

学号

$$(4) \int \frac{2+3^x-5+2^x}{3^x} dx;$$

$$= \int [2 - 5 \cdot (\frac{2}{3})^x] dx$$

$$= 2x - 5 \int (\frac{2}{3})^x dx$$

$$= 2x - \frac{5}{\ln \frac{2}{3}} (\frac{2}{3})^x + C$$

$$(5) \int \cot^2 x dx;$$

$$= \int (\csc^2 x - 1) dx$$

$$= \int \csc^2 x dx - \int dx$$

$$= -\cot x - x + C$$

$$(6) \int \frac{1+\cos^2 x}{1+\cos 2x} dx;$$

$$= \int \frac{1+\cos^2 x}{2\cos^2 x} dx$$

$$= \frac{1}{2} \int \frac{1}{\cos^2 x} dx + \frac{1}{2} \int dx$$

$$= \frac{1}{2} \tan x + \frac{x}{2} + C$$

$$(7) \int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}} dx;$$

$$= \int \frac{\sqrt{1+x^2}}{\sqrt{(1-x^2)(1+x^2)}} dx$$

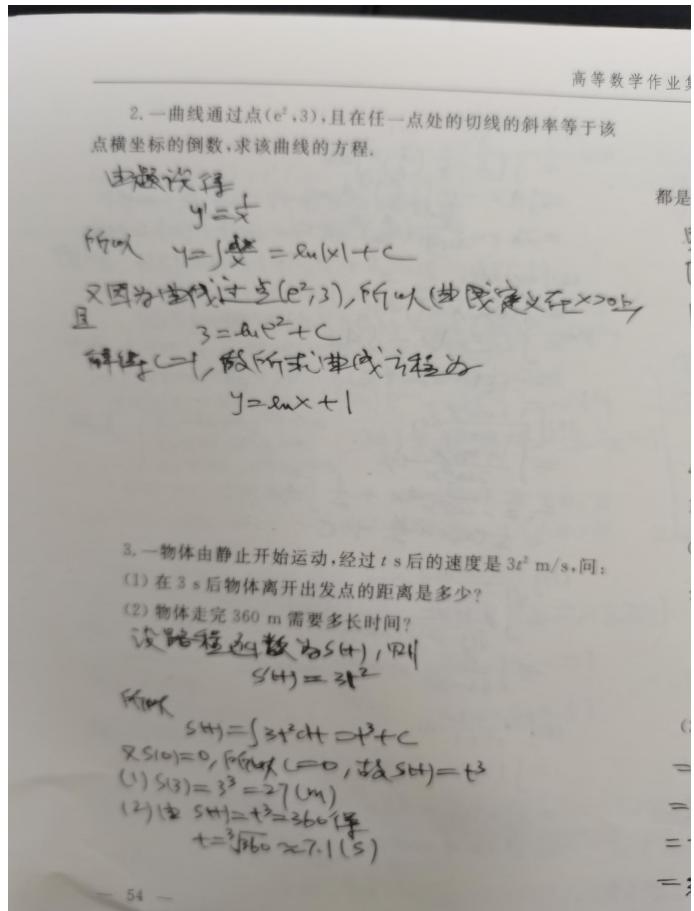
$$= \int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C$$

$$(8) \int \frac{3x^4+2x^2}{x^2+1} dx.$$

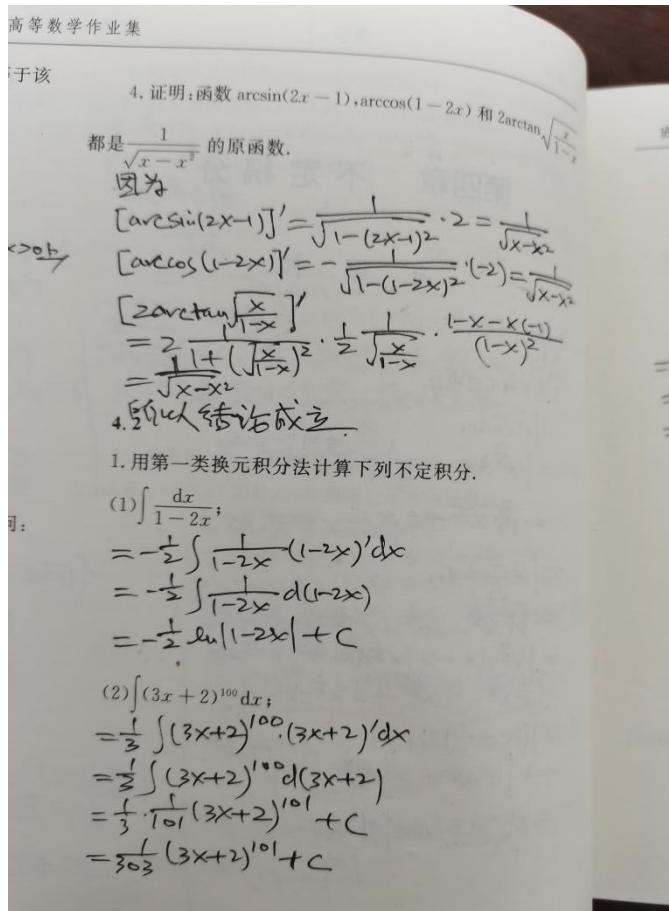
$$= \int (3x^2 - 1 + \frac{1}{x^2+1}) dx$$

