第一章：n阶行列式

① 一个定义 $D=\Delta(a_{ij})=\sum_{p_1\cdots p_n}(-1)^{t(p_1\cdots p_n)}a_{1p_1}+\cdots+a_{np_n}$

$$=\sum_{q_1\cdots q_n}(-1)^{t(q_1\cdots q_n)}a_{q_1 1}+\cdots+a_{q_n n}$$

② 五个性质

$D'=D$（转置为自身）n阶行列式

$D\xrightarrow[\text{2) 提公因式}(\frac{}{k},k\neq 0)]{\text{1) } r_i\to r_j}D$ $\quad =D$

$\tilde{D}=\frac{D}{k}$

3) r_i+kr_j $\quad \tilde{D}=D$

计算行列式

部分可加性

$D: R^{n^2}\to R$ 数表→数 ($n\times n$)

③ 展开定理 $D=\sum_{k} a_{i_0k}A_{i_0k}$

错位展开为0

④ 克莱姆法则

例1. $\begin{vmatrix} x+a & a & \cdots & a \\ a & x+a & \cdots & a \\ \vdots & \vdots & & \vdots \\ a & a & \cdots & x+a \end{vmatrix}$

法①. 原式 $\xlongequal[(x+na)]{C_1+C_j\ (j=2\cdots n)}$ $\begin{vmatrix} 1 & a & \cdots & a \\ 1 & x-a & \cdots & a \\ \vdots & \vdots & & \vdots \\ 1 & a & \cdots & x-a \end{vmatrix}$

$\xlongequal{C_i-aC_1}$ $\begin{vmatrix} 1 & 0 & \cdots & 0 \\ 1 & x & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 1 & 0 & \cdots & x \end{vmatrix}$

法② $\xlongequal[i=n-1]{r_i-r_n}$ $\begin{vmatrix} x & 0 & \cdots & -x \\ \vdots & \ddots & & \\ 0 & 0 & x & -x \\ a & a & a & x+a \end{vmatrix}$ → 展开

↓

公式.

法③ $\begin{vmatrix} x+a & a & \cdots & a \\ a & x+a & \cdots & a \\ \vdots & \vdots & & \vdots \\ a & a & \cdots & x+a \end{vmatrix}$

$\Leftrightarrow$ $\begin{vmatrix} x+a & a & \cdots & a & 0 \\ a & x+a & \cdots & a & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ a & a & \cdots & x+a & 0 \\ a & a & \cdots & a & 1 \end{vmatrix}$

$= \begin{vmatrix} x & 0 & \cdots & 0 & -1 \\ 0 & x & \cdots & 0 & -1 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & x & -1 \\ a & a & \cdots & a & 1 \end{vmatrix} = \cdots\cdots$

$f(x) = a_0 + a_1x + a_2x^2$

$\begin{cases} x_1 \to f(x_1) = f_1 \\ x_2 \to f(x_2) = f_2 \\ x_3 \to f(x_3) = f_3 \end{cases}$

$\begin{cases} a_0 + a_1x_1 + a_2x_1^2 = f_1 \\ a_0 + a_1x_2 + a_2x_2^2 = f_2 \\ a_0 + a_1x_3 + a_2x_3^2 = f_3 \end{cases}$

Vandermonde 行列式

$D_n = \begin{vmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_n \\ \vdots & \vdots & & \vdots \\ x_1^{n-2} & x_2^{n-2} & \cdots & x_n^{n-2} \\ x_1^{n-1} & x_2^{n-1} & \cdots & x_n^{n-1} \end{vmatrix}$ ⟵ $D = \begin{vmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{vmatrix}$

辗转相消法

$$= \begin{vmatrix} 1 & 1 & \cdots & 1 \\ x_1-x_1 & x_2-x_1 & \cdots & x_n-x_1 \\ \vdots & \vdots & & \vdots \\ x_1^{n-1}-x_1^{n-1} & x_2^{n-1}-x_1x_2^{n-2} & \cdots & x_n^{n-1}-x_1x_n^{n-2} \end{vmatrix} = (x_2-x_1)\cdots(x_n-x_1)\left|\quad\right| \cdots$$

例 $\begin{vmatrix} 1 & 1 & 1 & \cdots & 1 \\ x_1 & x_2 & x_3 & \cdots & x_n \\ \vdots & & & & \\ x_1^{n-2} & x_2^{n-2} & x_3^{n-2} & \cdots & x_n^{n-2} \\ x_1^n & x_2^n & x_3^n & \cdots & x_n^n \end{vmatrix}$

$\to$ 构造 $D_{n+1} = \begin{vmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ x_1 & x_2 & x_3 & \cdots & x_n & x \\ \vdots & \vdots & \vdots & & & \\ x_1^{n-2} & x_2^{n-2} & x_3^{n-2} & \cdots & x_n^{n-2} & x^{n-2} \\ x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \cdots & x_3^{n-1} & x^{n-1} \\ x_1^n & x_2^n & x_3^n & \cdots & x_n^n & x^n \end{vmatrix}$

$$= \left[\prod_{i>j}(x_i-x_j)\right](x-x_1)\cdots(x-x_n)$$

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$$\widetilde{D}_{n+1}(x) = A_{1,n+1} + xA_{2,n+1} + \cdots + x^n A_{n+1,n+1}$$

关于x的n次多项式

$A_{n,n+1}$是x^{n-1}的系数$=(-1)^{2n+1}D_n$

$$D_n = -A_{n,n+1} = -\left[-x_1 - x_2 - \cdots - x_n\right]\prod_{i>j}(x_i - x_j)$$

$$= \left(\sum_{k=1}^{n} x_n\right)\prod_{i>j}(x_i - x_j)$$

格式

$D = | \quad | = | \quad | = ?$

同理 $D_1 = | \quad | = ?$

……

D_k …

由克莱姆法则可得 $x_1 = \frac{D_1}{D}$ …