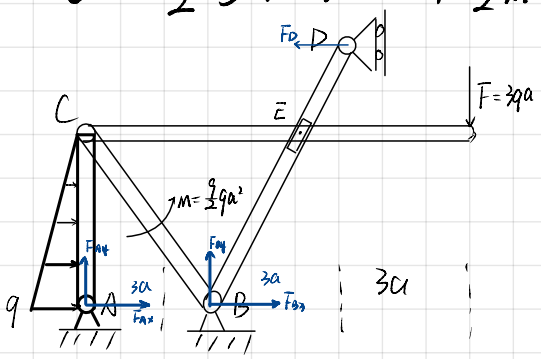
By Empty

1. 动量：$mwR + m(w\cdot 2R) = 3mwR$

动量矩：$\frac{1}{3}m(2R)^2\cdot w + m(w\cdot 2R)\cdot 2R = \frac{16}{3}mwR^2$

动能：$\frac{1}{2}\cdot\frac{1}{3}m(2R)^2\cdot w^2 + \frac{1}{2}m(w\cdot 2R)^2 = \frac{8}{3}mw^2R^2$

2.

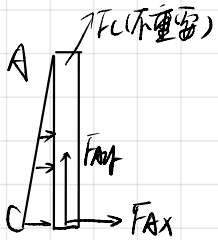


$\frac{1}{2}q\cdot 3\sqrt{3}a + F_{Ax} + F_{Bx} - F_D = 0$ ①

$F_{Ay} + F_{By} - 3qa = 0$ ②

$-\frac{9}{2}qa^2 + F_{By}3a + F_D\cdot\frac{3}{2}\sqrt{3}a + M - F\cdot 9a = 0$ ③

对 AC：

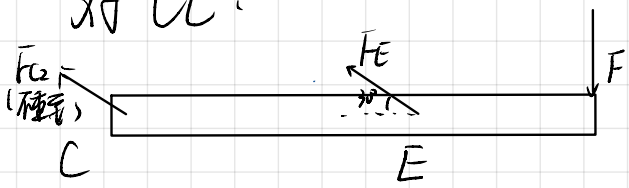


$F_{Ax}3\sqrt{3}a + \int_0^{3\sqrt{3}a}\left(\frac{y^2}{3\sqrt{3}a}q\,dy\right) = 0$

$F_{Ax}3\sqrt{3}a + \frac{\frac{1}{3}(3\sqrt{3}a)^3}{3\sqrt{3}a}q = 0$

$F_{Ax} = -\sqrt{3}qa$

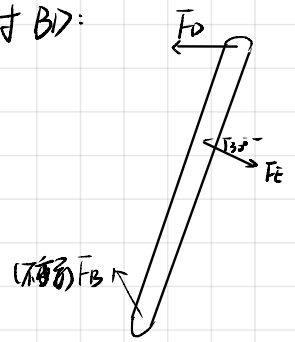
对 CE：



$\frac{1}{2}F_E\cdot 6a = F\cdot 9a$

得：$F_E = 9qa$

对 BD：



$F_E\cdot 6a = F_D\cdot\frac{\sqrt{3}}{2}\cdot 9a$

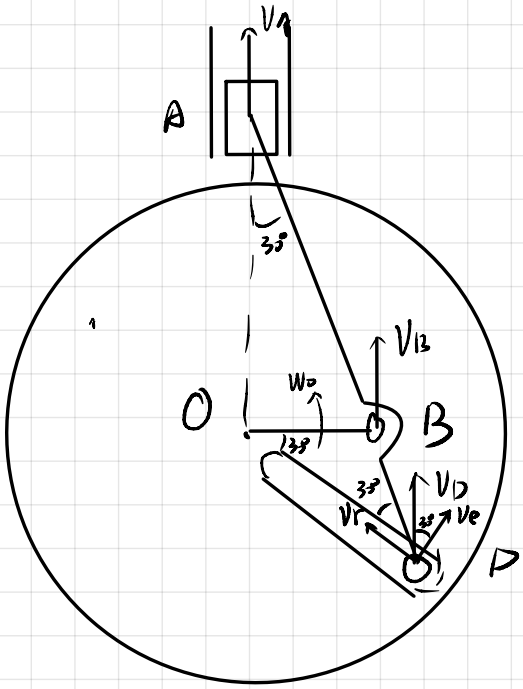
得：$F_D = 4\sqrt{3}qa$

由①②③得：

$$\begin{cases} F_{Bx} = \frac{7}{2}\sqrt{3}qa \\ F_{By} = -9qa \\ F_{Ay} = 12qa \end{cases}$$

综上，$\begin{cases} F_{Ax} = -\sqrt{3}qa,\ F_{Ay} = 12qa \\ F_{Bx} = \frac{7}{2}\sqrt{3}qa,\ F_{By} = -9qa \\ F_D = 4\sqrt{3}qa \end{cases}$

3.