$$= 3 \int x^2 dx - \int dx + \int \frac{dx}{1+x^2}$$

$$= x^3 - x + \arctan x + C$$



- 54 -



学号	姓名	学号
$\frac{x}{\sqrt{1-x^2}}$		
(3) $\int xe^{-x^2} dx;$	(7) $\int \frac{1}{1+9x^2} dx;$	
$= -\frac{1}{2} \int e^{-x^2} (-x^2)' dx$	$= \frac{1}{3} \arctan(3x)$	
$= -\frac{1}{2} \int x^{-2} d(-x^2)$	$= \frac{1}{3} \arctan(3x) + C$	
$= -\frac{1}{2} e^{-x^2} + C$		
(4) $\int \frac{\sin \lg x}{x} dx;$	(8) $\int \frac{x}{\sqrt{1+x^2}} dx;$	
$= \ln x \sin \lg x \cdot (\lg x)' dx$	$= \frac{1}{2} \int \frac{1}{\sqrt{1+x^2}} d(x^2)$	
$= \ln x \int \sin \lg x d \lg x$	$= \frac{1}{2} \arctan(x^2) + C$	
$= -\ln x \cos \lg x + C$		
(5) $\int \sqrt{\arcsin x} dx;$	(9) $\int \frac{1}{\sqrt{1-x^2}} dx;$	
$= \int \sqrt{\arcsin x} \cdot (\arcsin x)' dx$	$= \frac{1}{2} \arcsin x + C$	
$= \int \sqrt{\arcsin x} d \arcsin x$		
$= \frac{1}{2} (\arcsin x)^{\frac{1}{2}+1} + C = \frac{2}{3} (\arcsin x)^{\frac{3}{2}} + C$		
(6) $\int \frac{\sqrt{x}}{\sqrt{a^2-x^2}} dx (a > 0);$	(10) $\int \frac{dx}{x \ln x \ln \ln x};$	
$= \frac{2}{3} \int \frac{1}{\sqrt{(ax^2)^2-(x^2)^2}} (x^{\frac{3}{2}})' dx$	$= \int \frac{1}{\ln x \ln \ln x} d(\ln x)$	
$= \frac{2}{3} \int \frac{1}{\sqrt{(ax^2)^2-(x^2)^2}} d(x^{\frac{3}{2}})$	$= \int \frac{1}{\ln \ln x} d(\ln \ln x)$	
$= \frac{2}{3} \arcsin \frac{x}{ax} + C$		
$= \frac{2}{3} \arcsin \left(\frac{x}{a} \right)^{\frac{3}{2}} + C$		

