Teacher: **Yanjie Li** | Assignment Number: **3**
Course: **Linear Algebra in Control Theory** | Disclosure date: June 9, 2023

Problem 1

In $\mathbf{R}^4$, let

$$U = \operatorname{span}((1,1,0,0),(1,1,1,2)).$$

Find $u \in U$ such that $\|u-(1,2,3,4)\|$ is as small as possible.

Problem 2

Find $p \in \mathcal{P}_3(\mathbf{R})$ such that $p(0)=0, p'(0)=0$, and

$$\int_0^1 |2+3x-p(x)|^2 dx$$

is as small as possible.

Problem 3

Suppose $T \in \mathcal{L}(V)$ and U is a subspace of V.
(a) Prove that if $U \subset \operatorname{null} T$, then U is invariant under T.
(b) Prove that if $\operatorname{range} T \subset U$, then U is invariant under T.

Problem 4

Suppose $S, T \in \mathcal{L}(V)$ are such that $ST = TS$. Prove that $\operatorname{range} S$ is invariant under T.

Problem 5

Suppose $S, T \in \mathcal{L}(V)$ are such that $ST = TS$. Prove that null S is invariant under T.

Problem 6

Define $T \in \mathcal{L}\left(\mathbf{F}^3\right)$ by

$$T\left(z_1, z_2, z_3\right) = \left(2z_2, 0, 5z_3\right)$$

Find all eigenvalues and eigenvectors of T.

Problem 7

Define $T : \mathcal{P}(\mathbf{R}) \to \mathcal{P}(\mathbf{R})$ by $Tp = p'$. Find all eigenvalues and eigenvectors of T.

Problem 8

Suppose $T \in \mathcal{L}(V)$. Suppose $S \in \mathcal{L}(V)$ is invertible. (a) Prove that T and $S^{-1}TS$ have the same eigenvalues. (b) What is the relationship between the eigenvectors of T and the eigenvectors of $S^{-1}TS$?

Problem 9

Find all eigenvalues and eigenvectors of the backward shift operator $T \in \mathcal{L}\left(\mathbf{F}^\infty\right)$ defined by

$$T\left(z_1, z_2, z_3, \ldots\right) = \left(z_2, z_3, \ldots\right).$$

Problem 10

If A is a matrix with $m \times n$ dimension, please show that $A^T A$ and AA^T have the same nonzero eigenvalues.

Pay Attention

a) **Mark your class number, student number and name on the homework**.
b) Try to write your homework on **A4** size paper.
c) Please hand in your homework to your TA before class next Thursday (June 15). If you really cannot hand in your homework by the time mentioned above, please bring it to office A415 by yourself.

References

[1] Axler, S. (1997). Linear algebra done right. Springer Science & Business Media.

[2] Lay, D. C. . Linear algebra and its applications. Academic Press.

[3] Leon, S. J., de Pillis, L., & De Pillis, L. G. (2015). Linear algebra with applications (pp. 337-350). Boston: Pearson.