Calculus IA Exercises - 不定积分

硝基苯

1

$$\int \frac{dx}{1+\sqrt{x^2+2x+2}}$$

原式 $= \displaystyle\int \frac{dx}{1+\sqrt{(x+1)^2+1}}$

令 $x+1=\tan t \quad (-\pi/2<t<\pi/2)$

则 $dx=\sec^2 t\ dt$

且 $\sqrt{(x+1)^2+1}=\sqrt{\tan^2 t+1}=|\sec t|=\sec t$

上式 $= \displaystyle\int \frac{\sec^2 tdt}{1+\sec t}$

$$= \int \frac{dt}{\cos t(1+\cos t)}$$

> 分母积化和差

$$= \int \left(\frac{1}{\cos t}-\frac{1}{1+\cos t}\right) dt$$

> $1+\cos 2x = 2\cos^2 x$

$$= \int \sec tdt - \int \frac{1}{\cos^2 \frac{t}{2}} d\frac{t}{2}$$

$$= \ln|\sec t+\tan t| - \tan\frac{t}{2} + C$$

$\because \tan t = x+1, \sec t = \sqrt{x^2+2x+2}$

$$\tan\frac{t}{2} = \frac{1-\cos t}{\sin t} = \frac{\sec t - 1}{\tan t} = \frac{\sqrt{x^2+2x+2}-1}{x+1}$$

$$\therefore \int \frac{dx}{1+\sqrt{x^2+2x+2}} = \ln\left|x+1+\sqrt{x^2+2x+2}\right| - \frac{\sqrt{x^2+2x+2}-1}{x+1} + C$$

2

$$\int \sec^3 x dx$$

> 分部积分，解方程

$$\begin{aligned}
\int \sec^3 x dx &\triangleq I \\
&= \int \sec x d(\tan x) \\
&= \tan x \sec x - \int \tan x d(\sec x) \\
&= \tan x \sec x - \int \tan^2 x \sec x dx \\
&= \tan x \sec x - \int (\sec^2 x - 1) \sec x dx \\
&= \tan x \sec x - \int \sec^3 x dx + \int \sec x dx \\
&= \tan x \sec x - I + \ln|\tan x + \sec x|
\end{aligned}$$

$$\therefore \int \sec^3 x dx = \frac{1}{2}\tan x \sec x + \frac{1}{2}\ln|\tan x + \sec x| + C$$