)^2\cos^2\theta - v_1^2\right]\sin\theta\cos\theta}{2R^2}$

T8-39. 解：杆ABC作平面运动，以点A为基点，有 $\vec{v}_{B'} = \vec{v}_A + \vec{v}_{B'A}$，B'为杆ABC上与B重合的点

以B为动点，IV为动系：绝对运动：水平直线　相对运动：沿杆直线　牵连：复杂



$\vec{v}_a = \vec{v}_e + \vec{v}_r \Rightarrow \vec{v}_a = \vec{v}_A + \vec{v}_{BA} + \vec{v}_r$

	$\vec{v}_a$	$\vec{v}_A$	$\vec{v}_{BA}$	$\vec{v}_r$
大小	v_1	v_2	?	?
方向	✓	✓	✓	✓

沿 $\vec{v}_{BA}$ 轴投影，有 $-v_1\sin\theta = -v_2\cos\theta + v_{BA}$

其中：$\tan\theta = \frac{y}{x}$，$\sin\theta = \frac{y}{\sqrt{x^2+y^2}}$，$\cos\theta = \frac{x}{\sqrt{x^2+y^2}}$

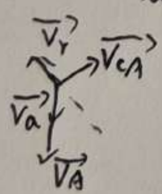
$\Rightarrow v_{BA} = v_2\cos\theta - v_1\sin\theta = \frac{v_2 x - v_1 y}{\sqrt{x^2+y^2}}$

$\omega_{IV} = \frac{v_{BA}}{BA} = \frac{v_{BA}}{\sqrt{x^2+y^2}} = \frac{v_2 x - v_1 y}{x^2+y^2}$ 为正时代表顺时针

以C为动点，IV为动系。

$\vec{v}_a = \vec{v}_e + \vec{v}_r \Rightarrow \vec{v}_a = \vec{v}_A + \vec{v}_{CA} + \vec{v}_r$

	$\vec{v}_a$	$\vec{v}_A$	$\vec{v}_{CA}$	$\vec{v}_r$
大小	?	v_2	✓	?
方向	✓	✓	✓	✓



沿 $\vec{v}_{CA}$ 投影，有 $-v_a\cos\theta = v_A\cos\theta + v_{CA}$

其中 $v_{CA} = \omega_{IV}\cdot CA = \frac{a}{\cos\theta}\frac{xv_2 - yv_1}{x^2+y^2}$

代入 $v_a = v_A - \frac{v_{CA}}{\cos\theta}$ 得

$v_a = v_2 - \frac{a}{\cos^2\theta}\cdot\frac{xv_2 - yv_1}{x^2+y^2}$

$= v_1\frac{ay}{x^2} - v_2\frac{a-x}{x}$.

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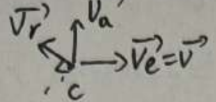
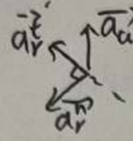
T9-2 解：取动点：C点；动系：凸轮 绝对运动：↕直线，相对：圆周运动；牵连：↔

由 $\vec{v_a}=\vec{v_r}+\vec{v_e}$ 知 $v_a=v_e=v$，$v_r=\sqrt{2}v$，有 $a_r^n=\frac{v_r^2}{R}=\frac{2v^2}{R}$



由 $\vec{a_a}=\vec{a_r}+\vec{a_e}=\vec{a_r^t}+\vec{a_r^n}$ 知 沿 $\vec{a_r^n}$ 方向投影，有

$-\frac{\sqrt{2}}{2}a_a=a_r^n \Rightarrow a_a=-0.2\sqrt{2}\,m/s^2$



由 $F_合=ma$ 知 $F_N-mg=ma_a \Rightarrow F_N=(mg+ma_a)=95.17N$

T9-8 解：设钢索静伸长量为 s_0 有 $ks_0=mg$，刹车后附加伸长量为 s，有

$F=k(s_0+s)$

微分方程：$mg-F=ma=m\frac{d^2s}{dt^2} \Rightarrow -ks=m\frac{dv}{dt}$

由 $\frac{dv}{dt}=\frac{ds}{dt}\cdot\frac{dv}{ds}=v\frac{dv}{ds}$ 得 $-ksds=mvdv$，积分有

$\int_0^s ksds=\int_v^0 mvdv \Rightarrow s=v\sqrt{\frac{m}{k}}$

故最大伸长量为 $s'=s+s_0=\frac{mg}{k}+v\sqrt{\frac{m}{k}}$

T9-11 解：由 $F=100(1-t)=ma$ 知 $a=10(1-t)=\frac{dv}{dt}$ 即 $10(1-t)dt=dv$

积分，有 $\int_0^t(10-10t)dt=\int_{v_0}^v dv \Rightarrow 10t-5t^2=v-v_0$

即 $v=v_0+10t-5t^2$

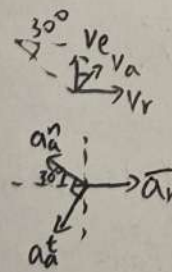
将 $v=0$ 时，有 $5t^2-10t-0.2=0$ 解得 ~~t=2.0~~ $t=2.02s$（负值舍去）

再有 $10t-5t^2+v_0=v=\frac{dx}{dt}$ 积分，有 $\int_0^t(10t-5t^2+v_0)dt=\int_0^x dx$

$\Rightarrow x=-\frac{5}{3}t^3+5t^2+v_0t$，将 $t=2.02s$ 代入，有 $x|_{v=0}=7.07m$

T9-16 解：取动点 销M，动系：水平杆 绝对运动：圆周，相对运动：↔，牵连：↕

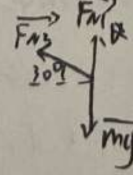
有 $\vec{v_a}=\vec{v_r}+\vec{v_e}$ 知 $v_e=v$，$v_r=\frac{1}{\sqrt{3}}v_e$，$v_a=\frac{2}{\sqrt{3}}v_e$，$a_e=0$



$\vec{a_a^n}+\vec{a_a^t}=\vec{a_r}$ 有 $a_a=\frac{2}{\sqrt{3}}a_a^n=\frac{2}{\sqrt{3}}\cdot\frac{4}{3}\frac{v_e^2}{R}=1.232\,m/s^2$

	$\vec{a_a^n}$	$\vec{a_a^t}$	$\vec{a_r}$
大小	✓	?	?
方向	✓	✓	✓

受力分析销M，有

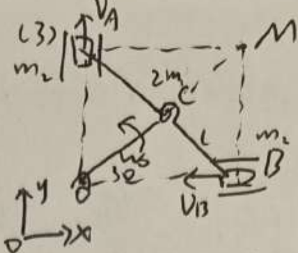


$F_{N2}\cos30^\circ=ma$

$\Rightarrow F_{N2}=0.284N$

解:

T10-3 (1) $p=\sum_i (mv_c)=0$

(3) 

