高等数学常用公式

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导数基本公式

导数基本公式
$C' = 0$（C 为常数）
$(x^\mu)' = \mu x^{\mu-1}$
$(a^x)' = a^x \ln a$（$a > 0$ 且 $a \neq 1$）
$(e^x)' = e^x$
$(\log_a x)' = \frac{1}{x \ln a}$（$a > 0$ 且 $a \neq 1$）
$(\ln x)' = \frac{1}{x}$
$(\sin x)' = \cos x$
$(\cos x)' = -\sin x$
$(\tan x)' = \sec^2 x$
$(\cot x)' = -\csc^2 x$
$(\sec x)' = \sec x \tan x$
$(\csc x)' = -\csc x \cot x$
$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$
$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$
$(\arctan x)' = \frac{1}{1+x^2}$
$(\text{arccot}\, x)' = -\frac{1}{1+x^2}$

高阶导数

常用 n 阶导数公式
$(x^\mu)^{(n)}=\mu(\mu-1)\cdots(\mu-n+1)x^{\mu-n}$
$(e^x)^{(n)}=e^x$
$(\frac{1}{x})^{(n)}=\frac{(-1)^n\cdot n!}{x^{n+1}}$
$(\ln x)^{(n)}=\frac{(-1)^{n-1}(n-1)!}{x^n}$
$(\sin x)^{(n)}=\sin(x+\frac{n\pi}{2})$
$(\cos x)^{(n)}=\cos(x+\frac{n\pi}{2})$
$[f(ax+b)]^{(n)}=a^n f^{(n)}(ax+b)$

麦克劳林公式

常用麦克劳林公式
$e^x=1+x+\frac{x^2}{2!}+\cdots+\frac{x^n}{n!}+\frac{e^{\theta x}}{(n+1)!}x^{n+1}$
$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots+(-1)^n\frac{x^{2n+1}}{(2n+1)!}+(-1)^{n+1}\frac{\cos\theta x}{(2n+3)!}x^{2n+3}$
$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots+(-1)^n\frac{x^{2n}}{(2n)!}+(-1)^{n+1}\frac{\cos\theta x}{(2n+2)!}x^{2n+2}$
$\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\cdots+(-1)^{n-1}\frac{x^n}{n}+(-1)^n\frac{x^{n+1}}{(n+1)(1+\theta x)^{n+1}}$
$(1+x)^\mu=1+\mu x+\frac{\mu(\mu-1)}{2}x^2+\cdots+\frac{\mu(\mu-1)\cdots(\mu-n+1)}{n!}x^n+\frac{\mu(\mu-1)\cdots(\mu-n)}{(n+1)!}(1+\theta x)^{\mu-n-1}x^{n+1}$
$\frac{1}{1-x}=1+x+x^2+\cdots+x^n+\frac{x^{n+1}}{(1-\xi)^{n+1}}$

斜渐近线 $y=ax+b$

$$a=\lim_{x\to\infty}\frac{f(x)}{x}=a\neq 0$$
$$b=\lim_{x\to\infty}(f(x)-ax)$$

弧微分

$$ds=\sqrt{1+(y')^2}dx$$
$$ds=\sqrt{(\varphi'(t))^2+(\psi'(t))^2}dt$$
$$ds=\sqrt{(r(\theta))^2+(r'(\theta))^2}d\theta$$

曲率

$$K=\left|\frac{d\alpha}{ds}\right|=\frac{|y''|}{[1+(y')^2]^{\frac{3}{2}}}$$
$$K=\frac{|\varphi'(t)\psi''(t)-\psi'(t)\varphi''(t)|}{[(\varphi'(t))^2+(\psi'(t))^2]^{\frac{3}{2}}}$$
$$K=\frac{\left|(r(\theta))^2+2(r'(\theta))^2-r(\theta)r''(\theta)\right|}{[(r(\theta))^2+(r'(\theta))^2]^{\frac{3}{2}}}$$

曲率半径

$$R=\frac{1}{K}$$

曲率中心

$$\begin{cases}\xi=x-\frac{y'\left[1+(y')^2\right]}{y''}\\ \eta=y+\frac{1+(y')^2}{y''}\end{cases}$$

一阶线性微分方程 $\frac{dy}{dx}+P(x)y=Q(x)$ 的通解为:

$$y=e^{-\int P(x)dx}\left(\int Q(x)e^{\int P(x)dx}dx+C\right)$$

不定积分基本公式

$$\int 0dx = C$$

$$\int 1dx = x + C$$

$$\int x^\mu dx = \frac{x^{\mu+1}}{\mu+1} + C \ \ (\mu \neq -1)$$

$$\int \frac{dx}{x} = \ln|x| + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C \ \ (a > 0, a \neq 1)$$

$$\int e^x dx = e^x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \csc^2 x = -\cot x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \csc x \cot x dx = -\csc x + C$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C = \arccos x + C$$

$$\int \frac{dx}{1+x^2} = \arctan x + C = -\text{arccot}\, x + C$$

$$\int \tan x dx = -\ln|\cos x| + C$$

$$\int \cot x = \ln|\sin x| + C$$

$$\int \sec x dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x dx = \ln|\csc x - \cot x| + C$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\arctan \frac{x}{a} + C \ \ (a > 0)$$

$$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C \ \ (a > 0)$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C \ \ (a > 0)$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C \ \ (a > 0)$$