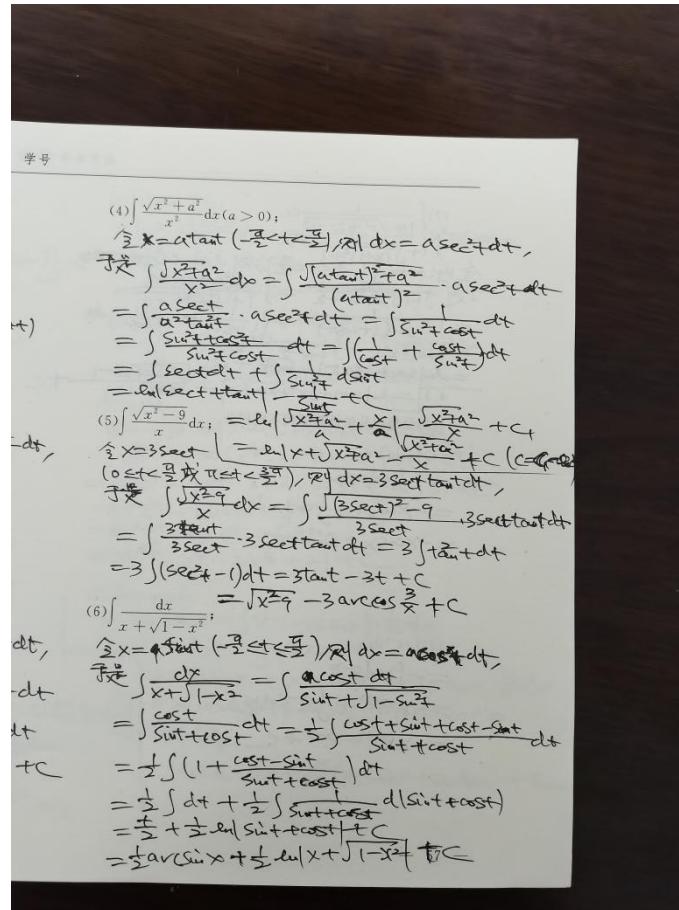
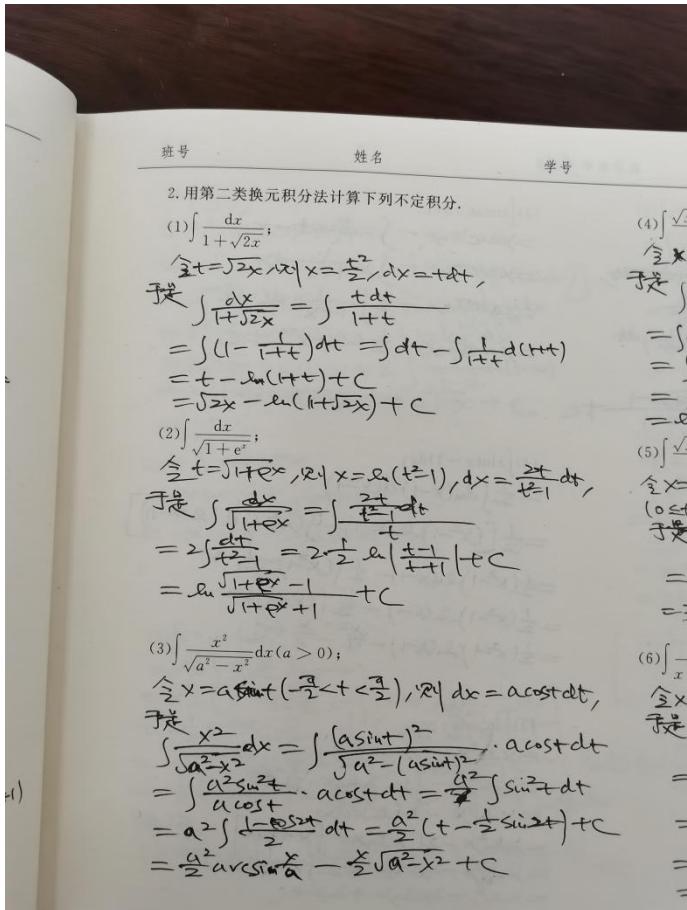
学号	
$\frac{3x}{1+9x^2}$	
(7) $\int \frac{3x}{1+9x^2} dx;$	
$= \frac{1}{3} \arctan(3x)$	
$= \frac{1}{3} \int \frac{1}{1+(3x)^2} d(3x)$	
$= \frac{1}{3} \arctan(3x) + C$	
(8) $\int \frac{\sin x + \cos x}{\sqrt{\sin x - \cos x}} dx;$	
$= \int \frac{1}{\sqrt{\sin x - \cos x}} (\sin x - \cos x)' dx$	
$= \int (\sin x - \cos x)^{-\frac{1}{2}} d(\sin x - \cos x)$	
$= \frac{1}{-\frac{1}{2}+1} (\sin x - \cos x)^{-\frac{1}{2}+1} = \frac{2}{3} (\sin x - \cos x)^{\frac{1}{2}} + C$	
(9) $\int \frac{1}{\cos^2 3x} dx;$	
$= \int \frac{1}{\cos^2 3x} dx + \int \frac{\sin 3x}{\cos^2 3x} dx$	
$= \frac{1}{3} \int \frac{1}{\cos^2 3x} d(3x) - \frac{1}{3} \int \frac{1}{\cos^2 3x} d(\cos 3x)$	
$= \frac{1}{3} \tan 3x + \frac{1}{3} \frac{1}{\cos 3x} + C$	
(10) $\int \frac{dx}{x \ln x \ln \ln x};$	
$= \int \frac{1}{\ln x \ln \ln x} d(\ln x)$	
$= \int \frac{1}{\ln \ln x} d(\ln \ln x)$	
$= \ln \ln \ln x + C$	