杆AB做平面运动,瞬心为M

$V_C=wl$, $w_{AB}=\frac{V_C}{l}=w$

$V_A=w\cdot 2l\cos30^\circ=\sqrt{3}wl$, $V_B=wl$

由 $\vec{p}=\vec{p_A}+\vec{p_B}+\vec{p_{OC}}+\vec{p_{AB}}$ 知

$p_x=-wl(\frac{5}{2}m_1+m_2)$, $p_y=\sqrt{3}wl(\frac{5}{2}m_1+m_2)$

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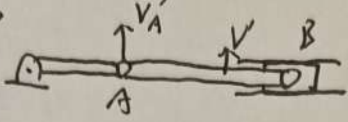
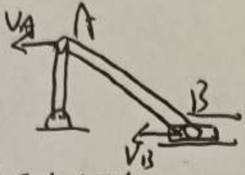
(2) $\varphi=0$ 时,



由杆AB知此时 $V_B=0$, $V'=\frac{1}{2}V_A=\frac{1}{2}rw$

$p=m_1\cdot\frac{1}{2}V_A+m_2\cdot V'=\frac{1}{2}(m_1+m_2)rw$

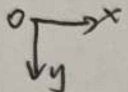
$\varphi=90^\circ$ 时,



杆AB瞬时平移,有 $V_B=V_A=rw$

$p=m_1\cdot\frac{1}{2}V_A+m_2V_A+m_3V_B=(\frac{1}{2}m_1+m_2+m_3)rw$

T10-4. 解: 建立如图所示坐标系



(1)

$$x_c=\frac{m_1\frac{r}{2}\sin30^\circ+m_2r\sin30^\circ}{m_1+m_2+m_3}=\frac{m_1r+2m_2r}{4(m_1+m_2+m_3)}$$

$$y_c=\frac{m_1\frac{r}{2}\cos30^\circ+m_2r\cos30^\circ+m_3(r\cos30^\circ-\frac{l}{2}+b)}{m_1+m_2+m_3}=\frac{\sqrt{3}r(3m_1+6m_2+4m_3)-12m_3b}{12(m_1+m_2+m_3)}$$

(2) 以滑块A为动点,滑杆为动系,有

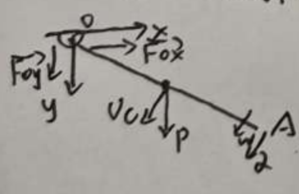
有 $\vec{v_a}=\vec{v_e}+\vec{v_r}\Rightarrow v_a=v_e=v_r=\omega r$. 滑杆速度为 $v_e=v=\omega r$

$$\vec{p}=(m_1w\tfrac{r}{2}\cos30^\circ+m_2wr\cos30^\circ)\vec{i}+[-m_1w\tfrac{r}{2}\sin30^\circ-m_2wr\sin30^\circ-m_3wr\sin30^\circ]\vec{j}$$
$$=(\tfrac{\sqrt{3}}{4}rm_1w+\tfrac{\sqrt{3}}{2}rm_2w)\vec{i}-\tfrac{wr}{4}(m_1+2m_2+2m_3)\vec{j}$$

受力分析知. $\frac{dp_y}{dt}=\sum F_{iy}^{(e)}=F_{Oy}+(m_1+m_2+m_3)g$

$$\Rightarrow F_{Oy}=-(m_1+m_2+m_3)g+\frac{d}{dt}[m_1w\tfrac{r}{2}\sin wt-m_2wr\sin wt-m_3wr\sin wt]$$
$$=-(m_1+m_2+m_3)g-w^2r\cos wt(\tfrac{m_1}{2}+m_2+m_3)$$
$$=-(m_1+m_2+m_3)g-\tfrac{\sqrt{3}}{4}rw^2(m_1+2m_2+2m_3)$$
(F_{Oy} 以向下为正)

T10-13. 解: 建如图系, y以竖直向下为正



$V_C=wl$, $\vec{p}=m\vec{v_c}=-wlm\sin\varphi\vec{i}+wlm\cos\varphi\vec{j}$

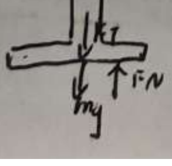
$p_x=-wlm\sin\varphi$, $p_y=wlm\cos\varphi$.

$$F_{Ox}=\frac{dp_x}{dt}=-\alpha lm\sin\varphi-wlmw\cos\varphi=-\frac{P}{g}(\alpha l\sin\varphi+w^2l\cos\varphi)$$

$$\sum F_y^{(e)}=F_{Oy}+P=\frac{dp_y}{dt}=\alpha lm\cos\varphi-wlmw\sin\varphi\Rightarrow F_{Oy}=-P+\frac{P}{g}(\alpha l\cos\varphi-w^2l\sin\varphi)$$

T10-17. 解: 建如图系, 有 $x_A=r+e\cos\varphi=r+e\cos wt$ 　$V_A=-ew\sin wt$ ← 也可用点的合成运动得出,动点C,动系AB.

$a_A=\ddot{x}_A=-ew^2\cos wt$, 由 $\sum F=ma_A$ 知 $F_N-mg-F_T=ma_A$



① 当导板在最高位置时, $F_T=(2e+\delta_0)k$, $\ddot{x}_A=-ew^2$

$F_N\geq0\Rightarrow k\geq\frac{mew^2-mg}{2e+\delta_0}$

② 当导板在最低位置时

$\ddot{x}_A=ew^2$, $F_T=\delta_0k$

$\Rightarrow F_N\geq0$ 恒成立

综上, $k\geq\frac{mew^2-mg}{2e+\delta_0}$

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T11-3 解:(a) $L_O = J_O\omega + 2m_1vr = \frac{1}{2}mr^2\cdot\frac{v}{r} + 2m_1vr = (\frac{1}{2}m+2m_1)vr$

(b) $L_O = J_O\omega + 2m_1vr = (\frac{1}{2}m + 2m_1)vr$

(2)(c) $L_{AB} = L_{CD}^{AB} + L_C^{AB} + L_D^{AB}$

其中 $L_{CD}^{AB} = \int_{-l}^{l}\frac{m_1}{2l}(x\sin\theta)^2\omega dx = \frac{m_1}{3}\omega l^2\sin^2\theta$, $L_C^{AB}=L_D^{AB}=m\omega l^2\sin^2\theta$

故 $L_{AB} = (\frac{m_1}{3}+2m)\omega l^2\sin^2\theta$

T11-9. 解: 通风机的动量矩为 $L=J\omega$, $L_0=J\omega_0$

动量矩定理 $J\alpha=-k\omega$ 即 $\frac{d\omega}{\omega}=-\frac{k}{J}dt$, 积分, 得

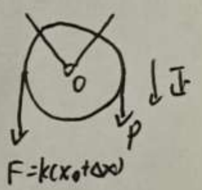
$\omega=\omega_0 e^{-\frac{k}{J}t}$ …①

将 $\omega=\frac{\omega_0}{2}$ 代入, 反解出 $t=\frac{J}{k}\ln2$

对①再积分有 $\int d\varphi=\int\omega_0 e^{-\frac{k}{J}t}dt \Rightarrow \varphi=-\frac{J}{k}\omega_0 e^{-\frac{k}{J}t}$, 再代入 $t=\frac{J}{k}\ln2$

