Calculus - 杂项

硝基苯

反三角函数

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

$$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$$

$$(\arctan x)' = \frac{1}{1+x^2}$$

$$(\mathrm{arccot} x)' = -\frac{1}{1+x^2}$$

反函数

$x = f(y)$ 单调，可导，且 $f'(y) \neq 0$

$[f^{-1}(x)]' = \dfrac{1}{f'(y)}$

高阶导数

$$(e^x)^{(n)} = e^x$$

$$(\sin x)^{(n)} = \sin(x + n \cdot \frac{\pi}{2})$$

$$(\cos x)^{(n)} = \cos(x + n \cdot \frac{\pi}{2})$$

$$(\ln x)^{(n)} = (-1)^{n-1} \frac{(n-1)!}{x^n}$$

$$(x^\mu)^{(n)} = \mu(\mu-1)(\mu-2)\cdots(\mu-n+1)x^{\mu-n}$$

$$\frac{d^n}{dx^n}f(ax+b)=a^n\cdot\frac{d^n}{d(ax+b)^n}f(ax+b)$$

$$(u\pm v)^{(n)}=u^{(n)}\pm v^{(n)}$$

$$(Cu)^{(n)}=Cu^{(n)}$$

$$(uv)^{(n)}=\sum_{k=0}^{n}C_n^k u^{(n-k)}v^{(k)}$$

隐函数

方程两边同时对 x 求导，解出 $\frac{dy}{dx}$

对数求导法

$y=u^v$

$y'=u^v(vlnu)'=u^v(v'\ln u+v\frac{u'}{u})$

多因式相乘除（考虑取绝对值）

曲线积分

$$ds=\sqrt{1+y'^2}dx$$

$$K=\frac{|y''|}{(1+y'^2)^{3/2}}$$

不定积分公式

$$(\arcsin e^x)'=(e^{-2x}-1)^{-1/2}$$

$$\int\arctan xdx=x\arctan x-\frac{1}{2}\ln(1+x^2)$$

$$\int\frac{dx}{(x^2+a^2)^{n+1}}=\frac{1}{2na^2}\frac{x}{(x^2+a^2)^n}+\frac{2n-1}{2na^2}\int\frac{dx}{(x^2+a^2)^n}$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\arctan\frac{x}{a} + C$$

$$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln|x+\sqrt{x^2 \pm a^2}| + C$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$$

$$\int \sqrt{x^2 \pm a^2}dx = \frac{x}{2}\sqrt{x^2 \pm a^2} \pm \frac{a^2}{2}\ln\left|x+\sqrt{x^2 \pm a^2}\right| + C$$

$$\int \sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\arcsin\frac{x}{a} + C$$

定积分公式

$$\int_0^{\frac{\pi}{2}} \sin^n x\,dx = \int_0^{\frac{\pi}{2}} \cos^n x\,dx = \begin{cases} \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdots\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2}, & n \text{ is even} \\ \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdots\frac{4}{5}\cdot\frac{2}{3}\cdot 1, & n \text{ is odd} \end{cases}$$

万能公式

$$u = \tan\frac{x}{2}$$

$$\sin x = \frac{2u}{1+u^2}$$

$$\cos x = \frac{1-u^2}{1+u^2}$$

$$dx = \frac{2}{1+u^2}du$$