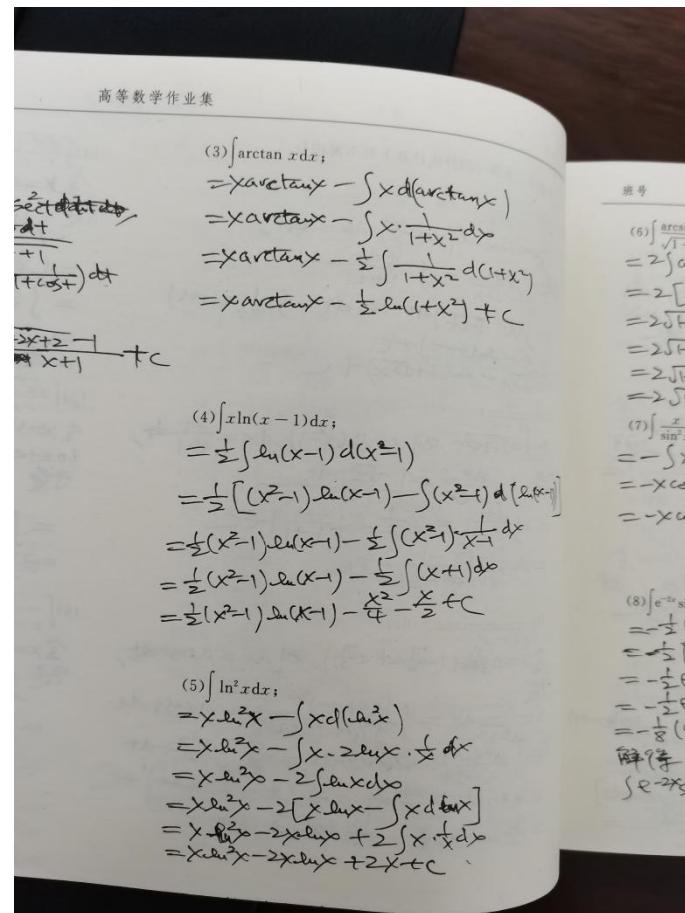
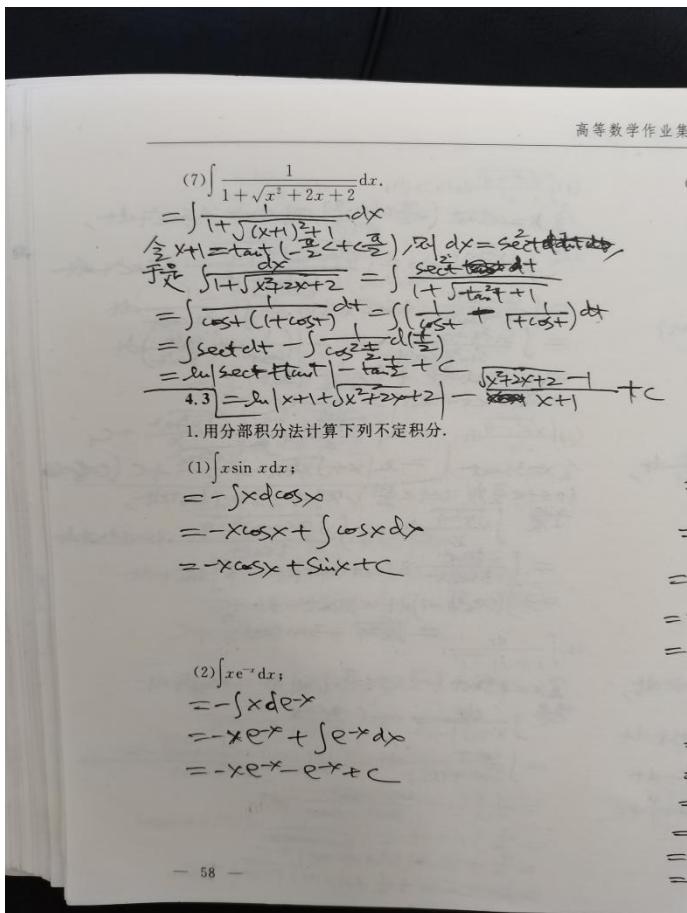
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$$\begin{aligned}
 (11) & \int \frac{\ln \tan x}{\cos x \sin x} dx; \\
 &= \int \frac{\ln \tan x}{\tan x} \cdot \frac{1}{\cos x} dx \\
 &= \int \frac{\ln \tan x}{\tan x} d(\ln \tan x) \\
 &= \int \ln \tan x d(\ln \tan x) \\
 &= \frac{1}{2} (\ln \tan x)^2 + C \\
 (12) & \int \tan^3 \frac{x}{3} \sec^2 \frac{x}{3} dx; \\
 &= 3 \int \tan^2 \frac{x}{3} \sec^2 \frac{x}{3} d\left(\frac{x}{3}\right) \\
 &= 3 \int \tan^2 \frac{x}{3} d(\tan \frac{x}{3}) \\
 &= \frac{3}{4} \tan^3 \frac{x}{3} + C \\
 (13) & \int \cos x \cos \frac{x}{2} dx; \\
 &= \int \frac{1}{2} (\cos \frac{x}{2} + \cos \frac{3x}{2}) dx \\
 &= \frac{1}{2} \int \cos \frac{x}{2} dx + \frac{1}{2} \int \cos \frac{3x}{2} dx \\
 &= \int \cos \frac{x}{2} d\left(\frac{x}{2}\right) + \frac{1}{3} \int \cos \frac{3x}{2} d\left(\frac{3x}{2}\right) \\
 &= \sin \frac{x}{2} + \frac{1}{3} \sin \frac{3x}{2} + C \\
 (14) & \int \sec^4 x dx; \\
 &= \int \sec^2 x \sec^2 x dx \\
 &= \int (1 + \tan^2 x) d \tan x \\
 &= \tan x + \frac{1}{3} \tan^3 x + C
 \end{aligned}$$