$V_B = \omega_0 \cdot OB = 50\,mm/s$

又 V_A、V_B 同向，

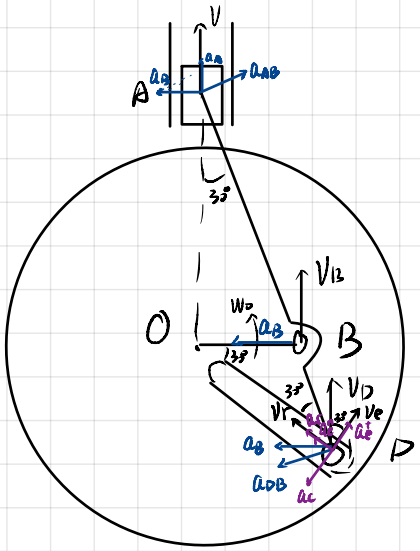
则 $V_A = V_D = V_B = 50\,mm/s$（瞬时平移）

以D为动点，转盘E为动系

$V_r = \frac{1}{2}V_D = 25\,mm/s$

$V_e = \frac{\sqrt{3}}{2}V_D = 25\sqrt{3}\,mm/s$

则 $\omega_E = \frac{V_e}{\sqrt{3}OB} = \frac{25\sqrt{3}\,mm/s}{50\sqrt{3}\,mm} = 0.5\,rad/s$



加速度：

$a_B = \omega_0^2 \cdot OB = 50\,mm/s^2$

以B为基点，A为动系，有：

$\frac{1}{2}\sqrt{3}a_{AB} = a_B$

得：$a_{AB} = \frac{100}{\sqrt{3}}\,mm/s^2$

则 $\alpha_{AD} = \frac{a_{AB}}{AB} = \frac{1}{\sqrt{3}}\,rad/s^2$

对D：$\vec{a_D} = \vec{a_B} + \vec{a_{DB}}$

以D为动点，

$\vec{a_D} = \vec{a_e^t} + \vec{a_e^n} + \vec{a_r} + \vec{a_c}$

$a_{DB} = \alpha_{AD} \cdot BD = \frac{50}{\sqrt{3}}\,mm/s^2$

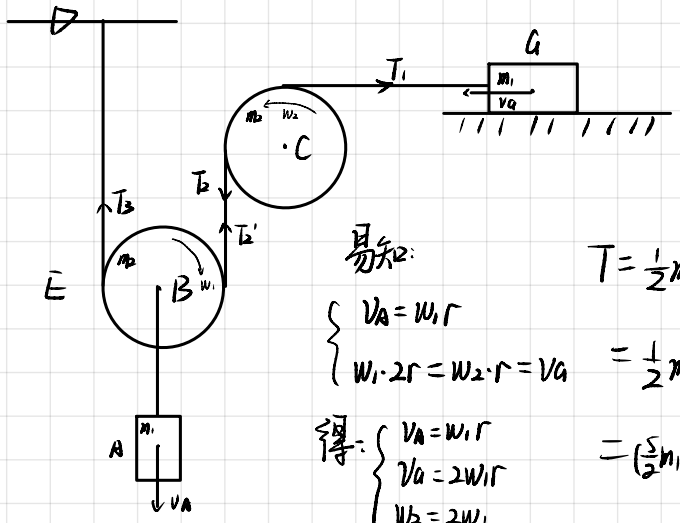
$a_c = 2\omega_E \cdot V_r = 25\,mm/s^2$

$a_e^t - a_c = \frac{1}{2}a_B + \frac{\sqrt{3}}{2}a_{DB}$

得：$a_e^t = 75\,mm/s^2$

则 $\alpha_E = \frac{a_e^t}{OD} = \frac{75\,mm/s^2}{50\sqrt{3}\,mm} = 0.866\,rad/s^2$

4、



易知：

$$\begin{cases} v_A = \omega_1 r \\ \omega_1 \cdot 2r = \omega_2 \cdot r = v_G \end{cases}$$

得：$\begin{cases} v_A = \omega_1 r \\ v_G = 2\omega_1 r \\ \omega_2 = 2\omega_1 \end{cases}$

$$T = \frac{1}{2}m_1 v_A^2 + \frac{1}{2}\cdot\left(\frac{1}{2}m_2 r^2\right)\omega_1^2 + \frac{1}{2}m_2 v_A^2 + \frac{1}{2}\left(\frac{1}{2}m_2 r^2\right)\omega_2^2 + \frac{1}{2}m_1 v_G^2$$

$$= \frac{1}{2}m_1 \omega_1^2 r^2 + \frac{1}{4}m_2 \omega_1^2 r^2 + \frac{1}{2}m_2 \omega_1^2 r^2 + m_2 \omega_1^2 r^2 + 2m_1 \omega_1^2 r^2$$

$$= \left(\frac{5}{2}m_1 + \frac{7}{4}m_2\right)v_A^2 = \frac{10m_1 + 7m_2}{4}v_A^2$$