转数 $n=|\frac{\varphi}{2\pi}|=\frac{J\omega_0}{4k\pi}$

T11-13. 解: 静止时, 设伸长了 x_0, 有 $P=kx_0$



运动时, 弹力 $F=k(x_0+\Delta x)$;

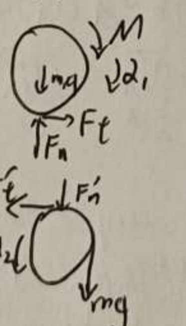
整体动量矩 $L_O=\frac{1}{2}r\frac{P}{g}\dot{x}+\frac{P}{g}\dot{x}r=\frac{3P}{2g}r\dot{x}$

$\Sigma M_O(F)=Pr-k(\Delta x+x)r=-kxr$

动量矩定理 $\dot{L}_O=\Sigma M_O(F)$, 即 $\frac{3P}{2g}r\ddot{x}=-kxr$

$\Rightarrow \ddot{x}+\frac{2gk}{3P}x=0$

T11-16. 解: AB: $L_1=J_1\omega_1$, CD: $L_2=J_2\omega_2+Rm\omega_2R=(J_2+mR^2)\omega_2$



主动轴 $\dot{L}_1=M-F_t r_1=J_1\alpha_1$

从动轴 $\dot{L}_2=(J_2+mR^2)\alpha_2=F_t r_2-mgR$

且有 $k=\frac{z_2}{z_1}=\frac{\alpha_1}{\alpha_2}=\frac{r_2}{r_1}$

$\Rightarrow \alpha_2=\frac{Mk-mgR}{J_1k^2+mR^2+J_2}$

T11-18. 解: 对整体分析, 由 $M_z(F)=0$, 有 $L_z=const$, 且 $L_{z初}=0 \Rightarrow L_z\equiv0$

$J_{台}=\frac{1}{12}m_{台}(0.4^2+0.15^2)=0.09125\ kg\cdot m^2$; $\omega_0=\frac{2\pi n_1}{60}=6\pi$ rad/s

(1) $L_{z末}=J_{台}\omega_1+2(\frac{1}{2}m_A r^2\cdot\omega_2+m_A v_A\cdot0.1)$, 其中 $\omega_2=\omega_1+\omega_0=\omega_1+6\pi$

$v_A=0.1\cdot\omega_1$, 代入上式, 有 $L_{z初}=L_{z末}=0$ 解得 $\omega_1=-2.18898$ rad/s

$\omega_2=16.66058$ rad/s

$\Rightarrow n_1=-20.9$ r/min, $n_2=159.1$ r/min

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$J_{台} = 0.09125 kg \cdot m^2$

A不绕A点转动，但仍绕花轴转动

(2) 拆去传动带，$\omega_A = 0$，有

1份是B提供的，另1份是A提供的

$L_{2末} = J_{台}\omega_1 + \frac{1}{2} m_B r^2 \omega_2 + 2 m_B v_B \cdot 0.1$

其中 $v_B = \omega_1 \cdot 0.1$，$\omega_2 = \omega_1 + 6\pi$

代入 $L_{2末} = 0$，有 $n_1 = -11.1 r/min$，$n_2 = 168.9 r/min$.

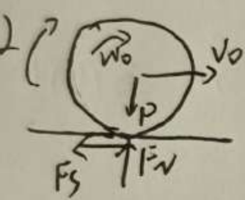
(3) A、B两盘等速反向运动，

$L_{2末} = J_{台}\omega_1 + \frac{1}{2} m_A r^2 (\omega_1 - 6\pi) + \frac{1}{2} m_B r^2 (\omega_1 + 6\pi) + 2 m_B v_B \cdot 0.1 \Rightarrow \omega_1 = 0$

$\Rightarrow n_1 = 0,\ n_B = 180 r/min,\ n_A = -180 r/min$

T11-22 解：

$\omega_0 r < v_0$



运动微分方程
$$\begin{cases} \Sigma F_x = -\frac{P}{g} a_0 = F_s \\ \Sigma F_y = F_N - P = 0 \\ F_s \cdot r = \frac{1}{2} m r^2 \alpha = \frac{1}{2}\frac{P}{g} r^2 \alpha \end{cases}$$

其中 $F_s = Pf$

$$\Rightarrow a_0 = -gf,\ \alpha = \frac{2gf}{r} \Rightarrow \begin{cases} v = v_0 - gft \\ \omega = \omega_0 + \frac{2gf}{r} t \end{cases}$$

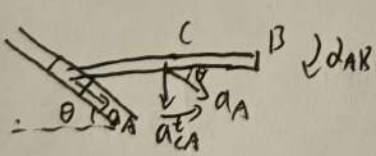
当圆柱纯滚动时，$v = \omega r$，代入上式，有

$$v_0 - gft = \omega_0 r + 2gft \Rightarrow t = \frac{v_0 - \omega_0 r}{3gf}$$

此时中心速度 $v = v|_{t = \frac{v_0 - \omega_0 r}{3gf}} = \frac{2}{3} v_0 + \frac{1}{3}\omega_0 r$

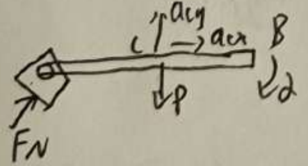
(2) 由 $s = v_0 t - \frac{1}{2} g f t^2 \Rightarrow s|_{t=t'} = \dfrac{5v_0^2 - 4v_0 r\omega_0 - r^2\omega_0^2}{18gf}$

T11-27 解：



杆AB作平面运动，设点A a_A，以A为基点，

$$\vec{a_C} = \vec{a_A} + \underset{\frac{l}{2}\alpha}{\vec{a_{CA}^t}} + \underset{0}{\vec{a_{CA}^n}}$$



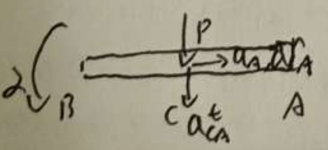
则 $a_{cx} = a_A \cos 30°$，$a_{cy} = -a_A \sin 30° - a_{CA}^t = -\frac{1}{2}a_A - \frac{l}{2}\alpha$

由于不计滑块质量，整体质心即点C，运动微分方程如下

$$\begin{cases} J_C \alpha_{AB} = \frac{1}{12}\frac{P}{g} l^2 = F_N \frac{l}{2} \sin 60° \\ X: \frac{P}{g} a_{cx} = F_N \cos 60°,\ Y: \frac{P}{g} a_{cy} = F_N \sin 60° - P \end{cases} \Rightarrow \begin{cases} \alpha_{AB} = \frac{18g}{13l} \\ F_N = 0.266P \end{cases}$$

T11-32 解：以点A为基点，分析点C

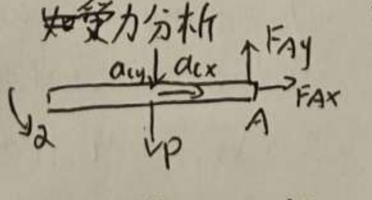
$$\vec{a_C} = \underset{\omega_0^2 \cdot \frac{l}{4}}{\vec{a_A}} + \underset{\alpha \cdot \frac{l}{2}}{\vec{a_{CA}^t}} + \underset{\omega_0^2 \cdot \frac{l}{2}}{\vec{a_{CA}^n}}$$