高等数学作业集

班号	
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(1) $\int \frac{1}{1+\sin x} dx$	
$\frac{1}{2} \int \frac{1}{1-\cos x} dx$	
$= \int \frac{2\sin x}{1-\cos^2 x} dx = - \int \frac{2\cos x}{1-\cos^2 x} dx$	
$= - \int \frac{1}{1-(\cos^2 x)^2} d(\cos^2 x)$	
$= -\arcsin(\cos^2 x) + C$	
(16) $\int \frac{\sin 2x}{\sqrt{1-\cos^4 x}} dx$	
$= \int \frac{2\sin x \cos x}{\sqrt{1-\cos^4 x}} dx = - \int \frac{2\cos x}{\sqrt{1-\cos^4 x}} dx$	
$= - \int \frac{1}{\sqrt{1-(\cos^2 x)^2}} d(\cos^2 x)$	
$= -\arcsin(\cos^2 x) + C$	
(17) $\int \frac{1}{1+\sin x} dx$	
$= \int \frac{1-\sin x}{1-\sin x} dx = \int \frac{1-\sin x}{\cos^2 x} dx$	
$= \int \frac{\cos x}{\cos^2 x} - \int \frac{\sin x}{\cos^2 x} dx$	
$= \int \frac{\cos x}{\cos^2 x} + \int \frac{1}{\cos^2 x} d(\cos x)$	
$= \tan x - \frac{1}{\cos x} + C$	
(18) $\int \frac{x-1}{x^2+2x+3} dx$	
$= \int \frac{x+1-2}{x^2+2x+3} dx$	
$= \int \frac{x+1}{x^2+2x+3} dx - 2 \int \frac{dx}{x^2+2x+3}$	
$= \frac{1}{2} \int \frac{1}{x^2+2x+3} d(x^2+2x+3) - 2 \int \frac{dx}{(x+1)^2+1^2}$	
$= \frac{1}{2} \ln(x^2+2x+3) - 2 \cdot \frac{1}{\sqrt{2}} \arctan \frac{x+1}{\sqrt{2}} + C$	
$= \frac{1}{2} \ln(x^2+2x+3) - \sqrt{2} \arctan \frac{x+1}{\sqrt{2}} + C$	





班号	姓名	学号

(6) $\int \arcsin \frac{x}{\sqrt{1+x^2}} dx$

$$= 2 \int \arcsin x d\sqrt{1+x^2}$$

$$= 2 [\sqrt{1+x^2} \arcsin x - \int \sqrt{1+x^2} d(\arcsin x)]$$

$$= 2 \sqrt{1+x^2} \arcsin x - 2 \int \sqrt{1+x^2} \frac{1}{\sqrt{1-x^2}} dx$$

$$= 2 \sqrt{1+x^2} \arcsin x - 2 \int \frac{1}{\sqrt{1-x^2}} dx$$

$$= 2 \sqrt{1+x^2} \arcsin x + 2 \int \frac{1}{\sqrt{1-x^2}} d(1-x)$$

$$\Rightarrow 2 \sqrt{1+x^2} \arcsin x + 2 \int \frac{1}{\sqrt{1-(1-x)^2}} dx + C$$

(7) $\int \frac{x}{\sin^2 x} dx$

$$= - \int x \cot x dx$$

$$= -x \cot x + \int \cot x dx$$

$$= -x \cot x + \ln |\sin x| + C$$

(8) $\int e^{-2x} \sin \frac{x}{2} dx$

$$= -\frac{1}{2} \int e^{-2x} \sin \frac{x}{2} d(e^{-2x})$$

$$= -\frac{1}{2} [e^{-2x} \sin \frac{x}{2} - \int e^{-2x} \cdot \cos \frac{x}{2} \cdot -\frac{1}{2} dx]$$

$$= -\frac{1}{2} e^{-2x} \sin \frac{x}{2} - \frac{1}{8} \int e^{-2x} \cos \frac{x}{2} d(e^{-2x})$$

$$= -\frac{1}{2} e^{-2x} \sin \frac{x}{2} - \frac{1}{8} [e^{-2x} \cos \frac{x}{2} - \int e^{-2x} \cdot (-\sin \frac{x}{2}) \cdot \frac{1}{2} dx]$$

$$= -\frac{1}{8} (4 \sin \frac{x}{2} + \cos \frac{x}{2}) e^{-2x} + C$$

解得
 $\int e^{-2x} \sin \frac{x}{2} dx = -\frac{2}{7} (4 \sin \frac{x}{2} + \cos \frac{x}{2}) e^{-2x} + C$