下降 h：

$$(m_1 + m_2)gh - f m_1 g \cdot 2h = \frac{10m_1 + 7m_2}{4}v_A^2$$

得：$v_A = \sqrt{\dfrac{4(m_1 + m_2 - 2f m_1)gh}{10m_1 + 7m_2}}$

对 v_A 求导有：

$$(m_1 + m_2)g v_A - f m_1 g 2v_A = \frac{10m_1 + 7m_2}{2}v_A \cdot a_A$$

$$a_A = \frac{2(m_1 + m_2 - 2f m_1)g}{10m_1 + 7m_2}$$

$$T_1 - f m_1 g = m_1 a_G = 2m_1 a_A$$

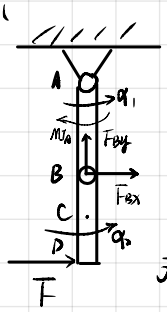
$$(T_2 - T_1)r = \frac{1}{2}m_2 r^2 \cdot \alpha_2 = m_2 a_A \cdot r$$

$$(T_3 - T_2)r = \frac{1}{2}m_2 r^2 \cdot \alpha_1 = \frac{1}{2}m_2 r a_A$$

得：$T_3 = f m_1 g + 2m_1 a_A + m_2 a_A + \frac{1}{2}m_2 a_A$

$$= f m g + \frac{4m_1 + 3m_2}{10m_1 + 7m_2}(m_1 + m_2 - 2f m_1)g$$

5.



对AB：

$M_{IA}=\frac{1}{3}ml^2\cdot\alpha_1$

$F_{BX}\cdot l-M_{IA}=0$

对BD：

$F\cdot\frac{l}{2}+F_{BX}\cdot\frac{l}{2}-M_{IC}=0$

$M_{IC}=\frac{1}{12}ml^2\alpha_2$

$F-F_{BX}-F_{IX}=0$

$F_{IX}=m(a_1l+\frac{1}{2}a_2l)$

联立上式，得

$$\begin{cases}F_{BX}=\frac{1}{3}ml\alpha_1\\ F+F_{BX}=\frac{1}{6}ml\alpha_2\\ F-F_{BX}=m(a_1l+\frac{1}{2}\alpha_2l)\end{cases}$$

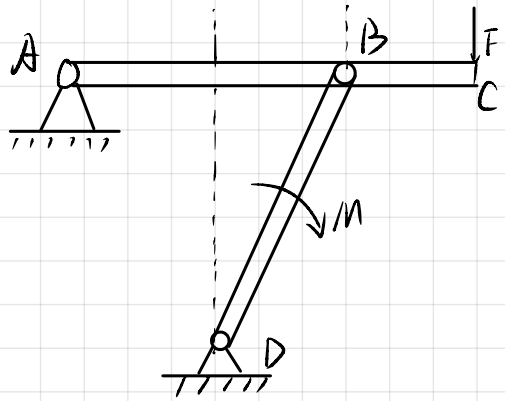
$\frac{1}{3}ml\alpha_1-\frac{1}{6}ml\alpha_2=-F$

$\frac{4}{3}ml\alpha_1+\frac{1}{2}ml\alpha_2=F$

$ml\alpha_1-\frac{1}{2}ml\alpha_2=-3F$

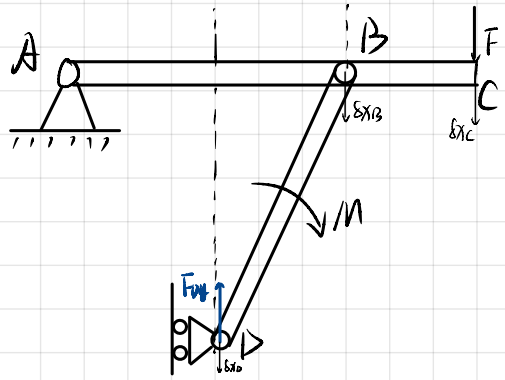
$$\begin{cases}\alpha_1=-\frac{9}{7}\frac{F}{ml}\\ \alpha_2=\frac{24}{7}\frac{F}{ml}\end{cases}$$

6、



D竖直约束力

解除D竖直方向约束力，用F_{Dy}代替



BD做瞬时平移，无转动

$\delta x_D = \delta x_B$

$\frac{\delta x_B}{2} = \frac{\delta x_C}{3}$

得：$\delta x_D = \frac{2}{3}\delta x_C$

虚功方程：

$-F_{Dy} \cdot \delta x_D + F\delta x_C = 0 \quad (\delta\theta = 0)$

得：$F_{Dy} = \frac{3}{2}F$