所以 $a_{cx} = a_A + a_{CA}^n = \frac{3}{4}\omega_0^2 l$

$a_{cy} = \alpha \cdot \frac{l}{2}$

~~如~~受力分析



微分方程
$$\begin{cases} X: \frac{P}{g} \cdot a_{cx} = F_{Ax} \\ Y: \frac{P}{g} a_{cy} = mg - F_{Ay} \\ \frac{1}{12}\frac{P}{g} l^2 \alpha = F_{Ay} \cdot \frac{l}{2} \end{cases}$$

$$\Rightarrow \alpha = \frac{3g}{2l},\ F_{Ax} = \frac{3P}{4g}\omega_0^2 l,\ F_{Ay} = \frac{P}{4}$$

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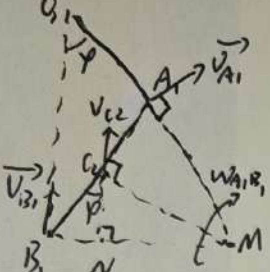
T12-10 解：分析整体，A_2O_2、A_1O_1定轴转动，A_2B_2、A_1B_1平面运动，D平移



$$T_1=0,\quad T_2=\underbrace{2\cdot\frac{1}{2}J_{O_1}\omega^2}_{\text{定轴转动}}+\underbrace{\frac{1}{2}m_2v_D^2}_{\text{平移}}+\underbrace{2\left(\frac{1}{2}m_1v_{C_2}^2+\frac{1}{2}J_{C_2}\omega_{A_1B_1}^2\right)}_{\text{平面运动}}$$

其中 $J_{O_1}=\frac{1}{3}m_1r^2,\ J_{C_2}=\frac{1}{12}m_1r^2$

分析杆A_1B_1运动，速度瞬心为M　由 $v_{A_1}=\omega O_1A_1=\omega_{A_1B_1}A_1M$ 且 $O_1A_1=A_1M$ 知

$\omega_{A_1B_1}=\omega$

由几何关系，$MC_2=\sqrt{NC_2^2+MN^2}=\sqrt{(\frac{r}{2}\cos\varphi)^2+(2r\sin\varphi-\frac{r}{2}\sin\varphi)^2}=\sqrt{\frac{r^2}{4}(1+8\sin^2\varphi)}$

故 $v_{C_2}=\omega_{A_1B_1}MC_2=\omega\sqrt{\frac{r^2}{4}(1+8\sin^2\varphi)}$

且 $v_D=v_{B_1}=\omega_{A_1B_1}\cdot B_1M=2r\omega\sin\varphi$ 代入上式，有

$T_2=\frac{2}{3}r^2\omega^2[m_1+3(m_1+m_2)\sin^2\varphi]$

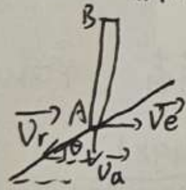
力的功为 $W=-2P_1\frac{r}{2}(1-\cos\varphi)-2P_1\frac{3r}{2}(1-\cos\varphi)-P_2\cdot 2r(1-\cos\varphi)-\frac{1}{2}k\cdot 4r^2\sin^2\varphi+2M\cdot\varphi$

$=2M\varphi-r(1-\cos\varphi)(4P_1+2P_2)-2kr^2\sin^2\varphi$

由动能定理，$W=T_2-T_1$，即 $W=T_2$，解得

$$\omega=\frac{1}{r}\sqrt{\frac{3[M\varphi-r(1-\cos\varphi)(2P_1+P_2)-kr^2\sin^2\varphi]}{m_1+3(m_1+m_2)\sin^2\varphi}}$$

T12-15 解：以点A为动点，楔块为动系，有 $v_a=v_e\tan\theta$，故 $a_A=a_C\tan\theta$，$v_A=v_C\tan\theta$

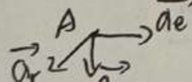


动能 $T=\frac{1}{2}mv_A^2+\frac{1}{2}m_Cv_C^2=\frac{1}{2}v_A^2(m+\frac{m_C}{\tan^2\theta})$　力的功为 $W=mgh$

由动能定理，$\frac{1}{2}v_A^2(m+\frac{m_C}{\tan^2\theta})-T_0=mgh$，两边对时间求导有

$$2\cdot\frac{1}{2}a_Av_A(m+\frac{m_C}{\tan^2\theta})=mgv_A\Rightarrow a_A=\frac{mg\tan^2\theta}{m\tan^2\theta+m_C}$$

由 $\vec{a_a}=\vec{a_e}+\vec{a_r}$ 知



$$a_C=a_e=a_A\frac{1}{\tan\theta}=\frac{mg\tan\theta}{m\tan^2\theta+m_C}$$

T12-18 解：

$T_1=0$，$T_2=\frac{1}{2}J_A\omega_{AB}^2+\underbrace{\frac{1}{2}m_2v_A^2+\frac{1}{2}\cdot\frac{1}{2}m_2R^2\cdot\omega_A^2}_{A\text{平面运动}}$　其中 $J_A=\frac{1}{3}m_1l^2$

$\omega_{AB}=\frac{v_A}{l\sin\theta}$，$\omega_A=\frac{v_A}{R}$，代入，有 $T_2=(\frac{3}{4}m_2+\frac{m_1}{6\sin^2\theta})v_A^2$

功为 $W=mg\frac{l}{2}(\sin45^\circ-\sin\theta)$　由动能定理 $W=T_2-T_1=T_2$，得

$(\frac{3}{4}m_2+\frac{m_1}{6\sin^2\theta})v_A^2=m_1g\frac{l}{2}(\sin45^\circ-\sin\theta)$

两端对时间求导，有

$$2a_Av_A(\frac{3}{4}m_2+\frac{m_1}{6\sin^2\theta})+v_A^2(-\frac{m_1\sin2\theta}{6\sin^4\theta})\dot{\theta}=\frac{m_1gl}{2}(-\cos\theta)\dot{\theta}$$

将 $\dot{\theta}=-\omega_{AB}=-\frac{v_A}{l\sin\theta}$，$v_A=0$，$\theta=45^\circ$代入，得

$$a_A=-\frac{3m_1}{4m_1+9m_2}g$$

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综-9 解：轮A、杆AB定轴转动，$J_A=\frac{1}{2}mr^2$；$J_{AB}=\frac{1}{3}m_1l^2$；物块C平移

设当轮转过角度为θ，$v_C=\omega_A r$，$\omega_A=\omega_{AB}$，有动能 $T_1=0$

$$T_2=\frac{1}{2}J_A\omega_A^2+\frac{1}{2}J_{AB}\omega_{AB}^2+\frac{1}{2}m_2(\omega r)^2=\omega^2\left(\frac{1}{4}mr^2+\frac{1}{6}m_1l^2+\frac{1}{2}m_2r^2\right)$$