(11) $\int \cos \ln x dx$

$$= x \cos \ln x - \int x \cdot (-\sin \ln x) \cdot \frac{1}{x} dx$$

$$= x \cos \ln x + \int \sin \ln x dx$$

$$= x \cos \ln x + x \sin \ln x - \int x \cos \ln x \cdot \frac{1}{x} dx$$

$$= x (\cos \ln x + \sin \ln x) - \int \cos \ln x dx$$

$$\int \cos \ln x dx = \frac{x}{2} (\cos \ln x + \sin \ln x) + C$$

学号

(9) $\int x^2 \cos^2 \frac{x}{2} dx$

$$= \int x^2 \cdot \frac{1+\cos x}{2} dx = \frac{1}{2} \int x^2 dx + \frac{1}{2} \int x^2 \cos x dx$$

$$= \frac{x^3}{6} + \frac{1}{2} \int x^2 d(\sin x)$$

$$= \frac{x^3}{6} + \frac{x^2}{2} \sin x - \int \sin x \cdot 2x dx$$

$$= \frac{x^3}{6} + \frac{x^2}{2} \sin x + x \cos x - \int \cos x dx$$

$$= \frac{x^3}{6} + \frac{x^2}{2} \sin x + x \cos x - \sin x + C$$

(10) $\int e^{3x} dx$

$$\text{令 } t = 3x \rightarrow dt = 3dx, dx = \frac{1}{3} dt, \sqrt{3} dx = dt$$

$$\int e^{3x} dx = \int e^{3t} \cdot 3t^2 dt = \int 3t^2 dt$$

$$= 3t^2 t - \int t^2 \cdot 6dt = 3t^2 t - \int 6t dt$$

$$= 3t^2 t - [6t^2 - \int t^2 \cdot 6dt]$$

$$= 3t^2 t - 6t^2 + 6t + C$$

$$= 3e^{3x} (x^3 - 2x^2 + 2) + C$$

(11) $\int \cos \ln x dx$

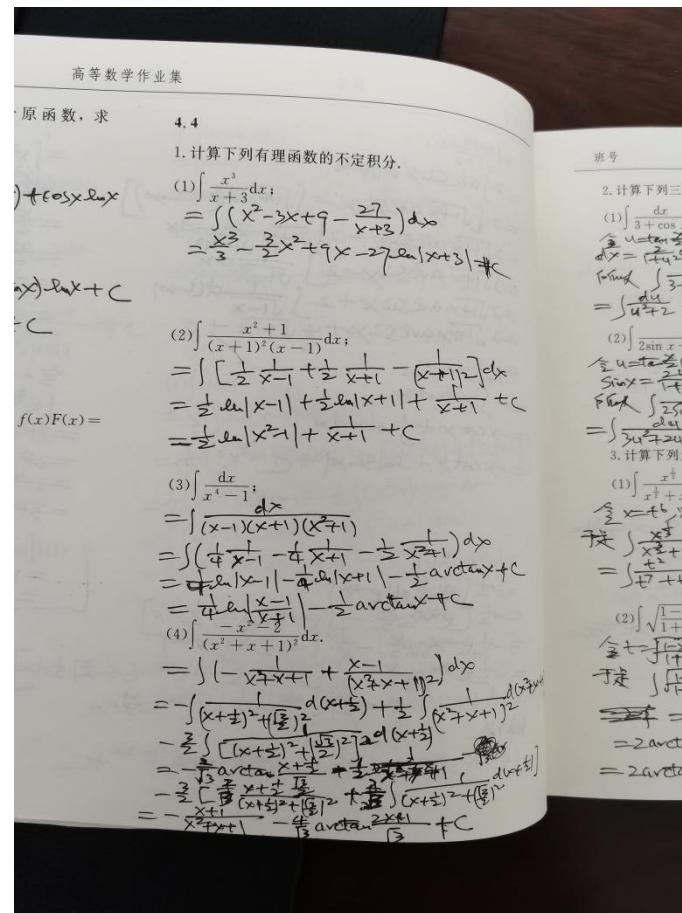
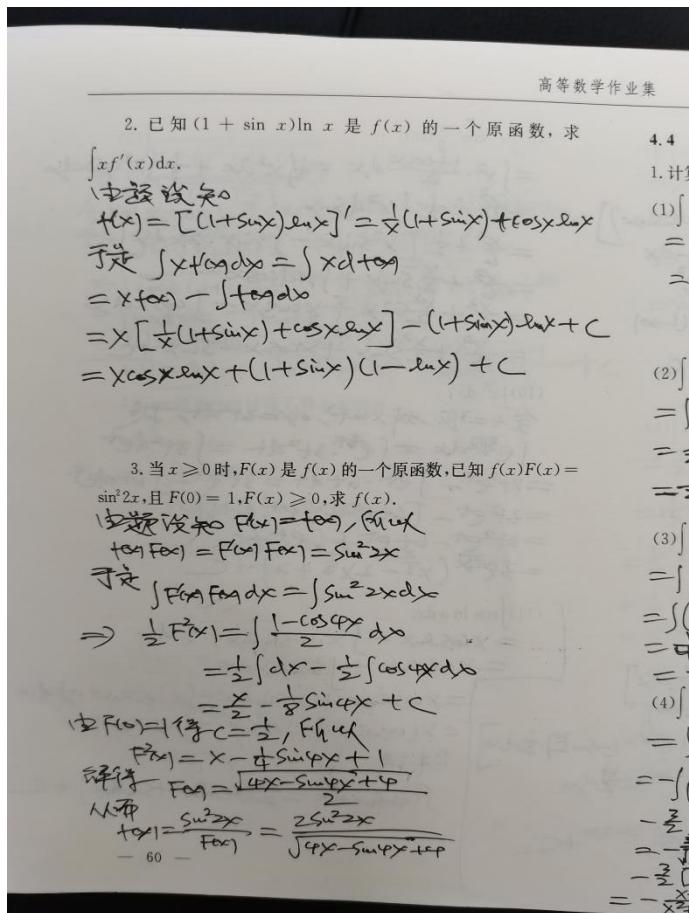
$$= x \cos \ln x - \int x \cdot (-\sin \ln x) \cdot \frac{1}{x} dx$$