力的功为 $\sum W_i=mgh-\frac{l}{2}\sin\theta m_1g=mg\theta r-\frac{l}{2}m_1g\sin\theta$

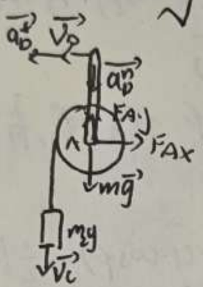
由动能定理，$\omega^2\left(\frac{1}{4}mr^2+\frac{1}{6}m_1l^2+\frac{1}{2}m_2r^2\right)=mg\theta r-\frac{l}{2}m_1g\sin\theta$

两边对时间求导有 $2\alpha\omega\left(\frac{1}{4}mr^2+\frac{1}{6}m_1l^2+\frac{1}{2}m_2r^2\right)=mg\omega r-\omega\frac{l}{2}m_1g\cos\theta$

$$\Rightarrow \alpha=\frac{6m_2gr-3m_1gl\cos\theta}{3mr^2+2m_1l^2+6m_1r^2}$$

将 $\theta=\frac{\pi}{2}$ 代入上两式有 $\omega=\sqrt{\frac{6g(m_2r\pi-m_1l)}{6m_2r^2+3mr^2+2m_1l^2}}$，$\alpha=\frac{6m_2gr}{3mr^2+2m_1l^2+6m_2r^2}$

受力情况如图



X：$F_{Ax}=-m_1a_D^t$

Y：$F_{Ay}-mg-m_1g-m_2g=-m_1a_D^n-m_2a_C$

其中 $a_D^t=\alpha_{AB}\frac{l}{2}$，$a_D^n=\omega^2\frac{l}{2}$，$a_C=\alpha_{AB}r$

$$\Rightarrow F_{Ax}=-\frac{1}{2}m_1\alpha_{AB}l,\quad F_{Ay}=mg+m_1g+m_2g-\frac{1}{2}m_1\omega^2l-m_2\alpha r$$

综-12. 解：$T=\frac{1}{2}J_{O1}\omega_1^2+\frac{1}{2}(J_{O2}+J_{O3})\omega_2^2+\frac{1}{2}m_Av_A^2$，其中 $\omega_2=\frac{\omega_1}{2}$，$v_A=\omega_2r=\frac{1}{2}\omega_1r$

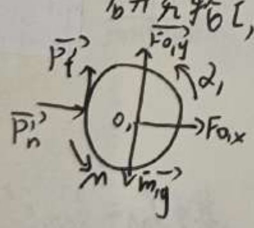
$J_{O1}=\frac{1}{2}m_1r^2$　$J_{O2}=2m_2r^2$，$J_{O3}=\frac{1}{2}m_3r^2$

$\sum W_i=M\varphi-\frac{1}{2}r\varphi m_Ag$　由动能定理，$T-T_0=\sum W_i$，知

$r^2\omega_1^2\left(\frac{1}{4}m_1+\frac{1}{4}m_2+\frac{1}{16}m_3+\frac{1}{8}m_A\right)=\left(M-\frac{1}{2}rm_Ag\right)\varphi$，两边对t求导有　$\dot{\varphi}=\omega_1$，

$\alpha_1=\frac{4(2M-m_Agr)}{r^2(2m_A+m_3+4m_1+4m_2)}$　由 $a_A=\frac{r\alpha_1}{2}$ 有 $a_A=\frac{2(2M-m_Agr)}{r(2m_A+m_3+4m_1+4m_2)}$

研究轮I，



$J_{O1}\alpha_1=M-P_t'r$，$J_{O1}=\frac{1}{2}m_1r^2\Rightarrow P_t'=\frac{M-m_1ra_A}{r}$

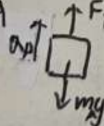
$P_n'=P_t'\tan20°=0.364P_t'$

X：$F_{O1x}+P_n'=0$

Y：$P_t'+F_{O1y}-m_1g=0$

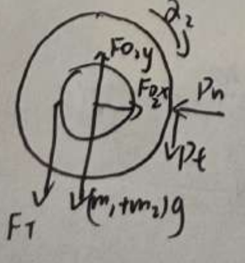
$$\Rightarrow\begin{cases}F_{O1x}=-0.364\frac{M-m_1ra_A}{r}\\F_{O1y}=m_1g-\frac{M-m_1ra_A}{r}\end{cases}$$

由物块A



可知，$F_T'=m_Aa_A+m_Ag$

研究轮II和III



X：$F_{O2x}-P_n=0$

Y：$F_{Oy}-P_t-(m_2+m_3)g-F_T=0$

$$\Rightarrow\begin{cases}F_{O2x}=0.364\frac{M-m_1ra_A}{r}\\F_{O2y}=(m_2+m_3+m_A)g+\frac{M}{r}+(m_1+m_A)a_A\end{cases}$$

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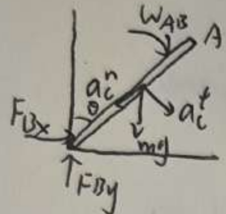
综-13. 解：(1) 杆脱离墙时，杆AB定轴转动 $T=\frac{1}{2}J_B\omega_{AB}^2=\frac{1}{6}ml^2\omega_{AB}^2$

重力的功 $W=mg\frac{l}{2}(1-\cos\theta)$，由动能定理，$T-0=W$，有

$\omega_{AB}^2=\frac{3g}{l}(1-\cos\theta)$ 即 $\omega_{AB}=\sqrt{\frac{3g}{l}(1-\cos\theta)}$



求导，有 $2\omega_{AB}\alpha_{AB}=\frac{3g}{l}\omega_{AB}\sin\theta \Rightarrow \alpha_{AB}=\frac{3g}{2l}\sin\theta$

从而 $a_C^t=\alpha\frac{l}{2}=\frac{3g}{4}\sin\theta$，$a_C^n=\omega^2\frac{l}{2}=\frac{3g}{2}(1-\cos\theta)$

由质心运动定理 X：$F_{Bx}=ma_C^t\cos\theta-ma_C^n\sin\theta$

Y：$F_{By}-mg=ma_C^t\sin\theta-ma_C^n\cos\theta$

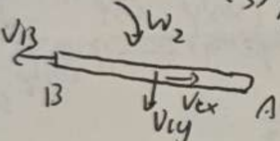
解得 $\begin{cases} F_{Bx}=\frac{3}{4}mg\sin\theta(3\cos\theta-2) \\ F_{By}=\frac{1}{4}mg(1-3\cos\theta)^2 \end{cases}$

(2) 当 $F_{Bx}=0$ 时，即 $\frac{3}{4}mg\sin\theta(3\cos\theta-2)=0 \Rightarrow \theta=48.19^\circ$

此时有 $\omega^2=\frac{g}{l}$，$v_{Cx}=\frac{l}{2}\omega\cos\theta=\frac{1}{3}\sqrt{gl}$