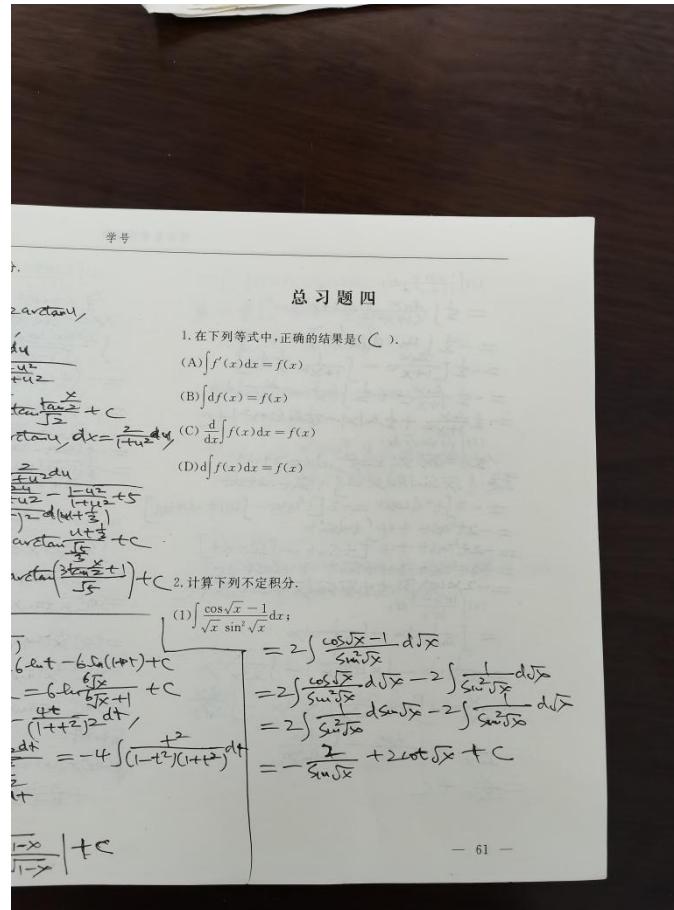
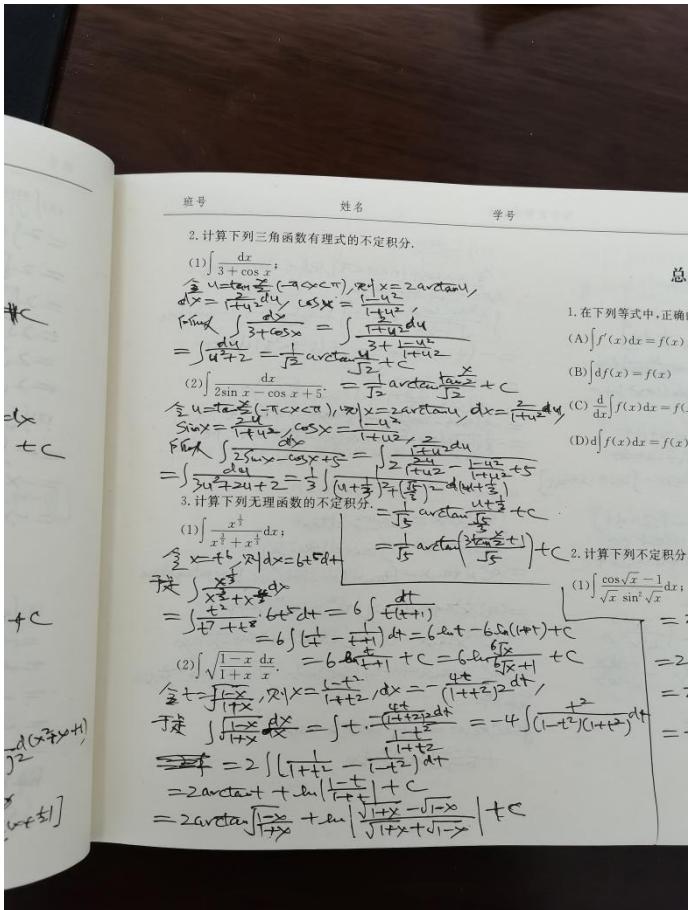
$$= x \cos \ln x + \int \sin \ln x dx$$

$$= x \cos \ln x + x \sin \ln x - \int x \cos \ln x \cdot \frac{1}{x} dx$$

$$= x (\cos \ln x + \sin \ln x) - \int \cos \ln x dx$$

$$\int \cos \ln x dx = \frac{x}{2} (\cos \ln x + \sin \ln x) + C$$





高等数

$$\begin{aligned}
 (2) & \int \frac{x \ln x}{(1+x^2)^2} dx \\
 &= \frac{1}{2} \int \frac{\ln x}{(1+x^2)^2} d(1+x^2) \\
 &= -\frac{1}{2} \int \ln x d\left(\frac{1}{1+x^2}\right) \\
 &= -\frac{1}{2} \left[\frac{\ln x}{1+x^2} - \int \frac{1}{1+x^2} \cdot \frac{1}{x} dx \right] \\
 &= -\frac{1}{2} \frac{\ln x}{1+x^2} + \frac{1}{2} \int \left(\frac{1}{x} - \frac{1}{1+x^2} \right) dx \\
 &= -\frac{1}{2} \frac{\ln x}{1+x^2} + \frac{1}{2} \ln|x| - \frac{1}{4} \ln(1+x^2) + C \\
 (3) & \int \sqrt{x} \sin \sqrt{x} dx \\
 & \quad \begin{aligned}
 t &= \sqrt{x}, \quad x = t^2, \quad dx = 2t dt, \\
 \int \sqrt{x} \sin \sqrt{x} dx &= \int t \sin t \cdot 2t dt \\
 &= -2 \int t^2 d \cos t = -2 \left[t^2 \cos t - \int \cos t \cdot 2t dt \right] \\
 &= -2t^2 \cos t + 4 \int t \sin t dt \\
 &= -2t^2 \cos t + 4 \left[t \sin t - \int \sin t dt \right] \\
 &= -2t^2 \cos t + 4t \sin t + 4 \cos t + C \\
 (4) & \int \frac{\ln x}{\ln^2 x} dx \\
 &= \int \frac{1}{\ln x} dx - \int \frac{1}{\ln^2 x} dx \\
 &= \frac{1}{\ln x} \cdot x - \int x d\left(\frac{1}{\ln x}\right) - \int \frac{1}{\ln^2 x} dx \\
 &= \frac{x}{\ln x} - \int x \left(-\frac{1}{x^2 \ln x}\right) \cdot \frac{1}{x} dx - \int \frac{1}{\ln^2 x} dx \\
 &= \frac{x}{\ln x} + \int \frac{dx}{x^2 \ln x} - \int \frac{dx}{\ln^2 x} \\
 &= \frac{x}{\ln x} + C
 \end{aligned}
 \end{aligned}$$

高等数学作业集

班号

$$\begin{aligned}
 (5) & \int \frac{x^3 \arccos x}{\sqrt{1-x^2}} dx. \\
 & \begin{aligned}
 x &= \cos t \quad (0 < t < \pi), \quad x \mapsto t = \arccos x, \\
 dx &= -\sin t dt, \quad \frac{d}{dt} x = -\sin t, \\
 \int x^3 \arccos x dx &= \int \frac{\cos^3 t \cdot t}{\sqrt{1-\cos^2 t}} \cdot (-\sin t) dt \\
 &= -\int t \cos^3 t dt = -\int (1-\sin^2 t) \sin t dt \\
 &= -\int t d\left(\sin t - \frac{\sin^3 t}{3}\right) \\
 &= -t \left(\sin t - \frac{\sin^3 t}{3}\right) + \int \left(\sin t - \frac{\sin^3 t}{3}\right) dt \\
 &= -t \left(\sin t - \frac{\sin^3 t}{3}\right) - \frac{1}{3} \cos t - \frac{1}{9} \sin 3t + C \\
 &= -\frac{1}{3} \sqrt{1-x^2} (2-x^2) \arccos x - \frac{1}{9} x (x^2+6) + C
 \end{aligned}
 \end{aligned}$$

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$$\begin{aligned}
 & \text{设 } n \text{ 为正整数, 证明: 递推公式 } \int \sec^n x dx = \frac{1}{n-1} \tan x \sec^{n-2} x + \frac{n-2}{n-1} \int \sec^{n-2} x dx. \\
 & \begin{aligned}
 \int \sec^n x dx &= \int \sec^{n-2} x d(\tan x) \\
 &= \sec^{n-2} x \tan x - \int \tan x d(\sec^{n-2} x) \\
 &= \sec^{n-2} x \tan x - \int \tan x (n-2) \sec^{n-3} x \cdot (\sec x \tan x) dx \\
 &= \sec^{n-2} x \tan x - (n-2) \int \tan^2 x \sec^{n-3} x dx \\
 &= \sec^{n-2} x \tan x - (n-2) \int (\sec^2 x - 1) \sec^{n-3} x dx \\
 &= \sec^{n-2} x \tan x - (n-2) \int \sec^n x dx + (n-2) \int \sec^{n-4} x dx
 \end{aligned}
 \end{aligned}$$

解得
 $\int \sec^n x dx = \frac{1}{n-1} \sec^{n-2} x \tan x + \frac{n-2}{n-1} \int \sec^{n-2} x dx$