此后，$\Sigma F_x=0$，有 $a_{Cx}=0$，故 $v_{Cx}=\frac{1}{3}\sqrt{gl}$ 不变

(3) 杆AB着地时 以B为基点，有 $v_{Cy}=\omega_2\cdot\frac{l}{2}$



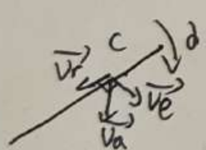
动能 $T=\frac{1}{2}J_B\omega_2^2+\frac{1}{2}mv_C^2=\frac{1}{2}mv_C^2+\frac{1}{2}\cdot\frac{1}{12}ml^2\omega_2^2=mg\frac{l}{2}=W$

其中 $v_C^2=v_{Cx}^2+v_{Cy}^2=\frac{1}{9}gl+\frac{\omega_2^2l^2}{4}$

$\frac{1}{18}mgl+\frac{1}{8}m\omega_2^2l^2+\frac{1}{24}ml^2\omega_2^2=\frac{1}{2}mgl \Rightarrow \omega_2=2\sqrt{\frac{2g}{3l}}$，代入得 $v_C=\frac{1}{3}\sqrt{7gl}$

综-16. 解：到水平位置时

(1) $W=mg\frac{l}{4}+mg\frac{b}{\sqrt{3}}$

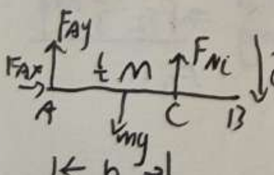


设AB为动系，点C为动点，绝对运动↕ 相对运动：沿AB 牵连：定轴转动

水平时，$\vec{v_e}=\vec{v_a}$ 即 $v_C=v_D$，$\omega_{AB}=\frac{v_C}{b}$

动能 $T=\frac{1}{2}J_A\omega_{AB}^2+\frac{1}{2}m_Dv_D^2$ 由动能定理得 $T=W \Rightarrow \omega=\sqrt{\frac{(3l+4\sqrt{3}b)g}{2l^2+6b^2}}$

(2)

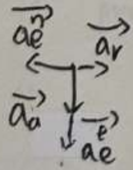


质心：$a_m^n=\frac{l}{2}\omega^2$，由质心运动定理，$F_{Ax}=-ma_m^n=-\frac{mgl(3l+4\sqrt{3}b)}{4l^2+12b^2}$

(3) 点C为动点：$\vec{a_a}=\vec{a_e^t}+\vec{a_e^n}+\vec{a_r}+\vec{a_C}$

大小 0

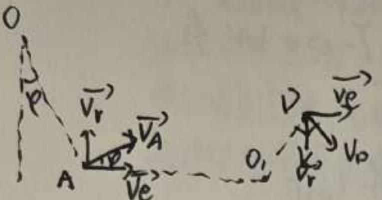
方向 ✓ ✓ ✓ ✓ ✓



$\left.\begin{array}{l} a_a=a_e^t=\alpha b \\ \text{由杆CD知 } ma_a=mg+F_{NC} \\ \text{由杆AB知 } J_A\alpha=mg\frac{l}{2}-F_{NC}b \end{array}\right\} \Rightarrow \begin{cases} a=\frac{(6b^2+3bl)g}{6b^2+2l^2} \\ F_N=\frac{mg(3bl-2l^2)}{6b^2+2l^2} \end{cases}$

综-19. 解：以点D为动点，AB为动系，有 $\vec{v_D}=\vec{v_e}+\vec{v_r} \Rightarrow v_e=v_D\cos\omega t=\omega r\cos\omega t$

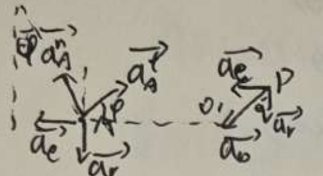
$a_{AB}=a_e=a_D\sin\omega t=\omega^2 r\sin\omega t$



以点A为动点，AB为动系有

$\vec{v_A}=\vec{v_e}+\vec{v_{r2}} \Rightarrow v_A=\frac{v_e}{\cos\varphi}=\frac{\omega r\cos\omega t}{\cos\varphi}$

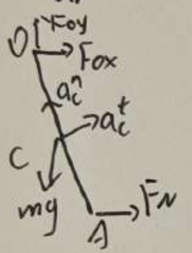
$\vec{a_A^n}+\vec{a_A^t}=\vec{a_e}+\vec{a_r}$ 且 $a_A^n=\frac{v_A^2}{l}$，$a_e=\omega^2 r\sin\omega t$



向 a_e 方向投影，有 $a_A^n\sin\varphi-a_A^t\cos\varphi=a_e$

$\Rightarrow a_A^t=\frac{1}{\cos\varphi}\left(\frac{\omega^2r^2\cos^2\omega t\sin\varphi}{\cos^2\varphi\cdot l}-\omega^2 r\sin\omega t\right)$

由于OA定轴转动，其质心C满足 $a_C^n=\frac{1}{2}a_A^n$，$a_C^t=\frac{1}{2}a_A^t$



分析杆OA，平面运动，微分方程如下：

$$\begin{cases} X: F_{Ox}+F_N=ma_{Cx}=m(a_C^t\cos\varphi-a_C^n\sin\varphi) \\ Y: F_{Oy}-mg=m(a_C^t\cdot\sin\varphi+a_C^n\cos\varphi) \\ J_{AO}\alpha=\Sigma M_O(F): \frac{1}{3}ml^2\alpha=F_N l\cos\varphi-mg\frac{l}{2}\sin\varphi \end{cases} \Rightarrow \begin{cases} F_N=\frac{2}{3}ma_A^t\sec\varphi+mg\frac{1}{2}\tan\varphi \\ F_{Ox}=-\frac{mr\omega^2}{6}\left[2\frac{r}{l}\frac{\sin\varphi}{\cos^4\varphi}\cos^2\omega t+(3-\frac{2}{\cos^2\varphi})\sin\omega t+\frac{3g}{\omega^2 r}\tan\varphi\right] \\ F_{Oy}=mg+\frac{mr\omega^2}{2}\left(\frac{r}{l}\frac{\cos^2\omega t}{\cos^3\varphi}-\tan\varphi\sin\omega t\right) \end{cases}$$

综-23. 解：设A的速度为 v，则轮C：$\omega_C=\frac{v}{R}$，轮D：$\omega_D=\frac{v}{2R}$，B：$v_B=\frac{v}{2}$

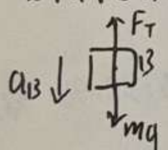
动能 $T=\frac{1}{2}mv_A^2+\frac{1}{2}J_C\omega_C^2+\frac{1}{2}2mv_D^2+\frac{1}{2}J_D\omega_D^2+\frac{1}{2}mv_B^2$

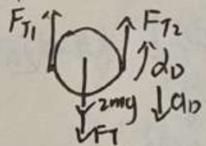
$=\frac{1}{2}mv^2+\frac{1}{2}mv^2+\frac{1}{4}mv^2+\frac{1}{8}mv^2+\frac{1}{8}mv^2=\frac{3}{2}mv^2$

功 $W=3mg\frac{h}{2}-mgh=\frac{1}{2}mgh$ 由动能定理，$T-0=W \Rightarrow \frac{3}{2}mv^2=\frac{1}{2}mgh$

两边求导，有 $3mva=\frac{1}{2}mgv \Rightarrow \underline{a_A=\frac{1}{6}g}$

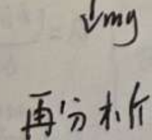
(2) 分析物块B $a_B=\frac{1}{2}a_A=\frac{g}{12}$，$mg-F_T=ma \Rightarrow F_T=\frac{11}{12}mg$

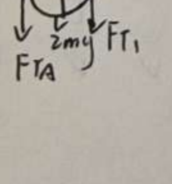




再分析轮D，$a_D=\frac{g}{12}$，$\alpha_D=\frac{g}{12R}$ 微分方程 $\begin{cases} F_{T1}R-F_{T2}R=J_D\frac{g}{12R}=\frac{1}{12}mgR \\ F_T+2mg-(F_{T1}+F_{T2})=\frac{1}{6}mg \end{cases} \Rightarrow \underline{F_{T1}=\frac{4}{3}mg}$

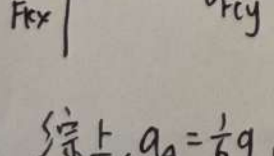
(3) 分析物块A $F_{TA}=mg+ma_A=\frac{7}{6}mg$ 分析轮C





$\begin{cases} X: F_{Cx}=0 \\ Y: F_{Cy}=F_{TA}+F_{T1}+2mg=\frac{9}{2}mg \end{cases}$

再分析无重杆CK



$F_{Ky}=F'_{Cy}=\frac{9}{2}mg$，

$F_{Kx}=0$

$\Sigma M_K=M_K-3RF'_{Cy}=0 \Rightarrow M_K=\frac{27}{2}mgR$

综上，$a_A=\frac{1}{6}g$，(2) $F_{T1}=\frac{4}{3}mg$ (3) $F_{Kx}=0$，$F_{Ky}=\frac{9}{2}mg$，$M_K=\frac{27}{2}mgR$(逆时针)

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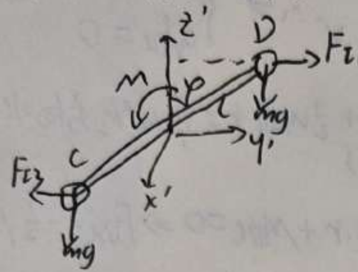
T13-6 解：分析杆(1)与两小球，$F_{I1}=F_{I2}=ma=ma_n=m\omega^2 l\sin\varphi$（$\omega$固定，无$a_t$）



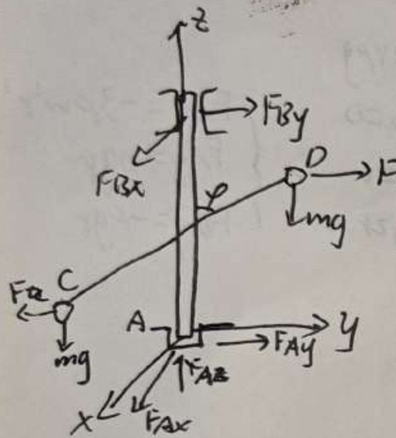
由达朗贝尔原理，$\Sigma M_x(\vec F)=M-mgl\sin\varphi+mgl\sin\varphi-2F_{I1}l\cos\varphi=0$

即$M=2F_{I1}l\cos\varphi$，其中$M=k(\varphi-\varphi_0)$，代入，得

$k\varphi-k\varphi_0=2m\omega^2l^2\sin\varphi\cos\varphi=m\omega^2l^2\sin2\varphi$

$$\Rightarrow \omega=\sqrt{\frac{k(\varphi-\varphi_0)}{ml^2\sin2\varphi}}$$

(2) 对整体分析，由达朗贝尔原理

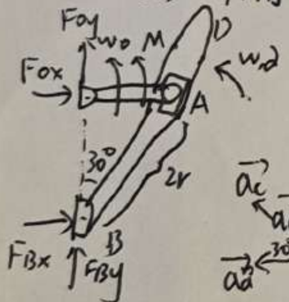


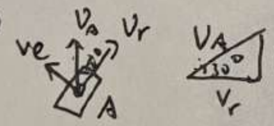
$$\begin{cases}\Sigma F_x=F_{Bx}+F_{Ax}=0,\ \Sigma F_y=F_{I1}+F_{By}+F_{Ay}-F_{I2}=0,\ \Sigma F_z=F_{Az}-2mg=0\\ \Sigma M_x(\vec F)=-F_{By}\cdot2b-mgl\sin\varphi+mgl\sin\varphi\ \cancel{\ldots}+F_{I2}(b-l\cos\varphi)-F_{I1}(b+l\cos\varphi)=0\\ \quad 即-2bF_{By}-2F_{I1}l\cos\varphi=0\\ \Sigma M_y(\vec F)=F_{Bx}\cdot2b=0\\ \Sigma M_z(\vec F)=0\ 恒成立\end{cases}$$

内力矩不要加上去

联立上式，解得 $\begin{cases}F_{Ax}=0\\ F_{Ay}=\dfrac{ml^2\omega^2\sin2\varphi}{2b}\\ F_{Az}=2mg\end{cases}$ $F_{Bx}=0$ $F_{By}=-\dfrac{ml^2\omega^2\sin2\varphi}{2b}$

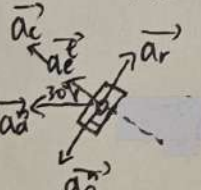
T13-14 解：取滑块A为动点，杆BD为动系，分析





由$v_A=\omega_0 r$，$\vec{v_A}=\vec{v_e}+\vec{v_r}$ 知 $v_e=\frac12\omega_0 r$，$v_r=\frac{\sqrt3}{2}\omega_0 r$

$\omega=\frac{v_e}{2r}=\frac14\omega_0$

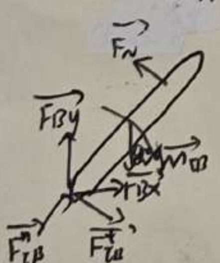


	$\vec{a_a^t}$ +	$\vec{a_a^n}$ =	$\vec{a_e^t}$ +	$\vec{a_e^n}$ +	$\vec{a_c}$ +	$\vec{a_r}$
大小	0	$\omega_0^2 r$	?	$\omega^2 2r$	$2\omega\cdot v_r$	?
方向	✓	✓	✓	✓	✓	✓

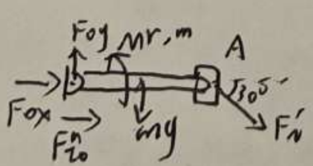
向$\vec{a_c}$方向投影，有

$a_a^n\cdot\frac{\sqrt3}{2}=a_e^t+a_c\Rightarrow a_e^t=\frac{\sqrt3}{2}a_a^n-a_c=\frac{\sqrt3}{4}\omega_0^2r\Rightarrow\alpha=\frac{a_e^t}{2r}=\frac{\sqrt3}{8}\omega_0^2$

先分析杆BD，由$\Sigma M_D(\vec F)=0$知 $F_N2r-M_{IB}-8mg\cdot\frac34r=0\Rightarrow F_N=3mg+\frac32\sqrt3 mr\omega_0^2$



再分析杆OA，



$F_{IO}^n=ma_c^n=\frac12 rm\omega_0^2$

由达朗贝尔原理，$\Sigma F_x=F_{Ox}+F_{IO}^n+\frac{\sqrt3}{2}F_N'=0$

$\Sigma F_y=F_{Oy}-\frac12F_N'-mg=0$

$\Sigma M_O(\vec F)=+M-mg\cdot\frac r2-r\cdot\frac12F_N'=0$

$$\Rightarrow\begin{cases}M=2mgr+\frac{3\sqrt3}{4}mr^2\omega_0^2\\ F_{Ox}=-\frac{3\sqrt3}{2}mg-\frac{11}{4}mr\omega_0^2\\ F_{Oy}=\frac52mg+\frac{3\sqrt3}{4}mr\omega_0^2\end{cases}$$

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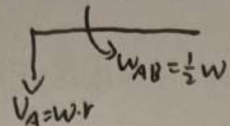
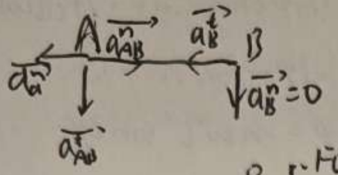
T 13-21 解: 杆AB: 以点B为瞬心, $\omega_{AB}=\frac{v_A}{2r}=\frac{1}{2}\omega$, $\omega_{BC}=0$



以点B为基点, $\overrightarrow{a_A}=\overrightarrow{a_A^n}=\overrightarrow{a_B^t}+\overrightarrow{a_{AB}^n}+\overrightarrow{a_{AB}^t}$

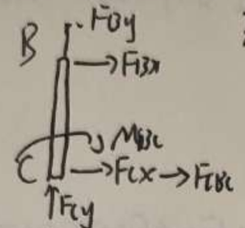
大小 $\omega^2 r$? $\omega_{AB}^2 2r$?

方向 ✓ ✓ ✓ ✓ $\Rightarrow \begin{cases} a_B^t=\frac{3}{2}\omega^2 r \\ a_{AB}^t=0 \end{cases} \Rightarrow \begin{cases} \alpha_{AB}=0 \\ \alpha_{BC}=\frac{3}{2}\omega^2 \end{cases}$



设AB中点为D $a_D=a_A^n+a_{DA}=\omega^2 r+\omega_{AB}^2\cdot r=\frac{5}{4}\omega^2 r$, 方向水平向左

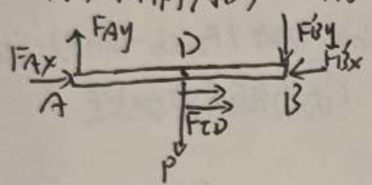
分析杆BC



其中 $M_{IBC}=\frac{1}{3}(\rho r)r^2\cdot\alpha_{BC}=\frac{1}{2}\rho\omega^2 r^3$

由达朗贝尔原理, $\sum M_C(\vec{F})=F_{Bx}\cdot r+M_{IBC}=0 \Rightarrow F_{Bx}=-\frac{1}{2}\rho\omega^2 r^2$

再分析杆AB, 由$\alpha_{AB}=0$, 其中 $F_{ID}=2r\rho\cdot a_D=\frac{5}{2}\rho\omega^2 r^2$, $P=mg=2r\rho g$



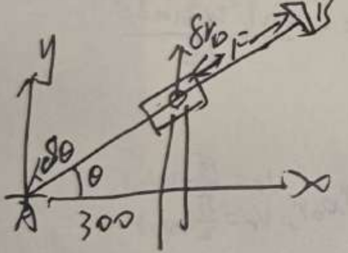
由达朗贝尔原理, $\sum F_x=F_{Ax}-F'_{Bx}+F_{ID}=0$

$\sum F_y=F_{Ay}-F'_{By}-P=0$

$\sum M_A(\vec{F})=-Pr-F'_{By}2r=0$ $\Rightarrow \begin{cases} F_{Ax}=-3\rho\omega^2 r^2 \\ F_{Ay}=\rho g r \\ F_{By}=-\rho g r \end{cases}$

T14-8 解: 虚功方程 $M\delta\theta-F\delta r_D\sin\theta=0$

弹簧伸长量 $\Delta l=l_0(\frac{1}{\cos\theta}-1)$, $F=k\Delta l=kl_0(\frac{1}{\cos\theta}-1)$

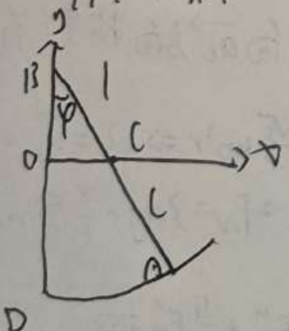


由 $y_D=l_0\tan\theta$, $\delta r_D=\delta y_D=\frac{l_0\delta\theta}{\cos^2\theta}$ 知

$(M-\frac{Fl_0\sin\theta}{\cos^2\theta})\delta\theta=0$ 故 $M=\frac{Fl_0\sin\theta}{\cos^2\theta}=\frac{kl_0^2\sin\theta(1-\cos\theta)}{\cos^3\theta}$

代入数据, $M=450\frac{\sin\theta(1-\cos\theta)}{\cos^3\theta}$ N·m

T14-14 解: 虚功方程 $-P\delta y_C=0$ $\delta y_C=0 \Rightarrow y_C=\text{const}$

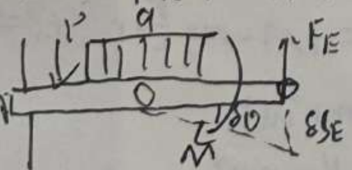


由于 $\varphi=0$ 时 $y_C=0$, 故 $y_C\equiv 0$

PE: $x_A=2l\sin\varphi$, $y_A=-l\cos\varphi$

从而 $\frac{x_A^2}{4l^2}+\frac{y_A^2}{l^2}=1$

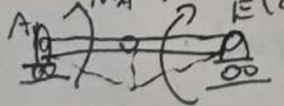
T14-19 解: (1) 以约束力F_E代替支座E处约束有



$M\delta\theta+\frac{ql}{4}\delta s_1-F_E\delta s_E=0$ 代入几何关系 $\delta s_1=\frac{1}{6}\delta s_E$, $\delta\theta=\frac{2}{l}\delta s_E$

$\Rightarrow F_E=2450\text{N}$

(2) 以约束力偶M_A代替固定端A处的转动约束有



$-(M+M_A)\delta\theta+\frac{ql}{2}\delta s_1+P\delta s_2=0$ 代入 $\delta s_1=\frac{1}{4}\delta s_L$, $\delta s_2=\frac{1}{6}\delta s_L$, $\delta\theta=\frac{2}{l}\delta s_L$

$\Rightarrow M_A=29400\text{N·m}$